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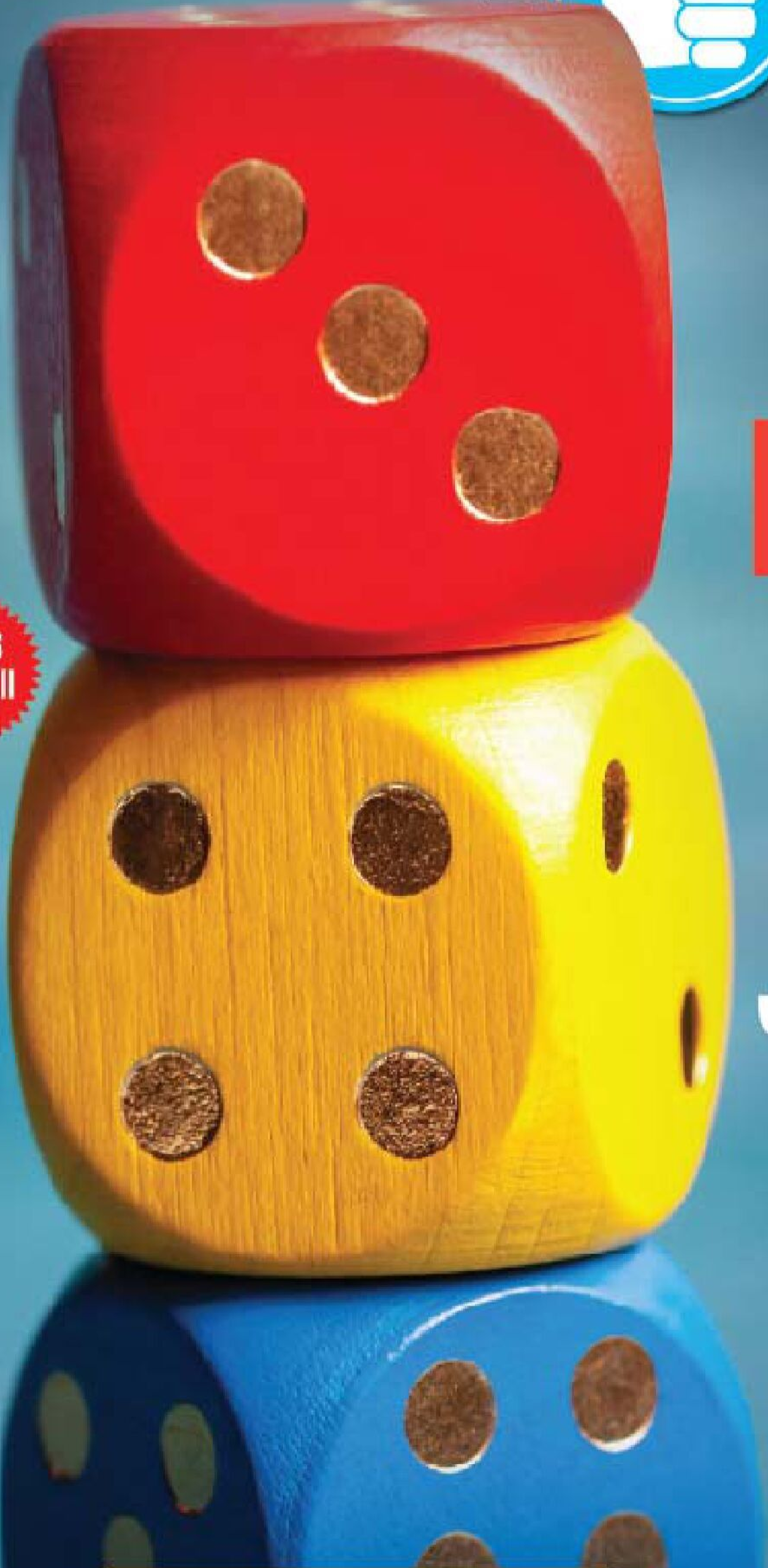

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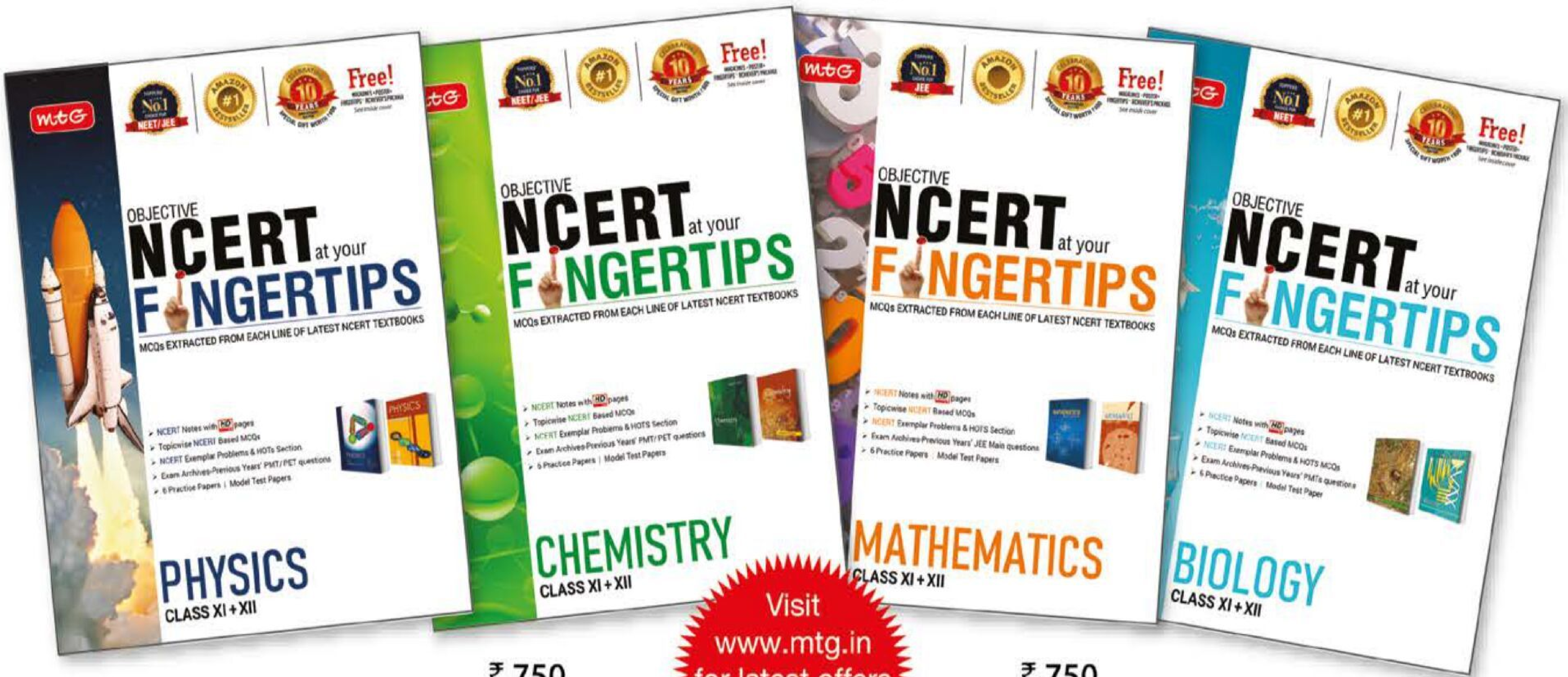

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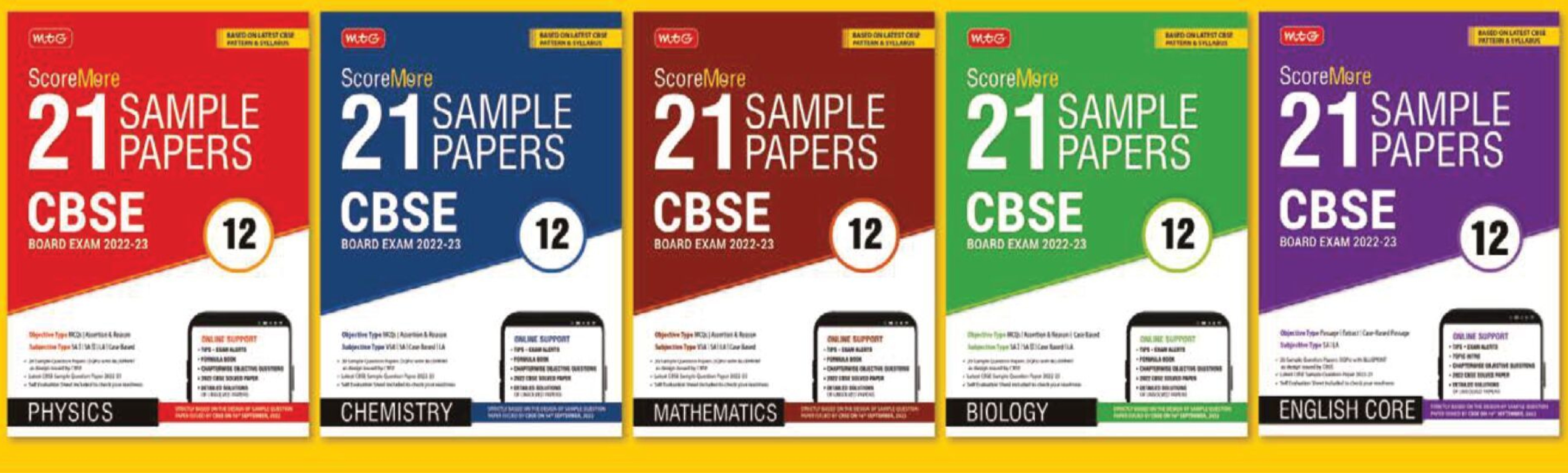
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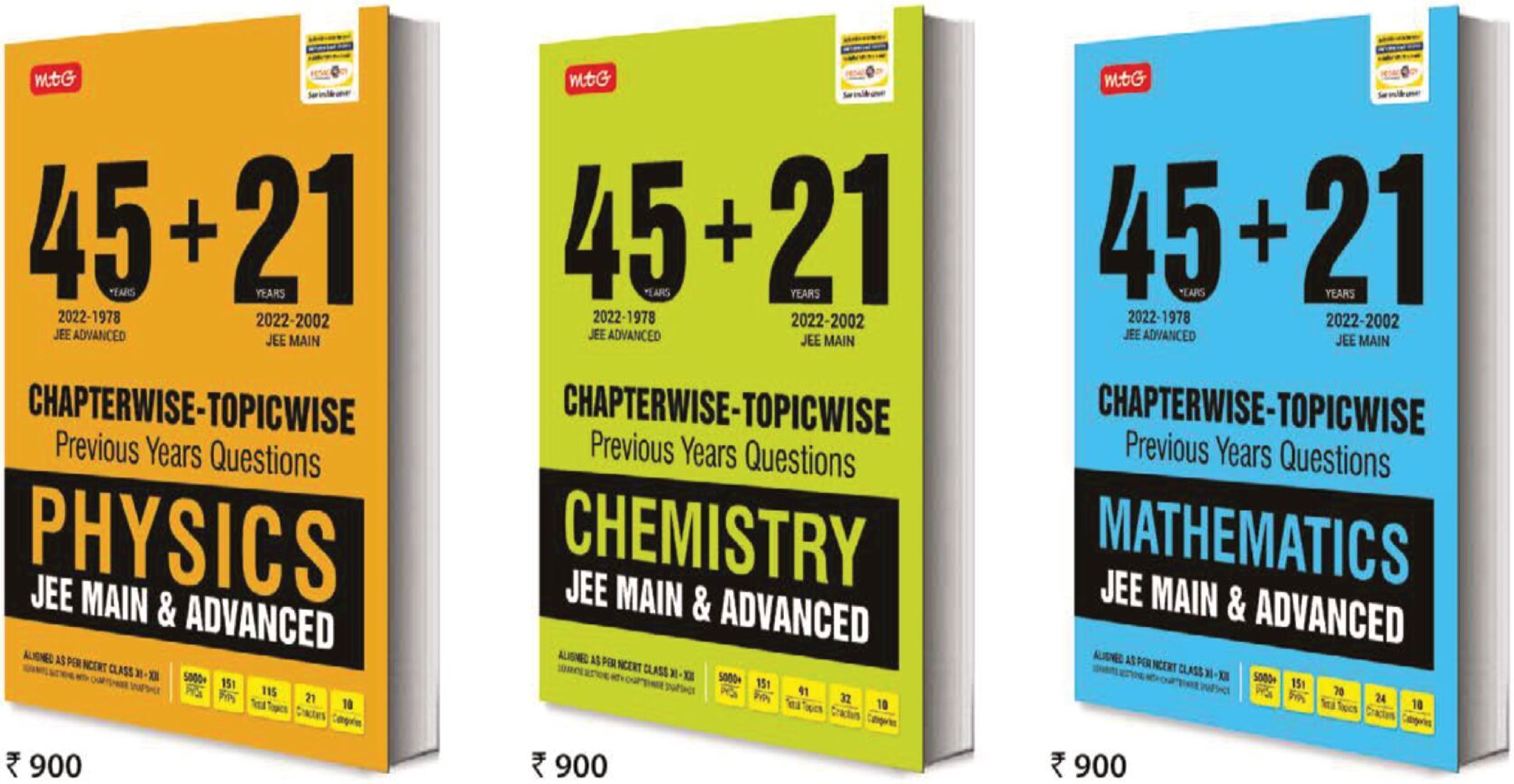
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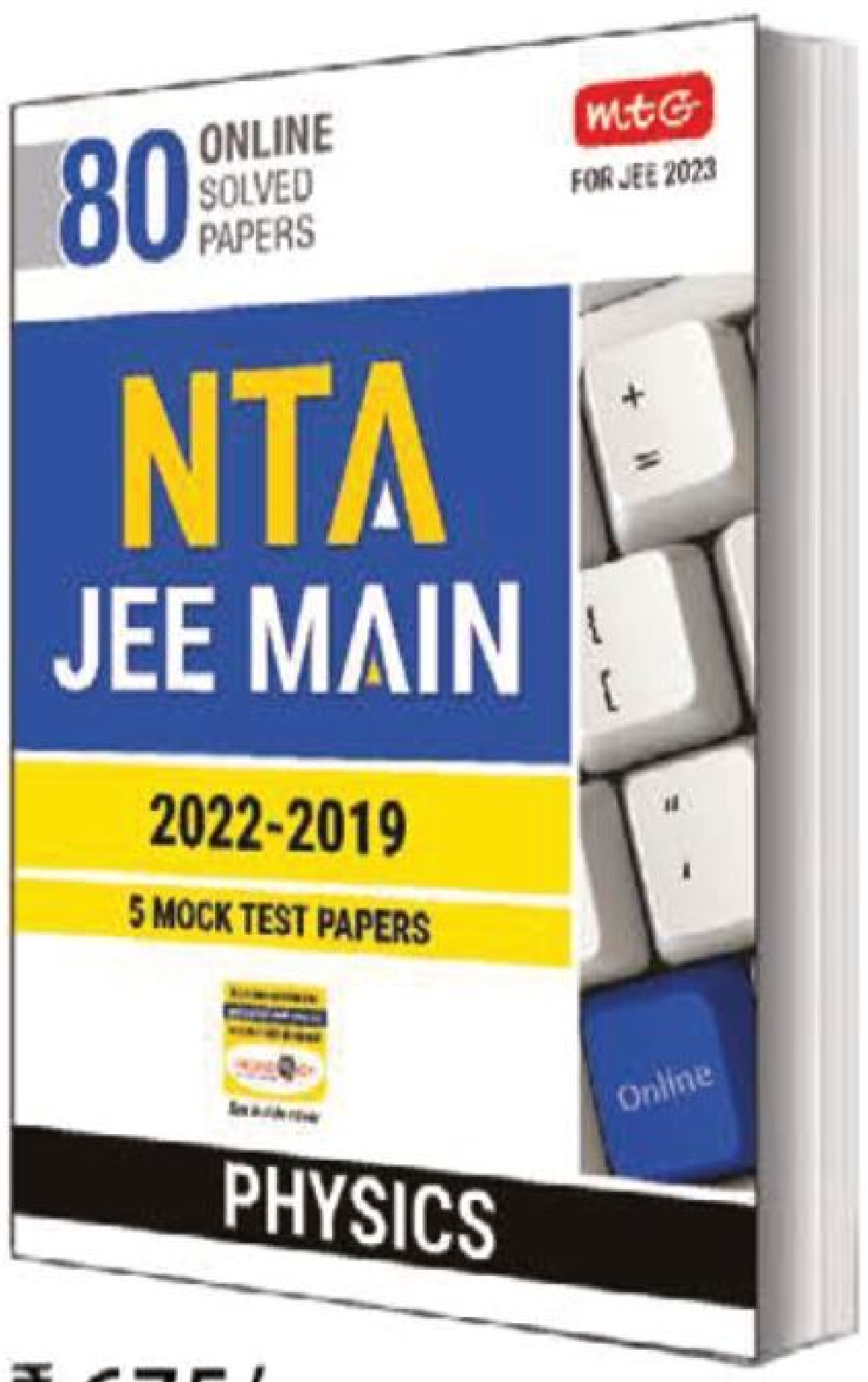
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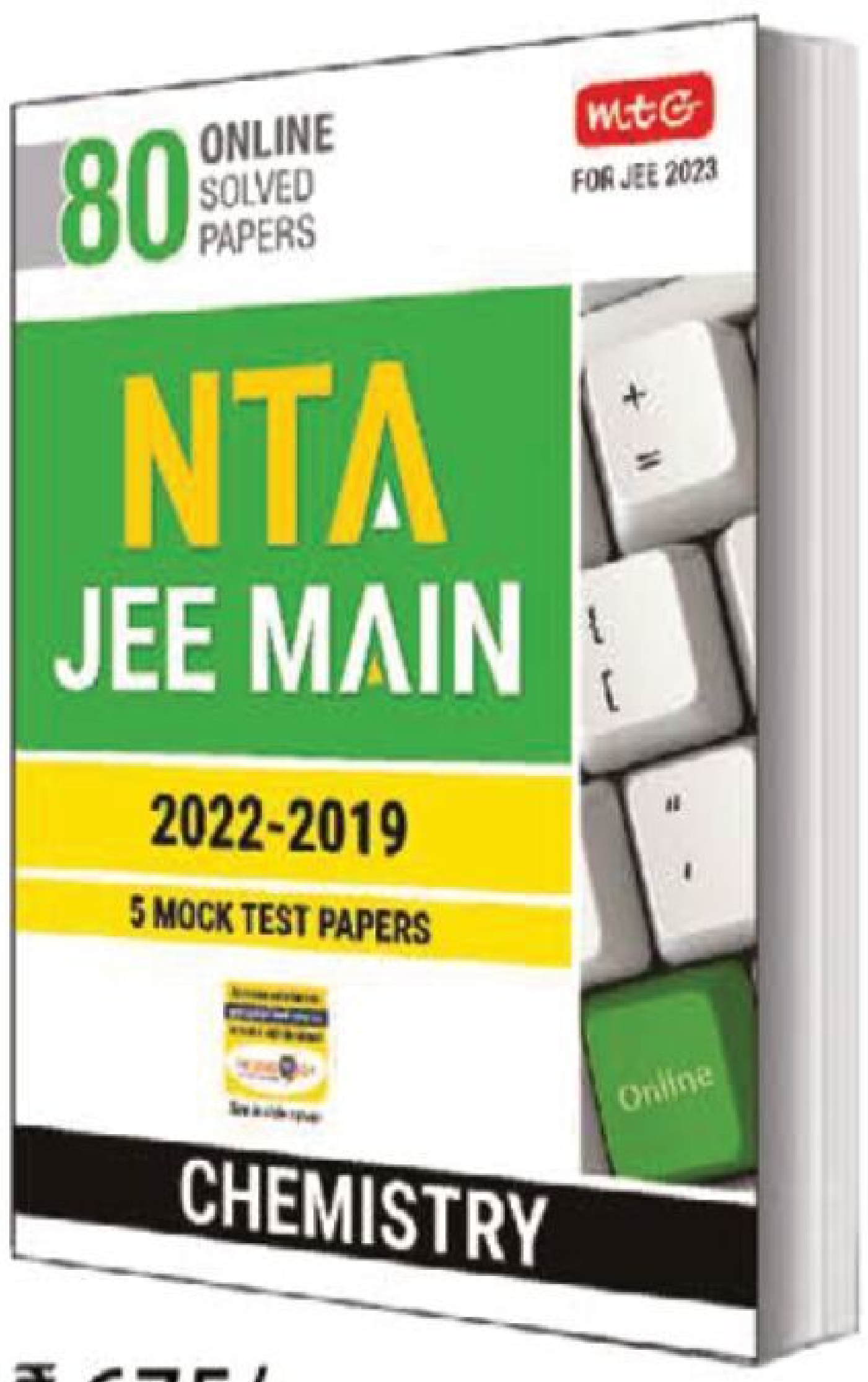
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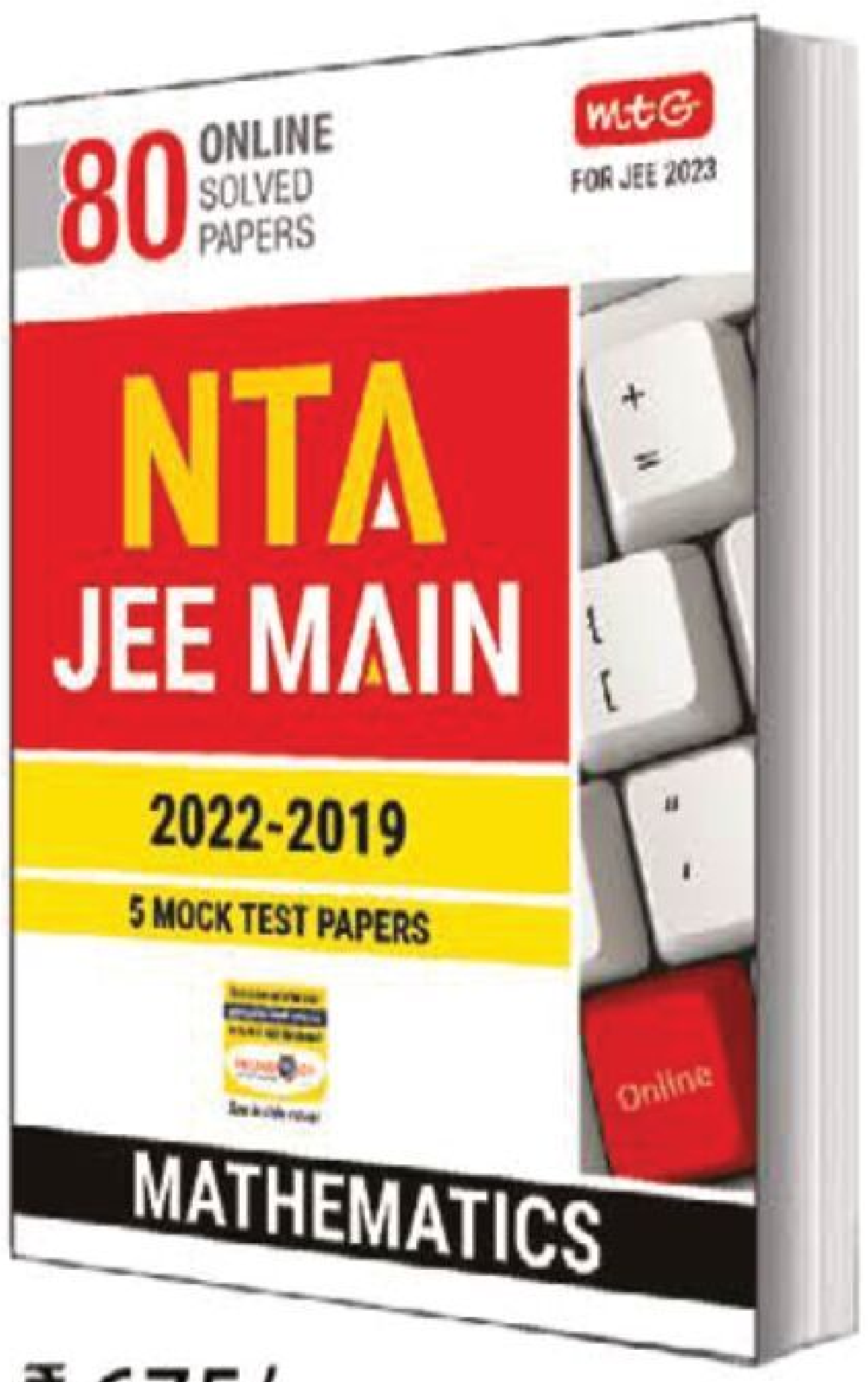
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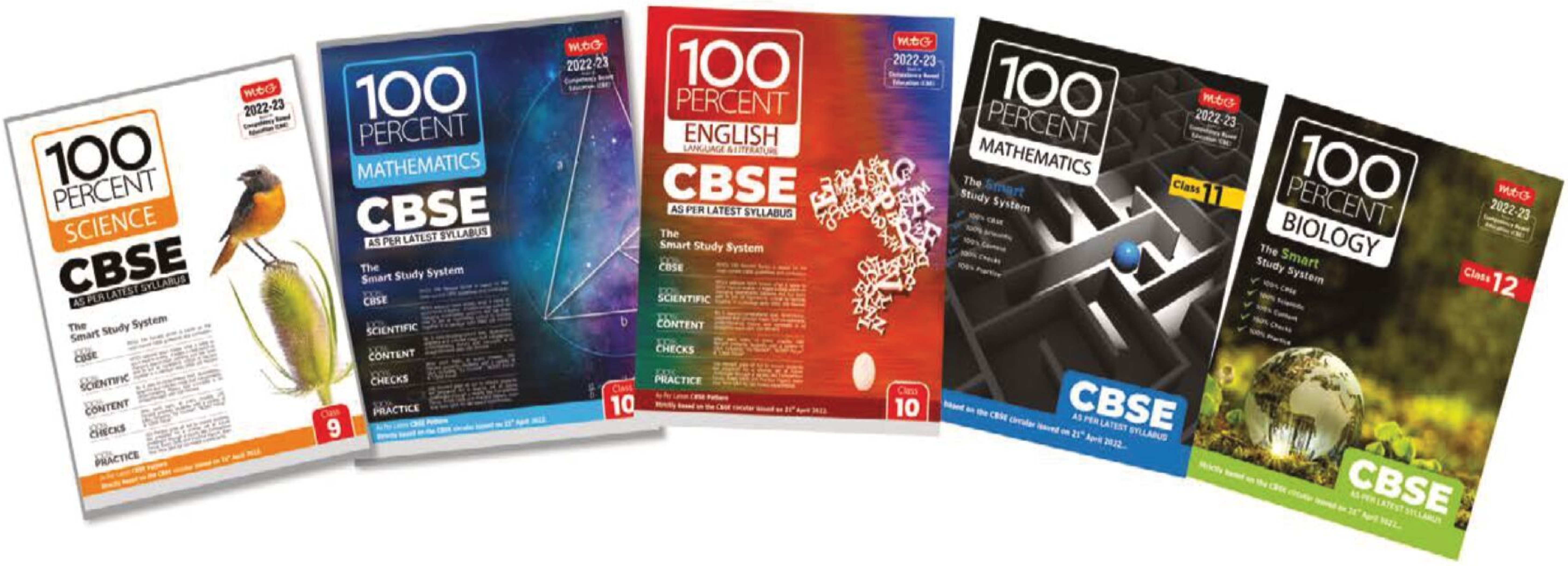

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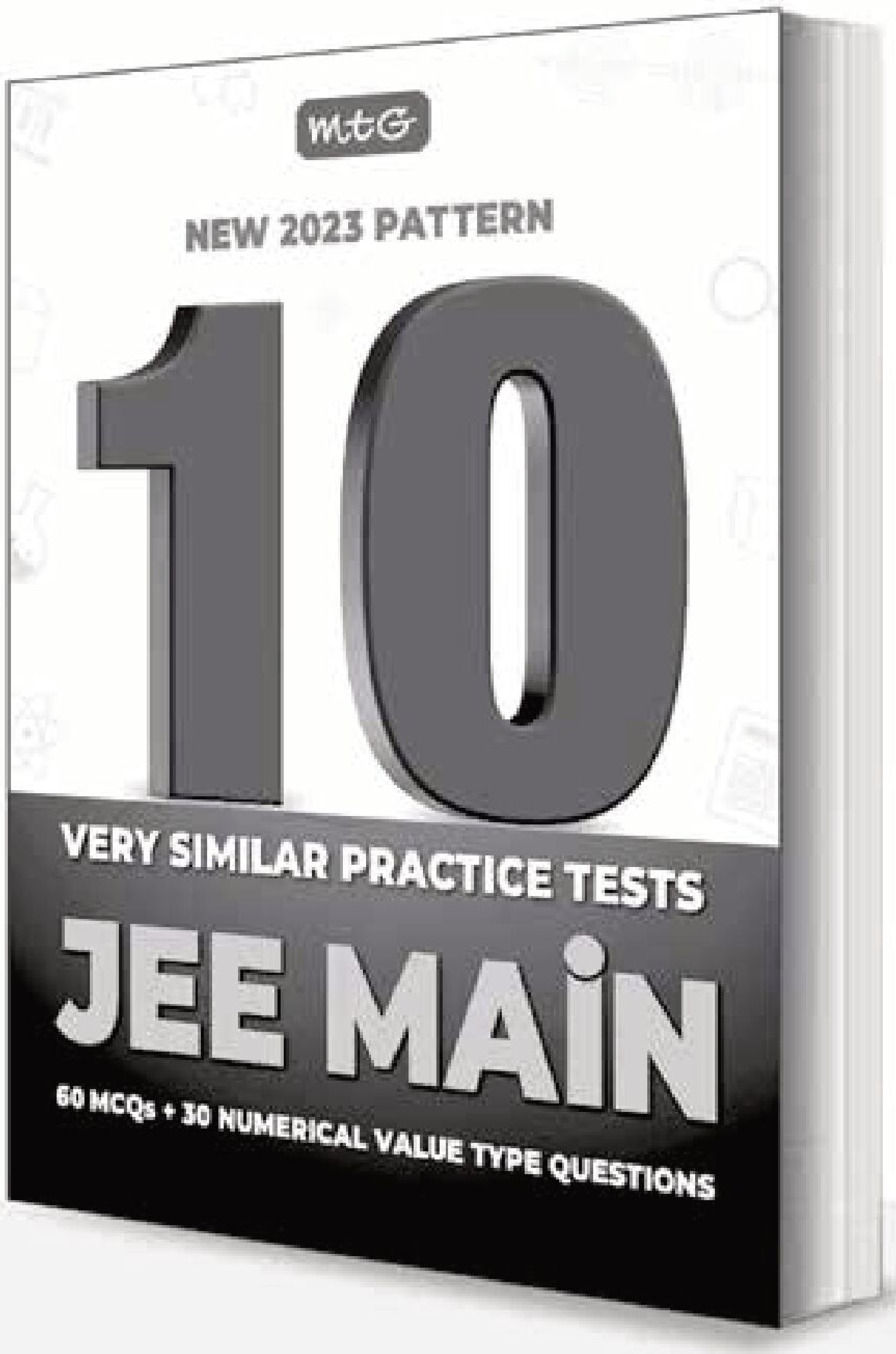
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Session-1
24, 25, 27, 28, 29, 30, 31 January 2023
Session-2
06, 08, 10, 11, 12 April 2023

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. If $\operatorname{Re}\left(\frac{z-8i}{z+6}\right)=0$, then z lies on the curve

(a) $x^2+y^2+6x-8y=0$
(b) $4x-3y+24=0$
(c) $x^2+y^2-8=0$
(d) none of these

2. If the ratio of the roots of $ax^2+2bx+c=0$ is same as the ratio of the $px^2+2qx+r=0$, then

(a) $\frac{2b}{ac}=\frac{q^2}{pr}$ (b) $\frac{b}{ac}=\frac{q}{pr}$
(c) $\frac{b^2}{ac}=\frac{q^2}{pr}$ (d) none of these

3. If the matrices, A, B, $(A+B)$ are non-singular, then $[A(A+B)^{-1}B]^{-1}$ is

(a) $A^{-1}B^{-1}$ (b) $B^{-1}+A^{-1}$
(c) $B^{-1}A^{-1}$ (d) none of these

4. Total number of 6-digit numbers that can be formed, having the property that every succeeding digit is greater than the preceding digit, is equal to

(a) ${}^9C_3$ (b) ${}^{10}C_3$ (c) ${}^9P_3$ (d) ${}^{10}P_3$

5. Let $g(x)$ be a polynomial of degree one and $f(x)$ be defined by $f(x)=\begin{cases} g(x), & x\le 0 \\ |x|^{\sin x}, & x>0 \end{cases}$. If $f(x)$ is continuous satisfying $f'(1)=f(-1)$, then $g(x)$ is

(a) $(1+\sin 1)x+1$ (b) $(1-\sin 1)x+1$
(c) $(1-\sin 1)x-1$ (d) $(1+\sin 1)x-1$

6. A relation ρ on the set of real numbers R is defined as follows: " $x\rho y$ if and only if $xy>0$. Then which of the following is true?

(a) ρ is reflexive and symmetric
(b) ρ is symmetric and transitive
(c) ρ is transitive only
(d) ρ is an equivalence relation.

7. The range of the function

$$y=f(x)=\log_2\frac{\sin x-\cos x+3\sqrt{2}}{\sqrt{2}}\text{ is}$$

(a) $0<y\le 3$ (b) $0<y<3$
(c) $1\le y\le 2$ (d) $1<y<2$

8. Let $f(x)=\log|x|$ and $g(x)=\sin x$. If A is the range of $f(g(x))$ and B is the range of $g(f(x))$, then

(a) $A\cup B=(-\infty,1]$ (b) $A\cup B=(-\infty,\infty)$
(c) $A\cap B=[-1,1]$ (d) $A\cap B=[0,1]$

9. The function

$$f(x)=\frac{4x^3-3x^2}{6}-2\sin x+(2x-1)\cos x$$

(a) increases in $\left[\frac{1}{2},\infty\right)$ (b) decreases in $\left(-\infty,\frac{1}{2}\right]$
(c) decreases in $\left[\frac{1}{2},\infty\right)$ (d) increases in $\left(-\infty,\frac{1}{2}\right]$

10. The statement among the following that is a tautology is

(a) $B\to[A\wedge(A\to B)]$ (b) $[A\wedge(A\to B)]\to B$
(c) $A\vee(A\wedge B)$ (d) $A\wedge(A\vee B)$

11. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is

(a) $\frac{5}{16}$ (b) $\frac{3}{16}$ (c) $\frac{1}{2}$ (d) $\frac{1}{32}$

12. Let B_i $(i=1,2,3)$ be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha-2\beta)\,p$

= αβ and (β – 3γ) $p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1)). Then $\dfrac{P(B_1)}{P(B_3)}$ is equal to

(a) 6 (b) 4 (c) 2 (d) 1

13. A variable straight line passes through the intersection of the lines $x + 2y = 1$ and $2x - y = 1$ meet the co-ordinate axes A and B, then the locus of middle point of AB is

(a) $x - 3y = 10xy$ (b) $x + 3y = 10xy$
(c) $3x + y = 10xy$ (d) none of these

14. $f(x) = x^2 - 4|x|$ and $g(x)$
$$= \begin{cases} \min\{f(t) : -6 \le t \le x\}, x \in [-6, 0] \\ \max\{f(t) : 0 < t \le x\}, x \in (0, 6] \end{cases}$$
, then $g(x)$ has

(a) exactly one point of local minima
(b) exactly one point of local maxima
(c) no point of local maxima but exactly one point of local minima
(d) neither a point of local maxima nor minima

15. If $f(x) = \dfrac{9^x}{9^x + 9}$, then $f\left(\dfrac{1}{2019}\right) + f\left(\dfrac{2}{2019}\right) + f\left(\dfrac{3}{2019}\right) + \ldots + \left(\dfrac{4037}{2019}\right) =$

(a) 1009 (b) $\dfrac{4037}{2}$ (c) 2018 (d) 2019

16. In a quadrilateral $ABCD$, let
$$\Delta = \begin{vmatrix} \cos A & \sin A & \cos(A+D) \\ \cos B & \sin B & \cos(B+D) \\ \cos C & \sin C & \cos(C+D) \end{vmatrix}$$
, then Δ is

(a) independent of A and B only
(b) independent of B and C only
(c) independent of A, B and C only
(d) independent of A, B, C and D all

17. The number of ways the letters of the word 'mathematics' be permuted so that vowels never come together is

(a) $\dfrac{11!}{2!2!2!}$ (b) $\dfrac{8!4!}{2!2!2!}$
(c) $\dfrac{7!5!}{2!2!2!}$ (d) None of these

18. Consider a quadratic equation given by $\sum_{r=1}^{n}\left(3x - \dfrac{1}{2r+1}\right)\left(2x - \dfrac{1}{2r+5}\right) = 0$. The product of roots of the equation is

(a) $\dfrac{n+1}{6n(2n+5)}$ (b) $\dfrac{n+1}{36(2n+5)}$
(c) $\dfrac{n+1}{36n(2n+5)}$ (d) $\dfrac{n+1}{72n(2n+5)}$

19. The general solution of the differential equation $y\{xy - (x^2 - y^2)^2\}dx = \{y^3 - x(x^2 - y^2)^2\}dy$ is equal to

(a) $\dfrac{y}{x} + \dfrac{1}{x^2 - y^2} = c$ (b) $\dfrac{x}{y} - \dfrac{1}{2}\ln\left|\dfrac{x-y}{x+y}\right| = c$
(c) $\dfrac{2x}{y} + \dfrac{1}{x^2 - y^2} = c$ (d) $\dfrac{2x}{y} - \dfrac{1}{x^2 - y^2} = c$

20. $\displaystyle\int_0^2 \left[\tan^{-1} x\right] dx + \int_0^2 \left[\cot^{-1} x\right] dx =$

(where [·] represents the greatest integer function)

(a) 1 – cot 2 (b) 1 + cot 2
(c) 2(1 + cot 2) (d) 2(1 – cot 2)

SECTION-B (NUMERICAL TYPE QUESTIONS)

Attempt any 5 questions out of 10.

21. If the area of the triangle on the complex plane formed by the points z, iz, $z + iz$ is 200 square units, then $|z| =$ ________.

22. Two parallel chords are drawn on the same side of the centre of a circle of radius 12 units. It is found that they subtend 72° and 144° at the centre of the circle. The perpendicular distance (in units) between the chords is ________.

23. If the image of the point (3, 8) in the line $px + 3y - 7 = 0$ is $(-1, -4)$, then value of p is ________.

24. Value of positive real parameter 'a' such that the area of the region bounded by the parabola $y = x - ax^2$, $ay = x^2$ attains its maximum value, is equal to ________.

25. Let f be a real valued function satisfying $f(x + y) = f(x) f(y)$ for all $x, y \in R$ such that $f(1) = 2$. If $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$, then $a =$ _______.

26. If $A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$, then $A + 2A^T = \lambda A^T$, where λ equals _______.

27. A random variable X has the probability distribution:

$X:$	1	2	3	4	5	6	7	8
$P(X):$	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events $E = \{X \text{ is a prime number}\}$ and $F = \{X < 4\}$, $P(E \cup F)$ is ______.

28. The least positive value of 'a' for which the equation, $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is ________.

29. If the function f given by $f(x) = x^3 - 3(a - 2)x^2 + 3ax + 7$, for some $a \in R$ is increasing in (0, 1] and decreasing in [1, 5), then a root of the equation, $\frac{f(x)-14}{(x-1)^2} = 0\ (x \neq 1)$ is ________.

30. Let $f(\theta) = \begin{vmatrix} \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \end{vmatrix}$

and the period of $f(\theta)$ is T, then the value of $\frac{4}{\pi} \times T$ is _____.

SOLUTIONS

1. (a) : Let $z = x + iy$. Then

$$\frac{z-8i}{z+6} = \frac{x+(y-8)i}{(x+6)+iy} = \frac{(x+(y-8)i)(x+6-iy)}{(x+6)^2+y^2}$$

$$= \frac{(x^2+6x+y^2-8y)+i(xy-8x+6y-48-xy)}{(x+6)^2+y^2}$$

$$\Rightarrow \operatorname{Re}\left(\frac{z-8i}{z+6}\right) = 0 \Rightarrow x^2 + y^2 + 6x - 8y = 0$$

2. (c) : Let roots of the equation $ax^2 + 2bx + c = 0$ are α and β and roots of the equation $px^2 + 2qx + r = 0$ are γ and δ.

Given, $\frac{\alpha}{\beta} = \frac{\gamma}{\delta} \Rightarrow \frac{\alpha}{\gamma} = \frac{\beta}{\delta}$

$$\Rightarrow \frac{\alpha+\beta}{\gamma+\delta} = \sqrt{\frac{\alpha\beta}{\gamma\delta}} \Rightarrow \frac{-\frac{2b}{a}}{-\frac{2q}{p}} = \sqrt{\frac{\frac{c}{a}}{\frac{r}{p}}} \Rightarrow \frac{b^2}{ac} = \frac{q^2}{pr}$$

3. (b) : $[A(A + B)^{-1}B]^{-1} = B^{-1}((A + B)^{-1})^{-1}A^{-1}$

$= B^{-1}(A + B)A^{-1} \qquad [\because (A^{-1})^{-1} = A]$

$= (B^{-1}A + I)A^{-1} = B^{-1}I + IA^{-1} = B^{-1} + A^{-1}$

4. (a) : $x_1 < x_2 < x_3 < x_4 < x_5 < x_6$, when the number is $x_1x_2x_3x_4x_5x_6$

Clearly, no digit can be zero.

Also, all the digits are distinct. So, let us first select six digits from the list of digits 1, 2, 3, 4, 5, 6, 7, 8, 9 which can be done in ${}^9C_6$ ways.

After selecting these digits, they can be put only in one order.

Thus, total number of such numbers

$= {}^9C_6 \times 1 = {}^9C_6 = {}^9C_3$

5. (b) : $f(0^+) = \lim_{x\to 0^+} |x|^{\sin x} = 1$

$f(0^-) = g(0) = 1$

Let $g(x) = ax + b$

$\Rightarrow \quad b = 1 \Rightarrow g(x) = ax + 1$

For $x > 0$, $f'(x) = e^{\sin x \ln(|x|)}\left[\cos x \ln(|x|) + \frac{\sin x}{x}\right]$

$f'(1) = 1[0 + \sin 1] = \sin 1$

$f(-1) = -a + 1 \Rightarrow a = 1 - \sin 1$

$g(1) = (1 - \sin 1)x + 1$

6. (b)

PUBLIC NOTICE

NTA Examination Calendar for Academic Year 2023-24

The Ministry of Education (MoE), Government of India (GoI) has established the National Testing Agency (NTA) as an independent, autonomous, and self-sustained premier testing organization under the Societies Registration Act (1860) for conducting efficient, transparent, and international standardized tests in order to assess the competency of candidates for admission to premier higher education institutions with a mission to improve equity and quality in education by developing and administering research-based valid, reliable, efficient, transparent, fair and international level assessments.

For the **Academic year 2023-24,** following is the Calendar for some major examinations to be conducted by the NTA:

S.No.	Name of Examination	Dates of Examinations	Reserve Dates
1.	Joint Entrance Examination [JEE (Main)] – 2023 Session 1	24, 25, 27, 28, 29, 30, 31 January 2023	01, 02, 03 February 2023
2.	Joint Entrance Examination [JEE (Main)] – 2023 Session 2	06, 08, 10, 11, 12 April 2023	13, 15 April 2023
3.	ICAR AIEEA 2023	26, 27, 28, 29 April 2023	
4.	National Eligibility cum Entrance Test [NEET (UG)] – 2023	07 May 2023	
5.	Common University Entrance Test (CUET) 2023	21 to 31 May 2023	01 to 07 June 2023

For further clarification related to the examination dates and other instructions, the Candidates are advised to visit the official website of NTA www.nta.ac.in for the latest updates.

7. (c) : We have, $y=\log_2\left(\dfrac{\sin x-\cos x+3\sqrt{2}}{\sqrt{2}}\right)$

$\Rightarrow \dfrac{1}{\sqrt{2}}(\sin x-\cos x)+3=2^y$

$\Rightarrow \sin\left(x-\dfrac{\pi}{4}\right)+3=2^y \quad \Rightarrow \quad \sin\left(x-\dfrac{\pi}{4}\right)=2^y-3$

We know, $-1\le\sin\left(x-\dfrac{\pi}{4}\right)\le 1$

$\Rightarrow -1\le 2^y-3\le 1 \Rightarrow 2\le 2^y\le 4$

$\Rightarrow 2^1\le 2^y\le 2^2 \quad \Rightarrow 1\le y\le 2$

8. (a) : The range of $|\sin x| = [0, 1]$

Now, A = range of $f(g(x))$

= range of $\log|\sin x| = (-\infty, 0]$

The range of $\log|x| = (-\infty, \infty)$

B = range of $g(f(x))$

= range of $\sin(\log|x|) = [-1, 1]$

$\therefore\ A\cup B=(-\infty,0]\cup[-1,1]=(-\infty,1]$

and $A\cap B=(-\infty,0]\cap[-1,1]=[-1,0]$

9. (a) : We have, $f(x)=\dfrac{4x^3-3x^2}{6}-2\sin x+(2x-1)\cos x$

$\Rightarrow f'(x) = (2x^2-x)-2\cos x+2\cos x-\sin x(2x-1)$

$= (2x-1)(x-\sin x)$

For $x > 0$, $x-\sin x > 0$ and $x < 0$, $x-\sin x < 0$

For $x\in\left[\dfrac{1}{2},\infty\right)$, $f'(x)\ge 0$ and for $x\in\left(0,\dfrac{1}{2}\right]$, $f'(x)\le 0$

$\therefore\ f(x)$ increases in $\left[\dfrac{1}{2},\infty\right)$.

10. (b): $(A\wedge(A\to B))\to B\equiv(A\wedge(\sim A\vee B))\to B$

$\equiv((A\wedge\sim A)\vee(A\wedge B)\to B\equiv(A\wedge B)\to B$

$\equiv(\sim A\vee\sim B)\vee B\equiv T$

11. (c) : P(getting an odd number twice)

= P(getting an even number thrice)

$\Rightarrow {}^nC_2\left(\dfrac{1}{2}\right)^n={}^nC_3\left(\dfrac{1}{2}\right)^n \Rightarrow {}^nC_2={}^nC_3 \Rightarrow n=2+3$

$\Rightarrow n=5$

Probability of getting an odd number for odd number of times $={}^5C_1\left(\dfrac{1}{2}\right)^5+{}^5C_3\left(\dfrac{1}{2}\right)^5+{}^5C_5\left(\dfrac{1}{2}\right)^5=\dfrac{1}{2^5}(5+10+1)$

$=\dfrac{16}{2^5}=\dfrac{1}{2}$

12. (a) : Let $P(B_1)=p_1$, $P(B_2)=p_2$, $P(B_3)=p_3$

Given, $p_1(1-p_2)(1-p_3)=\alpha$ ...(i)

$p_2(1-p_1)(1-p_3)=\beta$ ...(ii)

$p_3(1-p_1)(1-p_2)=\gamma$ ...(iii)

and $(1-p_1)(1-p_2)(1-p_3)=p$ ...(iv)

$\Rightarrow \dfrac{p_1}{1-p_1}=\dfrac{\alpha}{p}, \dfrac{p_2}{1-p_2}=\dfrac{\beta}{p}$ and $\dfrac{p_3}{1-p_3}=\dfrac{\gamma}{p}$

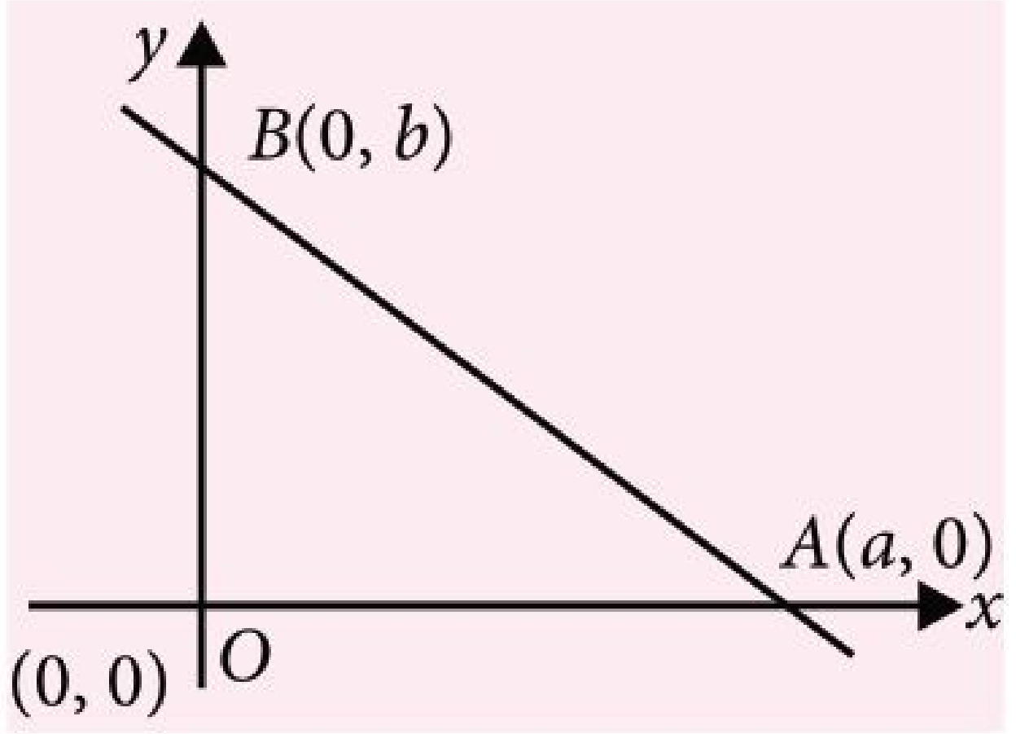
Also, $\beta=\dfrac{\alpha p}{\alpha+2p}=\dfrac{3\gamma p}{p-2\gamma}$

$\Rightarrow \alpha p-2\alpha\gamma=3\alpha\gamma+6p\gamma \Rightarrow \alpha p-6p\gamma=5\alpha\gamma$

$\Rightarrow \dfrac{p_1}{1-p_1}-\dfrac{6p_3}{1-p_3}=\dfrac{5p_1p_3}{(1-p_1)(1-p_3)} \quad \Rightarrow p_1-6p_3=0$

$\Rightarrow \dfrac{p_1}{p_3}=6$ *i.e.*, $\dfrac{P(B_1)}{P(B_3)}=6$

13. (b): The equation of the straight line passing through the point of intersection of the lines $x + 2y = 1$ and $2x - y = 1$ is



$(x+2y-1)+\lambda(2x-y-1)=0$

$\Rightarrow (1+2\lambda)x+(2-\lambda)y=(1+\lambda)$

$\Rightarrow \dfrac{x}{\frac{1+\lambda}{1+2\lambda}}+\dfrac{y}{\frac{1+\lambda}{2-\lambda}}=1$

$\therefore\ A\equiv\left(\dfrac{1+\lambda}{1+2\lambda},0\right)$ and $B\equiv\left(0,\dfrac{1+\lambda}{2-\lambda}\right)$

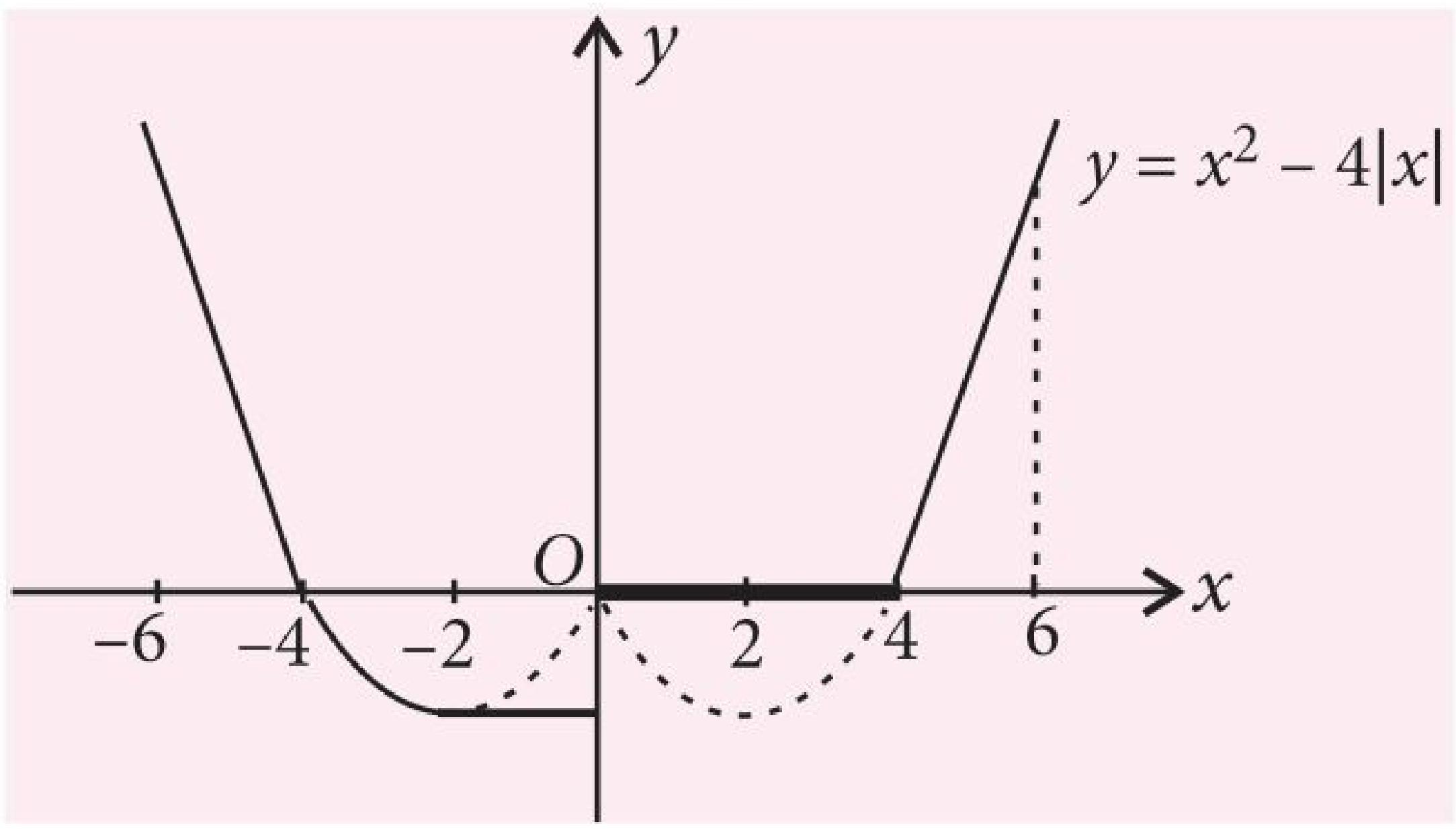
Let the mid-point of AB be (h, k)

$\therefore\ h=\dfrac{1}{2}\left(\dfrac{1+\lambda}{1+2\lambda}\right)$ and $k=\dfrac{1}{2}\left(\dfrac{1+\lambda}{2-\lambda}\right)$

Eliminating λ from h and k, we get $h+3k=10hk$.

Therefore the required locus is $x+3y=10xy$.

14. (d) : Bold line represents the graph of $y = g(x)$. Clearly, $g(x)$ has neither a point of local maxima nor a point of local minima.



15. (b): We have $f(x)=\dfrac{9^x}{9^x+9}$

$\therefore\ f(2-x)=\dfrac{9^{2-x}}{9^{2-x}+9}$

So, $f(x)+f(2-x)=\dfrac{9^x}{9^x+9}+\dfrac{9^{2-x}}{9^{2-x}+9}=\dfrac{9^x}{9^x+9}+\dfrac{9}{9^x+9}=1$

$\Rightarrow f(x)+f(2-x)=1$

$\therefore\ f\left(\dfrac{1}{2019}\right)+f\left(\dfrac{2}{2019}\right)+f\left(\dfrac{3}{2019}\right)+\ldots+f\left(\dfrac{4037}{2019}\right)$

$$=\left\{f\left(\frac{1}{2019}\right)+f\left(\frac{4037}{2019}\right)\right\}+\left\{f\left(\frac{2}{2019}\right)+f\left(\frac{4036}{2019}\right)\right\}+\dots$$
$$+\left\{f\left(\frac{2018}{2019}\right)+f\left(\frac{2020}{2019}\right)\right\}+f\left(\frac{2019}{2019}\right)$$
$$=\{1+1+\dots+1(2018\text{ times})\}+f(1)=2018+\frac{1}{2}=\frac{4037}{2}$$

16. (d) : Applying $C_3 \to C_3 - C_1\cos D + C_2 \sin D$, we get $\Delta = 0$, hence Δ is independent of A, B, C, D all.

17. (d)

18. (c) : Let $\frac{1}{2r+1}=\lambda$ and $\frac{1}{2r+5}=\mu$

$$\Rightarrow \sum_{r=1}^{n}(3x-\lambda)(2x-\mu)=0$$
$$\Rightarrow \sum_{r=1}^{n}[6x^2-x(3\mu+2\lambda)+\lambda\mu]=0$$

Since $\sum_{r=1}^{n}\lambda\mu=\sum_{r=1}^{n}\frac{1}{(2r+1)(2r+5)}$

$$\Rightarrow \frac{1}{4}\sum_{r=1}^{n}\left(\frac{1}{2r+1}-\frac{1}{2r+5}\right)$$
$$=\frac{1}{4}\left[\frac{1}{3}-\frac{1}{2n+5}\right]=\frac{n+1}{6(2n+5)}$$

$\therefore$ The product of roots of the equation $=\frac{n+1}{36n(2n+5)}$

19. (c) : $xy^2dx - y(x^2-y^2)^2dx = y^3dy - x(x^2-y^2)^2dy$

$$\Rightarrow y^2(xdx-ydy)=(x^2-y^2)^2(ydx-xdy)$$
$$\Rightarrow \frac{xdx-ydy}{(x^2-y^2)^2}=\frac{ydx-xdy}{y^2} \Rightarrow \frac{d(x^2-y^2)}{(x^2-y^2)^2}=2d\left(\frac{x}{y}\right)$$

Integrating, we get

$$\frac{2x}{y}+\frac{1}{(x^2-y^2)}=c$$

20. (c) : $[\tan^{-1}x]=\begin{cases}0;\ 0\le x<\tan 1\\ 1;\ x\ge \tan 1\end{cases}$

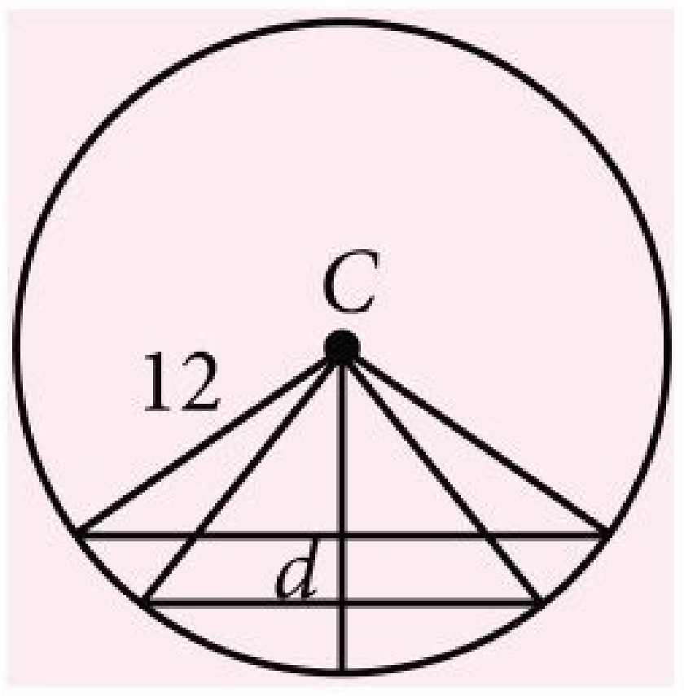
$$[\cot^{-1}x]=\begin{cases}1;\ 0\le x\le \cot 1\\ 0;\ x>\cot 1\end{cases}$$

$$\int_{\tan 1}^{2}1\,dx+\int_{0}^{\cot 1}1\,dx=2-\tan 1+\cot 1=2(1+\cot 2)$$

21. (20) : The area of the triangle formed by z, iz, $z + iz$ is given by $\frac{1}{2}|z|^2$.

$$\therefore\ \frac{1}{2}|z|^2=200 \Rightarrow |z|^2=400 \Rightarrow |z|=20$$

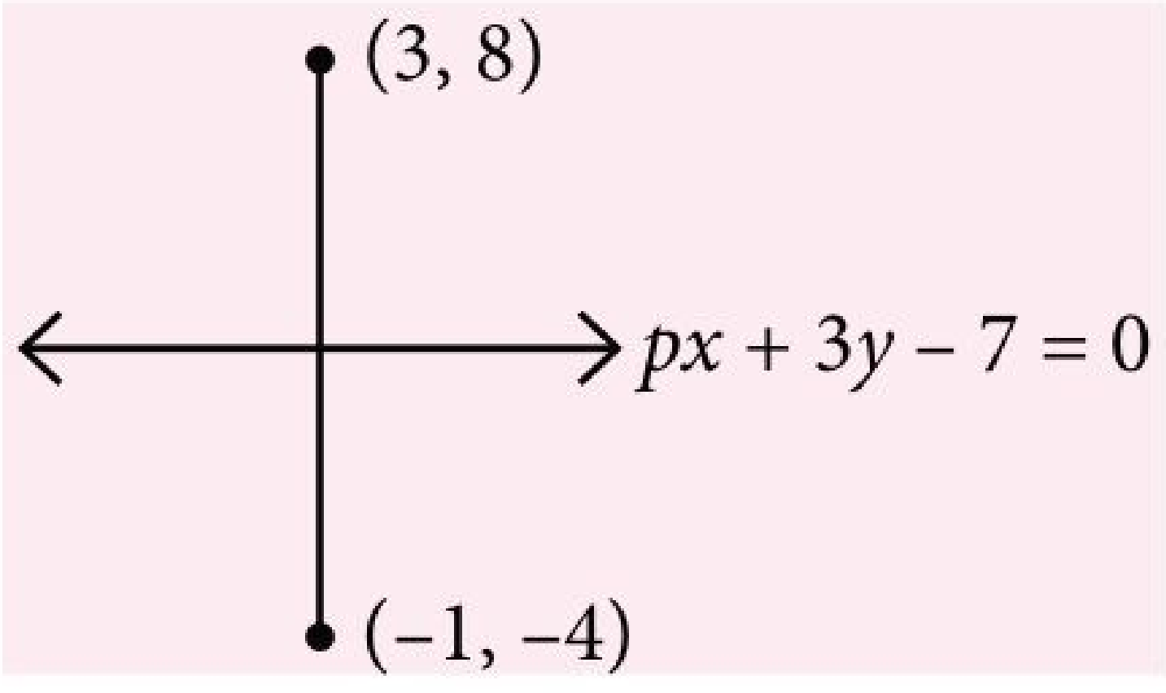
22. (6) : Distance, d



$= 12(\cos 36° - \cos 72°)$
$= 24\sin 54°\sin 18°$
$= 24\cos 36°\sin 18°$

$$=24\cdot\frac{\sqrt{5}+1}{4}\cdot\frac{\sqrt{5}-1}{4}=6\text{ units}$$

23. (1) :



Since, image of point (x, y) from line $ax + by + c = 0$ is given by

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{-2(ax_1+by_1+c)}{a^2+b^2}$$

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

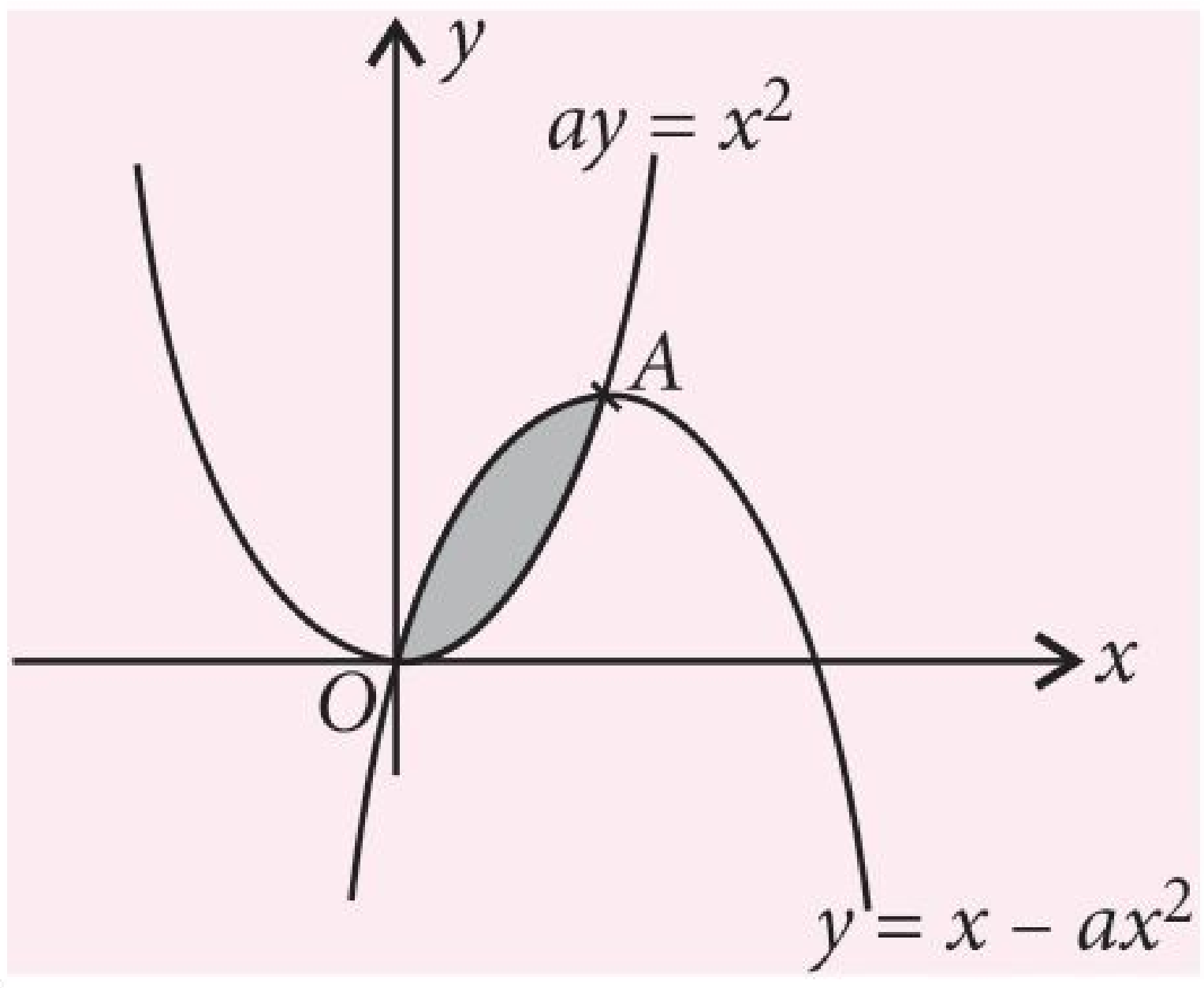
3	8		1		2	5		4				1		2	3		4		7	5
9		5		6			1						5			2		6		3
	7							9				4							9	
2								6				6								7
	3						2						9						8	
4								1				2								6
7							3						8							9
	6			4		1		8				5		6		4			3	
8		3	5		6		4	2		8		9	1		5		6	7		4
									8		7									
								3				4								
									2		5									
4		3	5		7		1	9		6		2	4		9		7	6		1
	5			6		3		7				8		1		2			9	
2							5						9							4
3								1				5								8
	9						6						8						6	
6								5				7								2
	4							3				4							7	
8		1		9			4						1			5		8		3
5	7		4		6	1		8				6		3	2		4		5	9

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

$$\therefore \quad \frac{-1-3}{p}=\frac{-4-8}{3}=\frac{-2(3p+24-7)}{p^2+9}$$

$$\therefore \quad \frac{-4}{p}=\frac{-12}{3} \Rightarrow p=1$$

24. (1) : At point A,



$$\frac{x^2}{a}=x-ax^2$$

$$\Rightarrow \quad x=0,\ x=\frac{a}{1+a^2}$$

$\therefore$ Required area,

$$\Delta=\int_0^{a/1+a^2}\left(x-ax^2-\frac{x^2}{a}\right)dx$$

$$\Rightarrow \quad \frac{d\Delta}{da}=\int_0^{a/1+a^2}\left(-x^2+\frac{x^2}{a^2}\right)dx+0\cdot\frac{d}{da}\left(\frac{a}{1+a^2}\right)$$

$$=\left[\frac{(1-a^2)}{3a^2}x^3\right]_0^{a/1+a^2}=\frac{a(1-a^2)}{3(1+a^2)^3}=\frac{a(1-a)(1+a)}{3\,(1+a^2)^3}$$

$$\Rightarrow \quad \frac{d\Delta}{da}>0\ \forall\ a\in(0,1)\ \text{and}\ \frac{d\Delta}{da}<0\ \forall\, a\in(1,\infty)$$

Thus, $a = 1$ is the point of maxima for Δ.

25. (3): We have, $f(x)=[f(1)]^x=2^x$ for all $x\in R$.

[If $f: R \to R$ is a function satisfying $f(x + y) = f(x)\, f(y)$ for all $x, y \in R$, then $f(x) = [f(1)]^x\ \forall\ x \in R$]

$$\therefore \quad \sum_{k=1}^{n} f(a+k)=16(2^n-1)$$

$$\Rightarrow \quad \sum_{k=1}^{n} 2^{a+k}=16(2^n-1) \quad \Rightarrow 2^a\sum_{k=1}^{n}2^k=16(2^n-1)$$

$$\Rightarrow \quad 2^a(2+2^2+2^3+\ldots+2^n)=16(2^n-1)$$

$$\Rightarrow \quad 2^a\times 2\left(\frac{2^n-1}{2-1}\right)=16(2^n-1)\Rightarrow 2^{a+1}=2^4 \ \Rightarrow\ a=3$$

26. (1): $A^T=\begin{bmatrix}0 & 1 & -2\\ -1 & 0 & -3\\ 2 & 3 & 0\end{bmatrix}, 2A^T=\begin{bmatrix}0 & 2 & -4\\ -2 & 0 & -6\\ 4 & 6 & 0\end{bmatrix}$

Thus, $2A^T+A=\begin{bmatrix}0 & 1 & -2\\ -1 & 0 & -3\\ 2 & 3 & 0\end{bmatrix}=A^T$

27. (0.8) : From the given table prime numbers are 2, 3, 5 and 7.

$\therefore$ $P(E) = P(2 \text{ or } 3 \text{ or } 5 \text{ or } 7)$

$= P(2) + P(3) + P(5) + P(7) = 0.62$

Now, $P(F) = P(1 \text{ or } 2 \text{ or } 3)$

$= P(1) + P(2) + P(3) = 0.50$

$\therefore$ $P(E \cap F) = P(2 \text{ or } 3) = P(2) + P(3) = 0.35$

Now, $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= 0.62 + 0.50 - 0.35 = 0.8$

28. (8) : Since, the given equation

$2x^2+(a-10)x+\frac{33}{2}=2a$ has real roots.

$$\therefore \quad D\ge 0 \Rightarrow \quad (a-10)^2-4(2)\left(\frac{33}{2}-2a\right)\ge 0$$

$\Rightarrow$ $a^2 + 100 - 20a - 132 + 16a \ge 0$

$\Rightarrow$ $a^2 - 4a - 32 \ge 0 \Rightarrow (a - 8)(a + 4) \ge 0$

$\Rightarrow$ $a \in (-\infty, -4] \cup [8, \infty)$

29. (7) : Here, $f(x) = x^3 - 3(a - 2)x^2 + 3ax + 7$

$\therefore$ $f'(x) = 3x^2 - 6(a - 2)x + 3a$

Since $f'(x) \ge 0\ \forall\ x \in (0, 1]$ and $f'(x) \le 0\ \forall\ x \in [1, 5)$

$\therefore$ $f'(x) = 0$ at $x = 1$

$\Rightarrow$ $3 - 6(a - 2) + 3a = 0 \Rightarrow -3a + 15 = 0 \Rightarrow a = 5$

So, $f(x) = x^3 - 9x^2 + 15x + 7$

$$\text{Now, } \frac{f(x)-14}{(x-1)^2}=0 \ \Rightarrow\ \frac{x^3-9x^2+15x-7}{(x-1)^2}=0$$

$$\Rightarrow \quad \frac{(x-1)^2(x-7)}{(x-1)^2}=0 \ \Rightarrow\ x=7$$

30. (2): $f(\theta)=\begin{vmatrix}-1 & 0 & 1\\ -1 & 1 & 0\\ 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta\end{vmatrix}$

$(R_1 \to R_1 - R_3;\ R_2 \to R_2 - R_3)$

$$=\begin{vmatrix}0 & 0 & 1\\ -1 & 1 & 0\\ 1+\sin^2\theta+4\sin 4\theta & \cos^2\theta & 4\sin 4\theta\end{vmatrix}(C_1\to C_1+C_3)$$

$= -(\cos^2\theta + 1 + \sin^2\theta + 4\sin 4\theta) = -2(1 + 2\sin 4\theta)$

which is periodic function with period $\frac{2\pi}{4}=\frac{\pi}{2}$

Hence the value of $\frac{4}{\pi}\times T=\frac{4}{\pi}\cdot\frac{\pi}{2}=2$

◆◆

CHAPTERWISE PRACTICE PAPER 2023

CUET (UG)

Exam on
21st to 31st
May 2023

SERIES - V Differential Equations

Section II of CUET (UG) is Domain specific. In this section of Mathematics 40 questions to be attempted out of 50.

Time Allowed : 45 Minutes **Maximum Marks :** 200

Multiple Choice Questions (MCQs)

1. Order and degree of the differential equation

$$\begin{vmatrix} x^3 & y^2 & 3 \\ 2x^2 & 3y\dfrac{dy}{dx} & 0 \\ 5x & 2\left(y\dfrac{d^2y}{dx^2}+\left(\dfrac{dy}{dx}\right)^2\right) & 0 \end{vmatrix}=0$$ are respectively

(a) 1, 1 (b) 1, 2 (c) 2, 2 (d) 2, 1

2. Find the degree of the given differential equation

$$e^{\left(\frac{dy}{dx}-\frac{d^3y}{dx^3}\right)}=\ln\left(\frac{d^5y}{dx^5}+1\right).$$

(a) 1 (b) 3
(c) 5 (d) Not defined

3. The order and degree of the differential equation of the family of parabolas with focus at the origin and axis of symmetry along the x-axis respectively are

(a) 1, 2 (b) 2, 10 (c) 2, 2 (d) 2, 3

4. The differential equation of all ellipses with centres at the origin and the ends of one axis of symmetry at (±1, 0), is

(a) $(x^2-1)y'-xy=0$
(b) $(x^2+1)y'-xy=0$
(c) $xy'+(x^2+1)y=0$
(d) $(x^2-1)y''+(x-1)y'=0$

5. The differential equation for the curve $x^3+y^3=4ax$ is

(a) $3xy\dfrac{dy}{dx}+2x^3+y^3=0$

(b) $3xy^2\dfrac{dy}{dx}+2x^3-y^3=0$

(c) $3xy^2\dfrac{dy}{dx}+2x^3+y^3=0$

(d) $3xy^2\dfrac{dy}{dx}-2x^3-y^3=0$

6. Solve the differential equation :
$(1+x)\,ydx=(y-1)\,xdy$

(a) $xy=c$ (b) $xy=ce^x$
(c) $xy=ce^y$ (d) $xy=ce^{y-x}$

7. Solve : $2\dfrac{y}{x}+\left(\left(\dfrac{y}{x}\right)^2-1\right)\dfrac{dy}{dx}=0$

(a) $y=1+ce^{1/x}$ (b) $x^2+y^2=yc'$
(c) $x^2+1=e^1$ (d) $x^2-y^2=c'y$

8. If $\dfrac{dy}{dx}=xy+2x+3y+6$, then $y(-1)-e^2y(-3)=$

(a) e^2-1 (b) e^2+1
(c) $2(e^2-1)$ (d) $-2(e^2-1)$

9. Solve : $(2x-y+1)\,dx+(2y-x+1)dy=0$

(a) $(y+1)^2-(y+1)(x+1)=c^2$
(b) $(y+1)^2-(y+1)(x+1)+(x+1)^2=c^2$
(c) $(y^2+xy+x^2)^2=c^2$
(d) None of these

10. Solve the differential equation $\dfrac{dy}{dx}-\dfrac{y}{x}+\operatorname{cosec}\dfrac{y}{x}=0$; given that $y=0$ when $x=1$.

(a) $\cos y=1+x$
(b) $\cos y-\cos x=1$
(c) $\cos\left(\dfrac{y}{x}\right)=1+\log|x|$
(d) $\cos y=1-\log|x|$

11. Solve the following differential equation

$$\frac{dy}{dx}=\frac{1}{x\cos y+\sin 2y}.$$

(a) $x=\sin y+Ce^{\sin y}+1$

(b) $x=-2(\sin y+1)+Ce^{\sin y}$

(c) $x=2\sin y+Ce^{\sin y}$

(d) $x=2(\sin y+1)-Ce^{\sin y}$

12. Find the solution of the differential equation

$$\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0.$$

(a) $\sin^{-1}x+\cos^{-1}y=c$ (b) $\sin^{-1}x+\sin^{-1}y=c$

(c) $x\sqrt{1-xy}+y^2=c$ (d) none of these

13. If $y(t)$ is a solution of $(1+t)\frac{dy}{dt}-ty=1$ and $y(0)=-1$, then $y(1)$ is equal to

(a) $-\frac{1}{2}$ (b) $e+\frac{1}{2}$ (c) $e-\frac{1}{2}$ (d) $\frac{1}{2}$

14. Solve the differential equation $\frac{dy}{dx}=\frac{2x(\log x+1)}{\sin y+y\cos y}$, given that $y=0$, when $x=1$.

(a) $2y=x\log x+1$

(b) $y=2x^2+x-1$

(c) $2y\sin y=2x^2\log x+x^2-1$

(d) $y=\sin y+2x$

15. The solution of the differential equation

$$(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0, \text{ is}$$

(a) $xe^{2\tan^{-1}y}=e^{\tan^{-1}y}+K$ (b) $(x-2)=Ke^{\tan^{-1}y}$

(c) $2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+K$ (d) $xe^{\tan^{-1}y}=\tan^{-1}y+K$

16. Solve the following differential equation :

$$x\log x\frac{dy}{dx}+y=2\log x$$

(a) $y(\log x)=(\log x)^2+c$ (b) $y(\log x)=\log x+c$

(c) $y\log x=c$ (d) None of these

17. Solve the initial value problem

$$xe^{y/x}-y+x\frac{dy}{dx}=0, y(e)=0$$

(a) $y=-x\log(\log|x|); x\in(-\infty,-1)\cup(1,\infty)$

(b) $y=\log(x), x\in(-\infty,-1)\cup(1,\infty)$

(c) $y=x, x\in(-\infty,-1)\cup(1,\infty)$

(d) $y=x+\log x, x\in(-\infty,-1)\cup(1,\infty)$

18. Which of the following is a homogeneous differential equation?

(a) $(x-y)^2\frac{dy}{dx}=a^2$

(b) $x\frac{dy}{dx}-2y=x^3$

(c) $(x+y-1)\,dy-(x-y+1)dx=0$

(d) $x\sin\left(\frac{y}{x}\right)dy=\left\{y\sin\left(\frac{y}{x}\right)-x\right\}dx$

19. The curve which passes through (1, 2) and whose tangent at any point has a slope that is half of slope of the line joining origin to the point of contact, is

(a) a rectangle hyperbola

(b) a circle

(c) a parabola

(d) a straight line through origin

20. If $y=f(x)$ satisfies the differential equation $x\frac{dy}{dx}+y=xe^x$ and $y(1)=1$, then the value of $y(2)$ is

(a) e^2 (b) $e^2/2$ (c) $e/2$ (d) e

21. The differential equation satisfied by the system of parabolas $y^2=4a(x+a)$ is

(a) $y\left(\frac{dy}{dx}\right)^2-2x\left(\frac{dy}{dx}\right)+y=0$

(b) $y\left(\frac{dy}{dx}\right)^2-2x\left(\frac{dy}{dx}\right)-y=0$

(c) $y\left(\frac{dy}{dx}\right)^2+2x\left(\frac{dy}{dx}\right)-y=0$

(d) $y\left(\frac{dy}{dx}\right)+2x\left(\frac{dy}{dx}\right)-y=0$

22. The differential equation which represents the family of curves $y=c_1e^{c_2x}$, where c_1 and c_2 are arbitrary constants, is

(a) $y''=y'y$ (b) $yy''=y'$

(c) $yy''=(y')^2$ (d) $y'=y^2$

23. If p and q are order and degree of differential equation $y^2\left(\frac{d^2y}{dx^2}\right)^2+3x\left(\frac{dy}{dx}\right)^{1/3}+x^2y^2=\sin x$, then

(a) $p>q$ (b) $\frac{p}{q}=\frac{1}{2}$

(c) $p=q$ (d) $p<q$

24. Find the differential equation whose solution is given by $y=x+2ax^2$.

(a) $2x=y(1+y_1)$ (b) $2y=x(1+y_1)$

(c) $2xy=1+y_1$ (d) $2y_1=x(1+y)$

25. The differential equation for which $y = a\cos x + b\sin x$ is a solution, is

(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$

(c) $\frac{d^2y}{dx^2} + (a+b)y = 0$ (d) $\frac{d^2y}{dx^2} + (a-b)y = 0$

26. If $\frac{dy}{dx} = \cos(x+y)$, $y\left(\frac{\pi}{2}\right) = 0$ then $y(0) =$

(a) $\tan^{-1}\left(\frac{\pi}{2}-1\right)$ (b) $\tan^{-1}\left(\frac{\pi}{2}+1\right)$

(c) $2\tan^{-1}\left(\frac{\pi}{2}-1\right)$ (d) $-2\tan^{-1}\left(\frac{\pi}{2}-1\right)$

27. Solve the differential equation $x\left(x^2-1\right)\frac{dy}{dx} = 1$, given that when $x = 2$, $y = 0$.

(a) $y = \frac{1}{2}\log|x^2+1| - \log|x| - \log 2 - \frac{1}{2}\log 3$

(b) $y = \frac{1}{2}\log|x^2-1| - \log|x| + \log 2 - \frac{1}{2}\log 3$

(c) $y = \frac{1}{2}\log|x^2-1| + \log|x| + \log 2 - \frac{1}{2}\log 3$

(d) $y = \frac{1}{2}\log|x^2-1| + \log|x| - \log 2 + \frac{1}{2}\log 3$

28. Suppose a solution of the differential equation $(x^7 - x^3y)\frac{dx}{dy} = 1$ satisfies the initial condition $y(0) = 4$. Then the value of $\frac{dy}{dx}$ when $y = \sqrt{2}$ is

(a) $1+\frac{2}{e}$ (b) $8\sqrt{2}\left(1+\frac{2}{e}\right)$

(c) $8\left(1-\frac{2}{e}\right)$ (d) $8\sqrt{2}\left(1-\frac{2}{e}\right)$

29. If $ye^y dx = (y^3 + 2xe^y)\,dy$, $y(0) = 1$, then the value of x when $y = 0$ is

(a) -1 (b) 0 (c) 1 (d) 2

30. If $y = ae^{-1/x} + b$ is a solution of $\frac{dy}{dx} = \frac{y}{x^2}$, then

(a) $a \in R, b = 0$ (b) $a = 2, b = 1$

(c) $a = 1, b = 1$ (d) $a = 2, b = 2$

31. If $x = x(y)$ is the solution of the differential equation $y\frac{dx}{dy} = 2x + y^3(y+1)e^y$, $x(1) = 0$; then $x(e)$ is equal to

(a) $e^3(e^e - 1)$ (b) $e^e(e^3 - 1)$

(c) $e^2(e^e + 1)$ (d) $e^e(e^2 - 1)$

32. Solve the differential equation :

$$\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$$

(a) $\sin(x+y) = x\cos(x+y) + c\cos(x+y) + 1$

(b) $\cos(x+y) = x\sin(x+y) + c\cos(x+y) + 1$

(c) $\sin(x-y) = x\cos(x-y) + \cos(x+y) + 1$

(d) $\cos(x-y) = x\cos(x+y) + c\cos(x+y)$

33. Solve the differential equation and find its particular solution $\frac{dx}{x+2} + \frac{dy}{y+2} = 0$, when $x = 1$, $y = 2$.

(a) $2xy + 2(x+y) = 8$ (b) $2xy + (x+y) = 8$

(c) $xy + 2(x+y) = 8$ (d) $xy + (x+y) = 8$

34. Solve $e^{dy/dx} = x + 1$, given that when $x = 0$, $y = 3$.

(a) $y = (x+1)\log(x+1) - (x+1)$

(b) $y = (x+1)\log(x+1) - (x+1) + 4$

(c) $y = \log(x+1) + 4$

(d) None of these

Assertion & Reason Based MCQs

For question numbers 35-38, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

(a) If both Statement-I and Statement-II are true and Statement-II is the correct explanation of Statement-I.

(b) If both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I.

(c) If Statement-I is true but Statement-II is false.

(d) If Statement-I is false and Statement-II is true.

35. Let a solution $y = y(x)$ of the differential equation $\frac{dy}{dx}\cos x + y\sin x = 1$ satisfy $y(0) = 1$.

Statement-I : $y(x) = \sqrt{2}\sin\left(\frac{\pi}{4} + x\right)$.

Statement-II : The integrating factor of the given differential equation is $\sec x$.

36. Let a solution $y = y(x)$ of differential equation, $\sqrt{1-x^2}\,dy - \sqrt{1-y^2}\,dx = 0$, satisfy $y\left(\frac{1}{2}\right) = 1$, then

Statement-I : $y(x) = \sin\left(\sin^{-1}x + \frac{\pi}{3}\right)$.

Statement-II : $y(x)$ is given by $y = \frac{x + \sqrt{3+3x^2}}{2}$.

37. **Statement-I :** The elimination of four arbitrary constants in $y = (c_1 + c_2 + c_3e^{c_4})x$ results into a differential equation of the first order $x\dfrac{dy}{dx} = y$.

Statement-II : Elimination of n arbitrary constants requires in general, a differential equation of the n^{th} order.

38. **Statement-I :** If $\dfrac{dy}{dx} + xy = x^3$, $x > 0$, $y \geq 0$ and $y(0) = 1$, then $y(1) = -1 + \dfrac{3}{\sqrt{2}}$.

Statement-II : The differential equation is linear in the dependent variable $1/y$.

Matrix-Match

(39-40) : Match Column I with Column II and select the correct answer by choosing an appropriate option.

39.

	Column - I		Column - II
(P)	The solution of $\dfrac{dy}{dx} = 2^{y-x}$ is	(1)	$x^2(y+3)^3 = e^{y+2}$
(Q)	The solution of $\dfrac{dy}{dx} = 1 + x + y^2 + xy^2$, when $y = 0$, $x = 0$ is	(2)	$2^{-x} - 2^{-y} = k$
(R)	Solution of $x^2\dfrac{dy}{dx} = x^2 + xy + y^2$ is	(3)	$y = \tan\left(x + \dfrac{x^2}{2}\right)$
(S)	The solution of $2(y+3) - xy\dfrac{dy}{dx} = 0$, given that $y(1) = -2$ is	(4)	$\tan^{-1}\left(\dfrac{y}{x}\right) = \log\lvert x\rvert + C$

	P	Q	R	S
(a)	2	3	1	4
(b)	3	2	4	1
(c)	3	2	1	4
(d)	2	3	4	1

40. Match the order and degree given in column II with corresponding differential equations given in column I.

	Column I		Column II
(P)	$\{y'^3 + y''\}^2 = ay'$	(1)	Order = 2
(Q)	$3y = 7xy + 5/y'$	(2)	Order = 1
(R)	$y'' - 5y' + 6y = 0$	(3)	Degree = 2
(S)	$(y' - 4x)^{3/2} = x + 5y'$	(4)	Degree = 1
		(5)	Degree = 3

	P	Q	R	S
(a)	1,2,3	3,4	1,4	2,5
(b)	1,3	2,3	1,4	2,5
(c)	3,4	1,3	1,2	3,5
(d)	2,5	2,3	1,4	4,5

Case Based MCQs

Case I : Read the following passage and answer the questions from 41 to 45.

°C
°F
50
120
40
100
30
80
20
60
10
40
0
20
10
0
20
20
30
40
40
60

A thermometer reading 80°F is taken outside. Five minutes later the thermometer reads 60°F. After another 5 minutes the thermometer reads 50 °F. At any time t the thermometer reading be T °F and the outside temperature be S °F.

41. If λ is positive constant of proportionality, then $\dfrac{dT}{dt}$ is

(a) $\lambda(T - S)$ (b) $\lambda(T + S)$
(c) λTS (d) $-\lambda(T - S)$

42. The value of $T(5)$ is

(a) 30°F (b) 40°F (c) 50°F (d) 60°F

43. The value of $T(10)$ is

(a) 50°F (b) 60°F (c) 80°F (d) 90°F

44. Find the general solution of differential equation formed in given situation.

(a) $\log T = St + c$ (b) $\log(T - S) = -\lambda t + c$
(c) $\log S = tT + c$ (d) $\log(T + S) = \lambda t + c$

45. Find the value of constant of integration c in the solution of differential equation formed in given situation.

(a) $\log(60 - S)$ (b) $\log(80 + S)$
(c) $\log(80 - S)$ (d) $\log(60 + S)$

Case II : Read the following passage and answer the questions from 46 to 50.

It is known that, if the interest is compounded continuously, the principal changes at the rate equal to the product of the rate of bank interest per annum and the principal. Let P denotes the principal at any time t and rate of interest be r % per annum.

46. Find the value of $\dfrac{dP}{dt}$.

(a) $\dfrac{Pr}{1000}$ (b) $\dfrac{Pr}{100}$ (c) $\dfrac{Pr}{10}$ (d) Pr

47. If P_0 be the initial principal, then find the solution of differential equation formed in given situation.

(a) $\log\left(\frac{P}{P_0}\right)=\frac{rt}{100}$ (b) $\log\left(\frac{P}{P_0}\right)=\frac{rt}{10}$

(c) $\log\left(\frac{P}{P_0}\right)=rt$ (d) $\log\left(\frac{P}{P_0}\right)=100rt$

48. If the interest is compounded continuously at 5% per annum, then in how many years will ₹ 100 double itself ? ($\log_e 2 = 0.6931$)

(a) 12.728 years (b) 14.789 years

(c) 13.862 years (d) 15.872 years

49. At what interest rate will ₹ 100 double itself in 10 years? ($\log_e 2 = 0.6931$).

(a) 9.66% (b) 8.239%

(c) 7.341% (d) 6.931%

50. How much will ₹ 1000 be worth at 5% interest after 10 years? ($e^{0.5} = 1.648$).

(a) ₹ 1648 (b) ₹ 1500

(c) ₹ 1664 (d) ₹ 1572

SOLUTIONS

1. **(d):** We have, $\begin{vmatrix} x^3 & y^2 & 3 \\ 2x^2 & 3y\frac{dy}{dx} & 0 \\ 5x & 2\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right) & 0 \end{vmatrix}=0$

Expanding along C_3, we get

$$3\left[4x^2\left\{y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right\}-15xy\frac{dy}{dx}\right]=0$$

$$\Rightarrow \quad 4x^2y\frac{d^2y}{dx^2}+4x^2\left(\frac{dy}{dx}\right)^2-15xy\frac{dy}{dx}=0$$

Now, highest order derivative is $\frac{d^2y}{dx^2}$.

So, its order is 2 and degree is 1.

2. **(d):** Since the given equation can not be expressed as a polynomial equation. So, its degree is not defined.

3. **(a) :** Equation of such parabolas is

$y^2 = 4A\,(A + x)$ ...(i)

where $(A, 0)$ is focus of parabola.

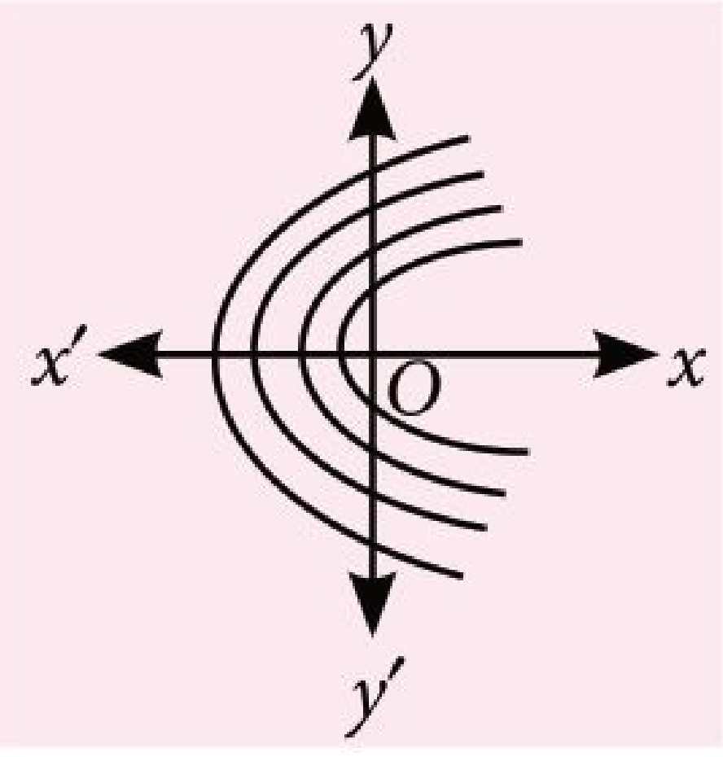
Differentiating (i) w.r.t. x, we get

$2y\frac{dy}{dx} = 4A \Rightarrow y\frac{dy}{dx} = 2A$ ...(ii)

Eliminating A from (i) and (ii), we get

$$y^2=\left(y\frac{dy}{dx}\right)^2+2y\frac{dy}{dx}\cdot x \Rightarrow y^2=y^2\left(\frac{dy}{dx}\right)^2+2xy\frac{dy}{dx},$$

which has order 1 and degree 2.



4. **(a) :** Let the ends of other axis of symmetry be $(0, \pm a)$. Then, the equation of the ellipse is

$$x^2+\frac{y^2}{a^2}=1 \Rightarrow a^2x^2+y^2=a^2 \quad \text{...(i)}$$

Differentiating w.r.t. x, we get

$$2a^2x+2y\frac{dy}{dx}=0 \Rightarrow a^2=-\frac{y}{x}\frac{dy}{dx} \quad \text{...(ii)}$$

Eliminating a^2 from (i) and (ii), we get

$$-xy\frac{dy}{dx}+y^2=-\frac{y}{x}\frac{dy}{dx} \Rightarrow (x^2-1)\frac{dy}{dx}-xy=0$$

5. **(b):** We have, $x^3 + y^3 = 4ax$ ...(i)

Differentiating (i) w.r.t. x, we get

$$3x^2+3y^2\frac{dy}{dx}=4a \quad \text{...(ii)}$$

From (i) and (ii), we get

$$x^3+y^3=3x^3+3xy^2\frac{dy}{dx}\Rightarrow 3xy^2\frac{dy}{dx}+2x^3-y^3=0$$

6. **(d):** The given differential equation can be written as $\left(\frac{1+x}{x}\right)dx=\left(\frac{y-1}{y}\right)dy$

$$\Rightarrow \int\left(\frac{1}{x}+1\right)dx=\int\left(1-\frac{1}{y}\right)dy$$

$\Rightarrow \log x + x = y - \log y + \log c$

$\Rightarrow \log y + \log x = y - x + \log c \Rightarrow xy = ce^{y-x}$

7. **(b):** Put $y = vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

The given differential equation can be written as,

$$2v+(v^2-1)\left(v+x\frac{dv}{dx}\right)=0$$

$$\Rightarrow v+x\frac{dv}{dx}=-\frac{2v}{v^2-1} \Rightarrow x\frac{dv}{dx}=\frac{-v(1+v^2)}{v^2-1}$$

$$\Rightarrow \int\frac{v^2-1}{v(1+v^2)}dv=-\int\frac{dx}{x}$$

$$\Rightarrow \int\left(\frac{2v}{1+v^2}-\frac{1}{v}\right)dv=-\log x+c$$

$$\Rightarrow \log(1+v^2)-\log v=-\log x+c$$

$$\Rightarrow \log\left|\frac{1+v^2}{v}\cdot x\right|=c \Rightarrow \log\left|\frac{x^2+y^2}{y}\right|=c$$

$\Rightarrow x^2 + y^2 = yc'$, where $c' = e^c$

8. (c) : We have, $\frac{dy}{dx} = (x+3)(y+2)$

Put $X = x + 3$, $Y = y + 2$

$\therefore$ The given differential equation becomes,

$$\frac{dY}{dX} = X \cdot Y \Rightarrow \int \frac{dY}{Y} = \int X\, dX \Rightarrow \log Y = \frac{X^2}{2} + C$$

$$\Rightarrow \quad Y = Ae^{X^2/2} \Rightarrow \quad y = -2 + Ae^{(x+3)^2/2}$$

So, $y(-1) = -2 + Ae^2$, $y(-3) = -2 + A$

$\therefore \quad y(-1) - e^2\, y(-3) = -2 + 2e^2 = 2(e^2 - 1)$.

9. (b): We have, $(2x - y + 1)dx = -(2y - x + 1)dy$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-(2x - y + 1)}{(2y - x + 1)}$$

Let $y = Y + k$ and $x = X + h$

$$\text{then} \quad \frac{dy}{dx} = \frac{dY}{dX} = \frac{-(2X - Y + 2h - k + 1)}{(2Y - X + 2k - h + 1)}$$

Now, 'h' and 'k' can be chosen such that $2h - k + 1 = 0$ and $2k - h + 1 = 0$

By solving these equations, we get $h = -1$, $k = -1$.

$$\text{So,} \quad \frac{dY}{dX} = \frac{-(2X - Y)}{(2Y - X)}$$

$$\text{Putting } Y = vX \Rightarrow \frac{dY}{dX} = v + X\frac{dv}{dX}$$

$$\therefore \quad v + X\frac{dv}{dX} = \frac{-(2 - v)}{(2v - 1)} \Rightarrow X\frac{dv}{dX} = -\left(\frac{2 - v}{2v - 1} + v\right)$$

$$\Rightarrow \quad X\frac{dv}{dX} = -\left(\frac{2 - v + 2v^2 - v}{2v - 1}\right)$$

$$\Rightarrow \quad \int \frac{(2v - 1)}{(2v^2 - 2v + 2)}\, dv = -\int \frac{dX}{X}$$

$$\Rightarrow \quad \frac{1}{2}\int \frac{(2v - 1)}{(v^2 - v + 1)}\, dv = -\int \frac{dX}{X}$$

$$\Rightarrow \quad \frac{1}{2}\log(v^2 - v + 1) = -\log X + \log c$$

$$\Rightarrow \quad \log\left(\sqrt{(v^2 - v + 1)}\; X\right) = \log c$$

$$\Rightarrow \quad X\sqrt{(v^2 - v + 1)} = c \Rightarrow \sqrt{(Y^2 - XY + X^2)} = c$$

$$\Rightarrow \quad (y + 1)^2 - (y + 1)(x + 1) + (x + 1)^2 = c^2$$

10. (c) : We have, $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\frac{y}{x} = 0$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x} - \operatorname{cosec}\frac{y}{x}$$

This is a homogeneous differential equation.

$$\text{Putting } y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = v - \operatorname{cosec}(v) \Rightarrow x\frac{dv}{dx} = -\operatorname{cosec} v$$

$$\Rightarrow \quad -\int \sin v\, dv = \int \frac{dx}{x} \Rightarrow \cos v = \log|x| + c$$

$$\Rightarrow \quad \cos\left(\frac{y}{x}\right) = \log|x| + c$$

It is given that $y = 0$ when $x = 1$

$\therefore \quad \cos(0) = \log(1) + c \Rightarrow 1 = 0 + c \Rightarrow c = 1$

$\Rightarrow \quad \cos\left(\frac{y}{x}\right) = \log|x| + 1$ is the required solution.

11. (b): We have, $\frac{dy}{dx} = \frac{1}{x\cos y + \sin 2y}$

$$\Rightarrow \frac{dx}{dy} = x\cos y + \sin 2y \Rightarrow \frac{dx}{dy} - (\cos y)x = (\sin 2y)$$

$$\therefore \quad \text{I. F.} = e^{-\int(\cos y)dy} = e^{-\sin y}$$

$$\therefore \quad xe^{-\sin y} = \int e^{-\sin y}\, 2\sin y\cos y\, dy \qquad \text{...(i)}$$

$$\text{Putting } \sin y = t \Rightarrow \cos y = \frac{dt}{dy}$$

$$\therefore \quad \int e^{-\sin y}\, 2\sin y\cos y\, dy = \int e^{-t}(2t)dt$$

$$= 2[-te^{-t} - e^{-t}] = -2e^{-t}(t + 1) = -2e^{-\sin y}(\sin y + 1)$$

$\therefore \quad$ From (i), we have $xe^{-\sin y} = -2e^{-\sin y}(\sin y + 1) + C$

$\Rightarrow \quad x = -2(\sin y + 1) + Ce^{\sin y}$

12. (b): We have, $\frac{dy}{dx} + \frac{\sqrt{1 - y^2}}{\sqrt{1 - x^2}} = 0$

$$\Rightarrow \quad \int \frac{dy}{\sqrt{1 - y^2}} = -\int \frac{dx}{\sqrt{1 - x^2}} + c$$

$$\Rightarrow \quad \sin^{-1} y = -\sin^{-1} x + c \Rightarrow \sin^{-1} y + \sin^{-1} x = c$$

13. (a) : We have, $(1 + t)\frac{dy}{dt} - ty = 1$

$$\Rightarrow \quad \frac{dy}{dt} - \frac{t}{1 + t}\cdot y = \frac{1}{1 + t},$$

which is a linear differential equation with

$$\text{I.F.} = e^{\int \frac{-t}{1+t}dt} = e^{-t + \log(1 + t)} = (1 + t)e^{-t}$$

The solution of differential equation is

$y(1 + t)e^{-t} = -e^{-t} + C$

$\Rightarrow \quad y(1 + t) = -1 + Ce^{t} \qquad \text{...(i)}$

It is given that $y(0) = -1 \Rightarrow C = 0$

Putting $C = 0$ in (i), we get

$$y(1 + t) = -1 \Rightarrow y = -\frac{1}{1 + t} \Rightarrow y(1) = -\frac{1}{2}$$

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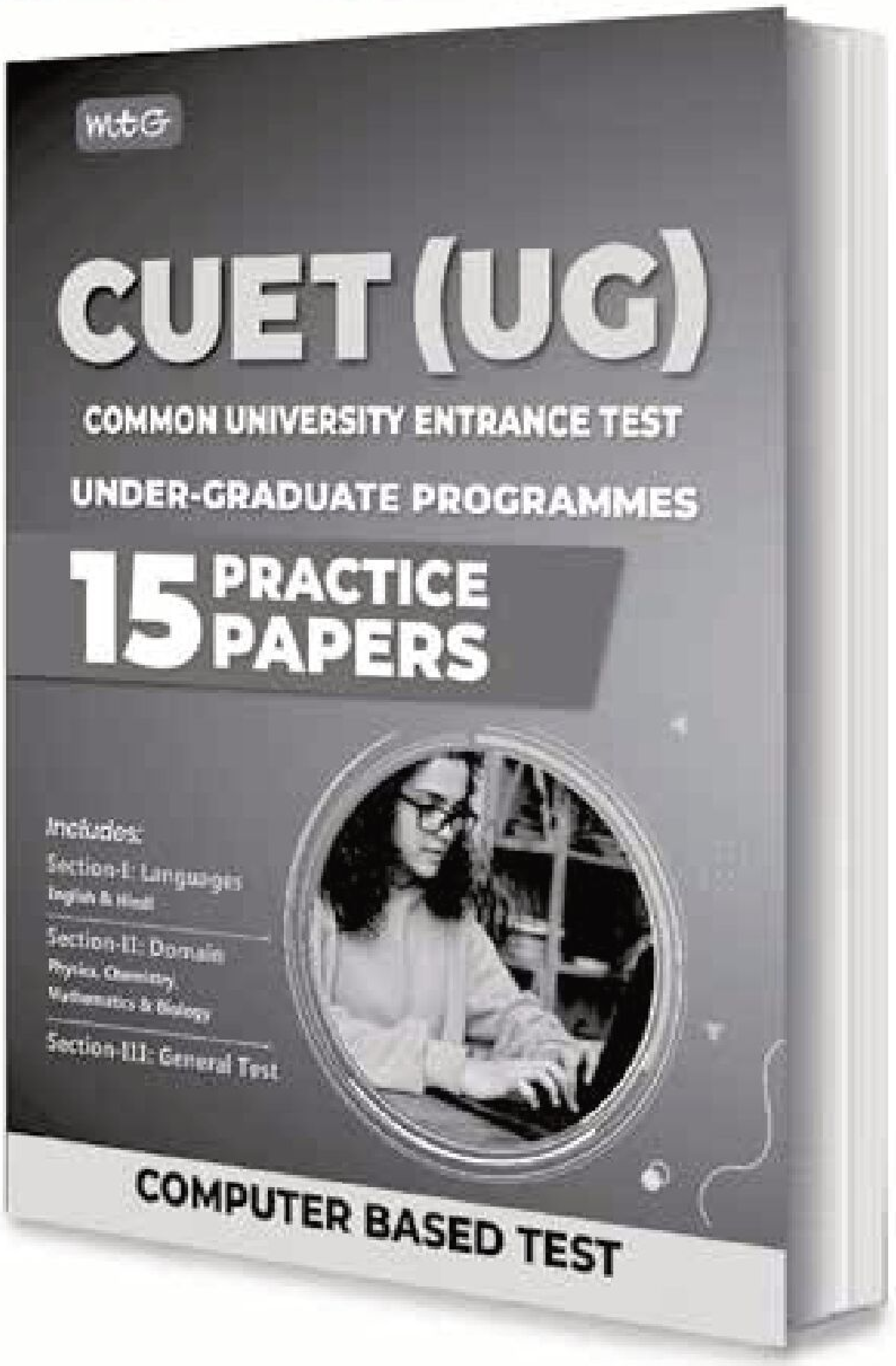

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14. (c) : We have $\frac{dy}{dx}=\frac{2x(\log x+1)}{\sin y+y\cos y}$

$$\Rightarrow \int(\sin y+y\cos y)dy=\int 2x(\log x+1)dx$$

$$\Rightarrow \int \sin y\ dy+y\sin y-\int 1\cdot\sin y\, dy$$

$$=2\left[\log x.\frac{x^2}{2}-\int\frac{1}{x}.\frac{x^2}{2}dx\right]+2\times\frac{x^2}{2}+c_1$$

$$\Rightarrow y\sin y=x^2\log x-\frac{x^2}{2}+x^2+c_1$$

$$\Rightarrow y\sin y=x^2\log x+\frac{x^2}{2}+c_1$$

It is given that $y = 0$, when $x = 1$

$$\Rightarrow 0=0+\frac{1}{2}+c_1\Rightarrow c_1=-\frac{1}{2}$$

$$\therefore\ 2y\sin y=2x^2\log x+x^2-1$$

15. (c) : We have, $(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$

$$\Rightarrow \frac{dx}{dy}+\frac{x}{1+y^2}=\frac{e^{\tan^{-1}y}}{1+y^2}$$

This is a linear differential equation with

$$\text{I.F.}=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$$

$\therefore$ Solution is given by $xe^{\tan^{-1}y}=\int\frac{e^{2\tan^{-1}y}}{1+y^2}dy$

$$\Rightarrow xe^{\tan^{-1}y}=\frac{1}{2}\int e^{2\tan^{-1}y}d(2\tan^{-1}y)$$

$$\Rightarrow xe^{\tan^{-1}y}=\frac{1}{2}e^{2\tan^{-1}y}+C$$

$\Rightarrow 2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+K$, where $K = 2C$

16. (a) : We have, $x\log x\frac{dy}{dx}+y=2\log x$

$$\Rightarrow \frac{dy}{dx}+\frac{y}{x\log x}=\frac{2}{x}$$

$$\text{I.F.}=e^{\int\frac{dx}{x\log x}}=e^{\log(\log x)}=\log x$$

So, $y(\log x)=\int\frac{2}{x}\log x\ dx=(\log x)^2+c$

17. (a) : We have, $xe^{y/x}-y+x\frac{dy}{dx}=0$

$$\Rightarrow \frac{dy}{dx}=\frac{y-xe^{y/x}}{x}$$

This is a homogeneous differential equation

Putting $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

$$\Rightarrow v+x\frac{dv}{dx}=\frac{vx-xe^v}{x}=v-e^v$$

$$\Rightarrow x\frac{dv}{dx}=-e^v\Rightarrow\int-\frac{1}{e^v}dv=\int\frac{dx}{x}$$

$$\Rightarrow e^{-v}=\log|x|+c\ \Rightarrow e^{-y/x}=\log|x|+c$$

It is given that $y(e) = 0$

$$\Rightarrow 1=(\log(e))+c\Rightarrow c=0$$

$$\Rightarrow e^{-y/x}=\log|x|\Rightarrow\frac{-y}{x}=\log(\log|x|)$$

$\Rightarrow y=-x\log(\log|x|),\ x\in(-\infty,-1)\cup(1,\infty)$ which is the required solution.

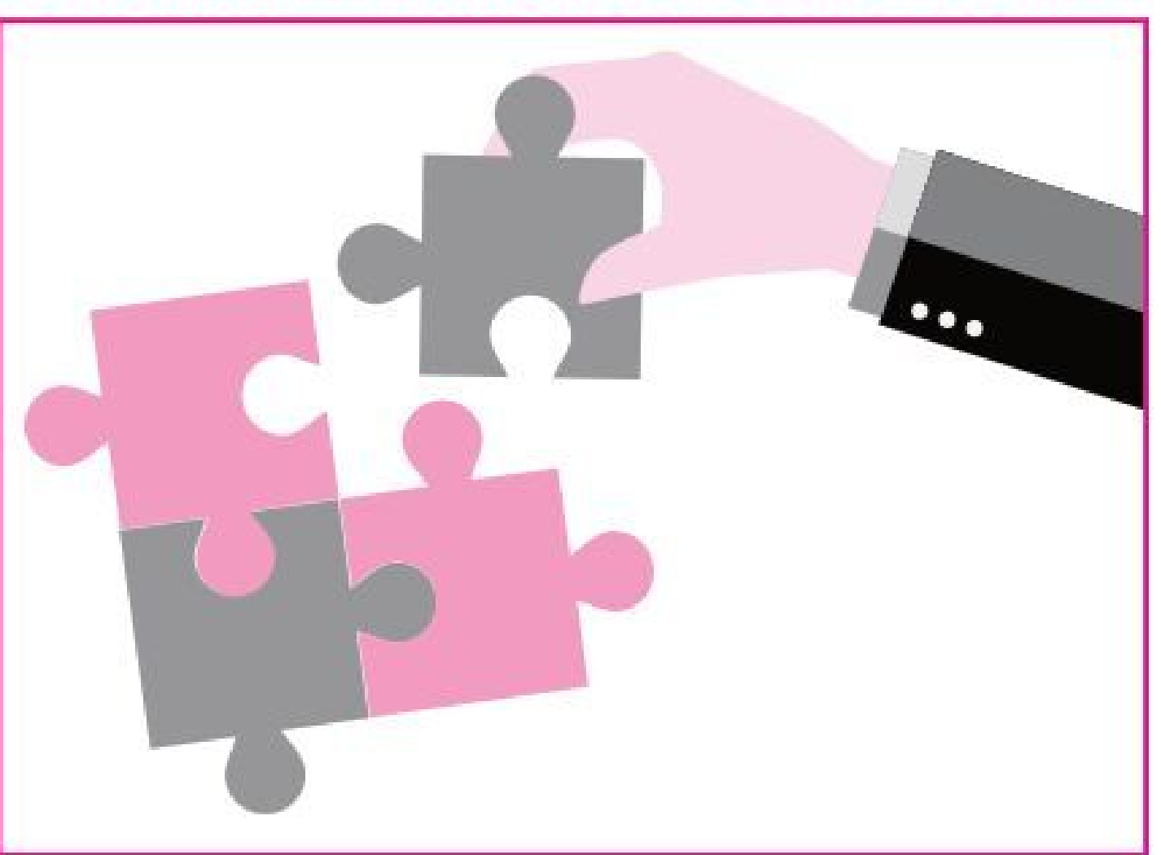
18. (d)

19. (c) : According to the question, $\frac{dy}{dx}=\frac{1}{2}\left(\frac{y}{x}\right)$

$$\Rightarrow \frac{dy}{y}=\frac{dx}{2x}\Rightarrow\log_e y=\frac{1}{2}\log_e x+\log_e c$$

$$\Rightarrow y^2=c^2x$$

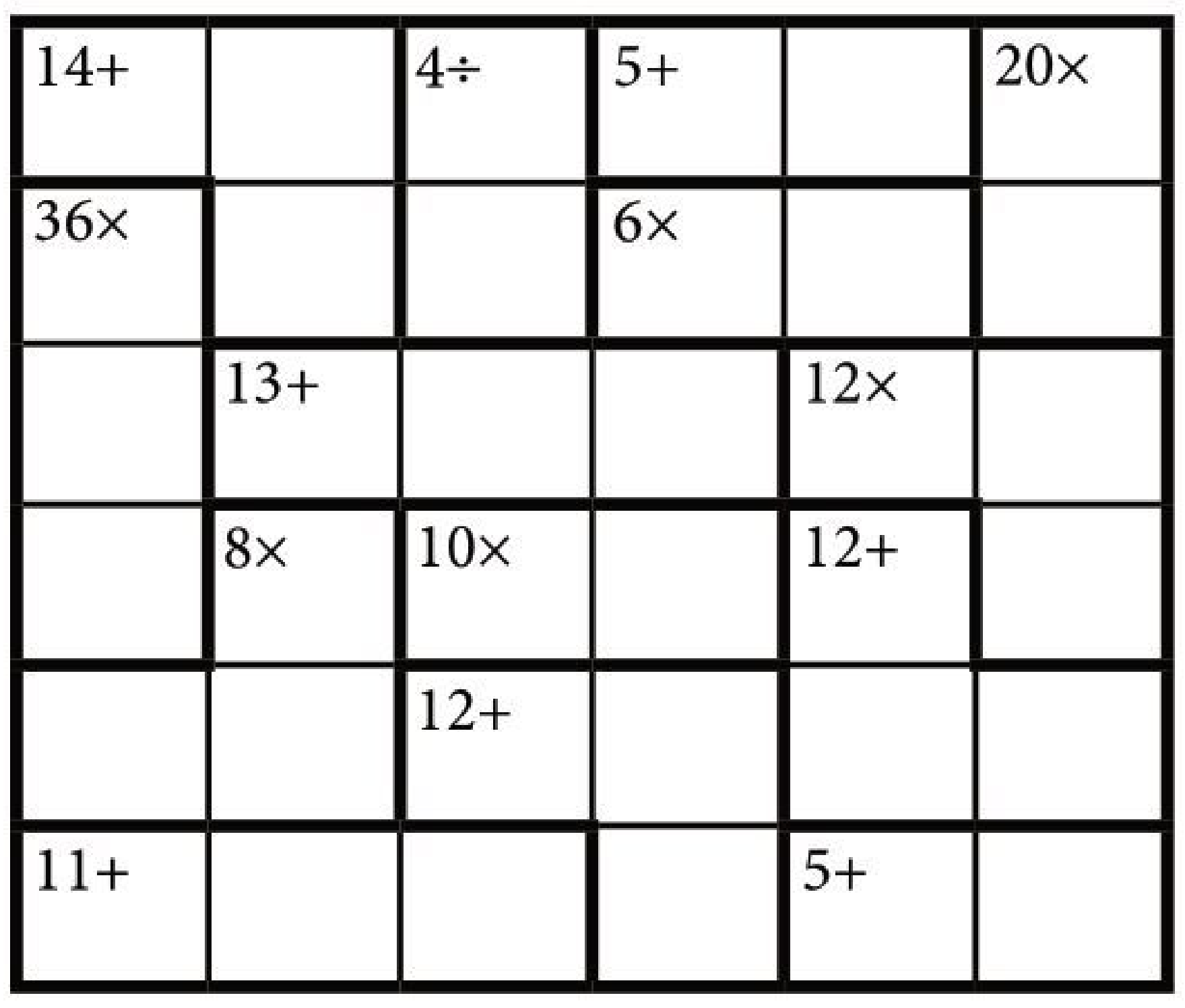
PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



14+		4÷	5+		20×
36×			6×		
	13+			12×	
	8×	10×		12+	
		12+			
11+				5+	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

20. (b): We have $x\frac{dy}{dx}+y=xe^x \Rightarrow \frac{dy}{dx}+\frac{y}{x}=e^x$

$\therefore$ I.F. $= e^{\int\frac{1}{x}dx}=x$

Hence, solution is, $y\cdot x=\int e^x\cdot x\,dx+c$

$\Rightarrow\ yx=e^x(x-1)+c$

We have, $y(1)=1\Rightarrow c=1$

$\therefore\ yx=e^x(x-1)$. Hence, $y(2)=e^2/2$

21. (c)

22. (c): $y=c_1e^{c_2x}$

Differentiating w.r.t. x, we get $y'=c_1c_2e^{c_2x}=c_2y$ ...(i)

Again differentiating w.r.t. x, we get $y''=c_2y'$ ...(ii)

From (i) and (ii) upon division, $\frac{y'}{y''}=\frac{y}{y'}\Rightarrow y''y=(y')^2$,

which is the desired differential equation of the family of curves.

23. (d): We have,

$$y^2\left(\frac{d^2y}{dx^2}\right)^2+x^2y^2-\sin x=-3x\left(\frac{dy}{dx}\right)^{1/3}$$

$$\Rightarrow\ \left(y^2\left(\frac{d^2y}{dx^2}\right)^2+x^2y^2-\sin x\right)^3=-27x^3\left(\frac{dy}{dx}\right)$$

So, order $=2=p$, degree $=6=q$

$\therefore\ p<q$

24. (b): We have, $y=x+2ax^2$

$\Rightarrow\ \frac{y-x}{x^2}=2a$ ...(i)

Differentiating (i) w.r.t. x, we get

$$\frac{x^2(y_1-1)-(y-x)\cdot 2x}{x^4}=0$$

$\Rightarrow\ x^2y_1-x^2-2xy+2x^2=0 \Rightarrow x^2y_1+x^2-2xy=0$

$\Rightarrow\ xy_1+x=2y \Rightarrow x(1+y_1)=2y$

25. (a): We have, $y=a\cos x+b\sin x$

$\Rightarrow\ \frac{dy}{dx}=-a\sin x+b\cos x$

$\Rightarrow\ \frac{d^2y}{dx^2}=-a\cos x-b\sin x\Rightarrow\frac{d^2y}{dx^2}=-y$

26. (d): Let $x+y=z\Rightarrow 1+\frac{dy}{dx}=\frac{dz}{dx}$

$\therefore$ The given differential equation can be written as

$\frac{dz}{dx}=1+\cos z$

$\Rightarrow\ \int dx=\int\frac{dz}{1+\cos z}\Rightarrow\int dx=\frac{1}{2}\int\sec^2\frac{z}{2}dz$

$\Rightarrow\ x=\tan\frac{z}{2}+c\Rightarrow x=\tan\left(\frac{x+y}{2}\right)+c$

Now, $y\left(\frac{\pi}{2}\right)=0\Rightarrow\frac{\pi}{2}=1+c\Rightarrow c=\frac{\pi}{2}-1$

$\therefore x=\tan\left(\frac{x+y}{2}\right)+\frac{\pi}{2}-1\Rightarrow y=-x+2\tan^{-1}\left(x+1-\frac{\pi}{2}\right)$

So, $y(0)=2\tan^{-1}\left(1-\frac{\pi}{2}\right)=-2\tan^{-1}\left(\frac{\pi}{2}-1\right)$.

27. (b): We have, $x(x^2-1)\frac{dy}{dx}=1\Rightarrow dy=\frac{1}{x(x^2-1)}dx$

Integrating both sides, we get

$$\int dy=\int\frac{1}{x(x^2-1)}dx=\int\left(-\frac{1}{x}+\frac{x}{x^2-1}\right)dx$$

$\therefore\ y=-\log|x|+\frac{1}{2}\log|x^2-1|+\log c$

It is given that $y=0$ when $x=2$

$\therefore\ 0=-\log|2|+\frac{1}{2}\log|3|+\log c$

$\Rightarrow\ \log c=\log 2-\frac{1}{2}\log 3$

$\therefore\ y=\frac{1}{2}\log|x^2-1|-\log|x|+\log 2-\frac{1}{2}\log 3$

28. (d): We have, $\frac{dy}{dx}+x^3y=x^7$ $\therefore$ I.F. $=e^{x^4/4}$

So, solution is, $ye^{x^4/4}=x^4e^{x^4/4}-4e^{x^4/4}+c$

$y=x^4-4+ce^{-x^4/4}$

When $y=4$, $x=0$, we have

$4=-4+c\Rightarrow c=8$

Now, $\left[\frac{dy}{dx}\right]_{x=\sqrt{2}}=8\sqrt{2}\left(1-\frac{2}{e}\right.$

29. (b): We have, $ye^ydx=(y^3+2xe^y)dy$

$\Rightarrow\ \frac{dx}{dy}=\frac{y^2}{e^y}+\frac{2x}{y}\ \Rightarrow\frac{dx}{dy}-\frac{2}{y}x=\frac{y^2}{e^y}$

I.F. $=e^{-2\log y}=\frac{1}{y^2}$

$\therefore$ General solution is, $x\frac{1}{y^2}=\int\frac{y^2}{e^y}\frac{1}{y^2}dy+c$

$\Rightarrow\ \frac{x}{y^2}=-e^{-y}+c$

At $x=0$, $y=1\Rightarrow c=e^{-1}$

$\Rightarrow\ x=-y^2(e^{-y}-e^{-1})$ $\therefore$ If $y=0$, then $x=0$

30. (a) : We have $\frac{dy}{dx}=\frac{y}{x^2} \Rightarrow \frac{dy}{y}=\frac{dx}{x^2}$

$\Rightarrow \quad \log y=-\frac{1}{x}+\log c \Rightarrow \frac{y}{c}=e^{\frac{-1}{x}} \Rightarrow y=c\cdot e^{\frac{-1}{x}}$

Comparing with $y = ae^{-1/x} + b$, we get $a \in R$, $b = 0$

31. (a) : The given differential equation can be rewritten as, $\frac{dx}{dy}-\frac{2x}{y}=y^2(y+1)e^y$,

which is a linear D.E.

$\therefore \quad \text{I.F.}=e^{-\int\frac{2}{y}dy}=\frac{1}{y^2}$

So, solution of given D.E. is, $\frac{x}{y^2}=\int(y+1)e^y dy+C$

$\Rightarrow \quad \frac{x}{y^2}=\int ye^y dy+\int e^y dy+C \Rightarrow \frac{x}{y^2}=y\cdot e^y+C$

$\Rightarrow \quad x = y^2 (y \cdot e^y + C)$ ...(i)

Now, $x(1) = 0 \Rightarrow 0 = 1(1 \cdot e + C) \Rightarrow C = -e$

So, (i) becomes, $x = y^2(y \cdot e^y - e)$

$\therefore \quad x(e) = e^2(e \cdot e^e - e) = e^3 (e^e - 1)$

32. (a) : The given differential equation can be written as $\frac{dy}{dx}=\sin(x+y)$

Putting $x + y = t \Rightarrow \frac{dy}{dx}=\frac{dt}{dx}-1$

$\therefore \frac{dt}{dx}-1=\sin t \Rightarrow \frac{dt}{dx}=1+\sin t \Rightarrow \frac{dt}{1+\sin t}=dx$

Integrating both sides, we get $\int\frac{dt}{1+\sin t}=\int dx$

$\Rightarrow \quad \int\frac{1-\sin t}{\cos^2 t}dt=x+c$

$\Rightarrow \quad \int(\sec^2 t-\sec t\ \tan t)\,dt=x+c$

$\Rightarrow \quad \tan t - \sec t = x + c \Rightarrow -\left(\frac{1-\sin t}{\cos t}\right)=x+c$

$\Rightarrow \quad \sin t - 1 = x\cos t + c\cos t$

Substituting the value of t, we get

$\sin(x + y) = x\cos(x + y) + c\cos(x + y) + 1$

33. (c) : We have, $\frac{dx}{x+2}=\frac{-dy}{y+2}$

Integrating both sides, we get

$\log(x + 2) = -\log(y + 2) + \log C$

At $x = 1$, $y = 2$, $\log 3 = -\log 4 + \log C \Rightarrow \log 12 = \log C$

$\therefore \quad \log(x + 2) + \log(y + 2) = \log 12$

$\Rightarrow \quad \log((x + 2) \times (y + 2)) = \log 12$

$\Rightarrow \quad xy + 2x + 2y + 4 = 12$

$\Rightarrow \quad xy + 2(x + y) = 8$, which is required particular solution.

34. (b)

35. (a) : We have, $\frac{dy}{dx}\cos x+y\sin x=1$

$\Rightarrow \quad \frac{dy}{dx}+y\tan x=\sec x$ ...(i)

This is a linear differential equation with I.F. given by

$e^{\int\tan x\,dx}=e^{\log\sec x}=\sec x$

So, Statement-2 is true.

$\therefore$ Solution is , $y \sec x = \tan x + C$ ...(ii)

It is given that $y = 1$ when $x = 0$.

$\therefore \quad 1 = C$

Putting $C = 1$ in (ii), we get $y \sec x = \tan x + 1$

$\Rightarrow \quad y=\sin x+\cos x=\sqrt{2}\sin\left(\frac{\pi}{4}+x\right)$

So, Statement - 1 is true and Statement - 2 is correct explanation for Statement - 1.

36. (c) : We have, $\sqrt{(1-x^2)}dy-\sqrt{(1-y^2)}dx=0$

$\Rightarrow \quad \int\frac{dy}{\sqrt{(1-y^2)}}=\int\frac{dx}{\sqrt{(1-x^2)}} \Rightarrow \sin^{-1}y=\sin^{-1}x+C$

At $x=\frac{1}{2}; y=1 \Rightarrow \frac{\pi}{2}=\frac{\pi}{6}+C \Rightarrow C=\frac{\pi}{3}$

$\therefore \quad \sin^{-1}y=\sin^{-1}x+\frac{\pi}{3}$. Thus, $y=\sin\left(\sin^{-1}x+\frac{\pi}{3}\right)$

$\Rightarrow \quad y=\sin(\sin^{-1}x)\cos\frac{\pi}{3}+\cos(\sin^{-1}x)\sin\frac{\pi}{3}$

$\Rightarrow \quad y=x\cdot\frac{1}{2}+\sqrt{(1-x^2)}\,\frac{\sqrt{3}}{2}$

Statement-I is true but statement-II is false.

37. (b): Let $c_1 + c_2 + c_3e^{c_4} = A$ (constant)

Then, $y = Ax$ ...(i)

$\Rightarrow \quad \frac{dy}{dx}=A \Rightarrow \frac{dy}{dx}=\frac{y}{x}$ [using(i)]

$\Rightarrow \quad x\frac{dy}{dx}=y$

Statement-I and statement-II are true but statement-II is not the correct explanation of statement-I.

38. (c) : We have, $\frac{dy}{dx}+xy=x^3$

It is a linear differential equation with I.F. = $e^{x^2/2}$

$\therefore \quad$ Solution is $ye^{x^2/2}=\int e^{x^2/2}x^3dx$

$\Rightarrow \quad y = x^2 - 2 + ce^{-x^2/2}$

Now, $y(0) = 1 \Rightarrow c = 3$

$\therefore \quad y = x^2 - 2 + 3e^{-x^2/2} \Rightarrow y(1)=-1+\frac{3}{\sqrt{e}}$

Statement-I is true but statement-II is false.

39. (d): P. We have, $\dfrac{dy}{dx}=2^{y-x} \Rightarrow \dfrac{dy}{2^y}=\dfrac{dx}{2^x}$

Integrating both sides, we get $\dfrac{-2^{-y}}{\log 2}=\dfrac{-2^{-x}}{\log 2}+C$

$\Rightarrow \quad -2^{-y}+2^{-x}=C\log 2=k(\text{say}) \quad \Rightarrow \quad 2^{-x}-2^{-y}=k$

Q. We have, $\dfrac{dy}{dx}=1+x+y^2+xy^2$

$\Rightarrow \dfrac{dy}{dx}=(1+y^2)(1+x) \Rightarrow \dfrac{dy}{1+y^2}=(1+x)dx$

Integrating both sides, we get $\tan^{-1} y=x+\dfrac{x^2}{2}+C$ ...(i)

When, $y = 0$, $x = 0$; $\tan^{-1}(0) = 0 + 0 + C \quad \therefore \quad C = 0$

Now, from (i), $\tan^{-1} y=x+\dfrac{x^2}{2} \Rightarrow y=\tan\left(x+\dfrac{x^2}{2}\right)$

R. We have, $x^2\dfrac{dy}{dx}=x^2+xy+y^2$

$$\Rightarrow \frac{dy}{dx}=1+\frac{y}{x}+\left(\frac{y^2}{x^2}\right) \qquad ...(ii)$$

Let $y=vx \Rightarrow \dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$

From (ii), we get $v+x\dfrac{dv}{dx}=1+v+v^2$

$\Rightarrow x\dfrac{dv}{dx}=1+v^2 \quad \Rightarrow \dfrac{dv}{1+v^2}=\dfrac{dx}{x}$

Integrating both sides, we get

$\tan^{-1}v = \log|x| + C \Rightarrow \tan^{-1}\left(\dfrac{y}{x}\right)=\log|x|+C$

S. We have, $2(y+3)-xy\dfrac{dy}{dx}=0$

$\Rightarrow 2(y+3)=xy\dfrac{dy}{dx} \Rightarrow 2\dfrac{dx}{x}=\left(\dfrac{y}{y+3}\right)dy$

$\Rightarrow 2\cdot\dfrac{dx}{x}=\left(1-\dfrac{3}{y+3}\right)dy$

Integrating both sides, we get

$2\log x = y - 3\log(y + 3) + C$ ...(i)

Given that $y(1) = -2$

$\therefore$ From (i), we have $2\log 1 = -2 - 3\log(-2 + 3) + C$

$\Rightarrow \quad 2\cdot 0 = -2 - 3\cdot 0 + C \quad \Rightarrow \quad C = 2$

On substituting the value of C in (i), we get

$2\log x = y - 3\log(y + 3) + 2$

$\Rightarrow \quad \log x^2 + \log(y + 3)^3 = (y + 2) \Rightarrow x^2(y + 3)^3 = e^{y+2}$

40. (b): P. $\{(y')^3 + y''\}^2 = ay'$

$\Rightarrow \quad (y')^6 + (y'')^2 + 2(y')^3y'' = ay'$

$\Rightarrow$ Order is '2' and degree is '2'.

Q. $3y=7xy'+\dfrac{5}{y'}$

$\Rightarrow \quad 3yy' = 7x(y')^2 + 5 \quad \Rightarrow$ Order is '1' and degree is '2'.

R. $y'' - 5y' + 6y = 0$

$\Rightarrow$ Order is '2' and degree is '1'

S. $(y' - 4x)^{3/2} = x + 5y'$

$\Rightarrow \quad (y')^3 - (4x)^3 - 3 \times 4x(y')(y' - 4x)$
$\qquad = x^2 + 25(y')^2 + 10xy'$

$\Rightarrow$ Order is '1' and degree is '3'.

41. (d): Given, at any time t, the thermometer reading be T°F and the outside temperature be S°F. Then, by Newton's law of cooling, we have

$$\frac{dT}{dt}\propto(T-S)\Rightarrow\frac{dT}{dt}=-\lambda(T-S)$$

42. (d): Since after 5 minutes, thermometer reads 60°F

$\therefore$ Value of $T(5) = 60$°F.

43. (a): Clearly from given information, value of $T(10)$ is 50°F.

44. (b): We have, $\dfrac{dT}{dt}=-\lambda(T-S)$

$\Rightarrow \dfrac{dT}{T-S}=-\lambda\,dt \Rightarrow \displaystyle\int\frac{1}{T-S}dT=-\lambda\int dt$

$\Rightarrow \quad \log(T - S) = -\lambda t + c$

45. (c): Since, at $t = 0$, $T = 80$°F

$\therefore \quad \log(80 - S) = 0 + c \Rightarrow c = \log(80 - S)$

46. (b): Here, P denotes the principal at any time t and the rate of interest be r% per annum compounded continuously, then according to the law given in the problem, we get $\dfrac{dP}{dt}=\dfrac{Pr}{100}$

47. (a)

48. (c): We have, $r = 5$, P_0 = ₹ 100 and $P = 2P_0$ = ₹ 200

Substituting these values in (2), we get

$\log 2=\dfrac{5}{100}t$

$\Rightarrow \quad t = 20\log_e 2 = 20 \times 0.6931$ years = 13.862 years

49. (d): We have,

P_0 = ₹ 100, $P = 2P_0$ = ₹ 200 and t = 10 years

Substituting these values in (2), we get

$\log 2=\dfrac{10r}{100} \Rightarrow r = 10\log 2 = 10 \times 0.6931 = 6.931$ %

50. (a): We have, P_0 = ₹ 1000, $r = 5$ and $t = 10$

Substituting these values in (2), we get

$\log\left(\dfrac{P}{1000}\right)=\dfrac{5\times 10}{100}\Rightarrow\log\left(\dfrac{P}{1000}\right)=0.5 \Rightarrow \dfrac{P}{1000}=e^{0.5}$

$\Rightarrow \quad P = 1000 \times 1.648$ = ₹ 1648

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Are you ready for Olympiads?

LEVEL 2

Exam on 12th February 2023

SOF
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CLASS XI

SYLLABUS*

Section – 1 : Verbal and Non-Verbal Reasoning.

Section – 2 : Sets, Relations and Functions, Principle of Mathematical Induction, Logarithms, Complex Numbers & Quadratic Equations, Linear Inequations, Sequences and Series, Trigonometry, Straight Lines, Conic Sections, Permutations and Combinations, Binomial Theorem, Statistics, Mathematical Reasoning, Limits and Derivatives, Probability, Introduction to 3-D Geometry.

Total Questions : 50 Time : 1 hr.

PATTERN & MARKING SCHEME

Section	(1) Logical Reasoning	(2) Mathematical Reasoning or Applied Mathematics	(3) Everyday Mathematics	(4) Achievers Section
No. of Questions	15	20	10	5
Marks per Ques.	1	1	1	3

OR

Section – 2 : Numbers, Quantification, Numerical Applications, Sets, Relations and Functions, Sequences and Series, Permutations and Combinations, Mathematical Reasoning, Limits, Continuity and Differentiability, Probability, Descriptive Statistics, Basics of Financial Mathematics, Straight Lines, Circles.

Section – 3 : The syllabus of this section will be based on the syllabus of Quantitative Aptitude.

Section – 4 : Sets, Relations and Functions, Sequences and Series, Permutations and Combinations, Limits and Derivatives, Straight Lines, Circles, Probability.

Practice Questions

1. Let $X = \{x \in R : |x+1| < 2\}$ and $Y = \{x \in R : |x-1| \geq 2\}$. Then which one of the following statements is not true?

A. $X - Y = (-1, 1)$ B. $Y - X = R - (-3, 1)$

C. $X \cap Y = (-3, -1]$ D. $X \cup Y = R - [1, 3)$

2. The range of the relation R given by $R = \{(x, y) : y = x + 6/x;$ where $x, y \in N$ and $x < 6\}$ is

A. $\{1, 2, 3\}$ and $\{3, 5\}$ B. $\{1, 2, 3\}$

C. $\{5, 7\}$ D. $\{3, 5, 7\}$

3. Let $f : R \to R$ be defined by $f(x) = \dfrac{x}{1+x^2}$, $x \in R$. Then the range of f is

A. $(-1, 1) - \{0\}$ B. $R - [-1, 1]$

C. $\left[-\frac{1}{2}, \frac{1}{2}\right]$ D. $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$

4. The ratio in which the point P whose y-coordinate is 3 divide the line segment joining the points $(3, -1, 4)$ and $(2, 5, -3)$ is

A. 2 : 3, internally B. 3 : 2, externally

C. 2 : 1, externally D. 2 : 1, internally

5. Let $\left(-2-\frac{1}{3}i\right)^3 = \dfrac{x+iy}{27}\,(i=\sqrt{-1})$, where x and y are real numbers, then $y - x$ equals

A. –91 B. –85 C. 85 D. 91

6. The set of admissible values of x such that $\dfrac{2x+3}{2x-9} < 0$ is

A. $\left(-\infty, \frac{-3}{2}\right) \cup \left(\frac{9}{2}, \infty\right)$ B. $(-\infty, 0) \cup \left(\frac{9}{2}, \infty\right)$

C. $\left(\frac{-3}{2}, 0\right)$ D. $\left(-\frac{3}{2}, \frac{9}{2}\right)$

7. If the minimum value of $f(x) = \dfrac{5x^2}{2} + \dfrac{\alpha}{x^5}, x > 0,$ is 14, then the value of α is equal to

A. 32 B. 64 C. 128 D. 256

8. If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha + \beta)$ and the quadrant in which $\alpha + \beta$ lies, respectively are

A. –1/7 and IVth quadrant

B. 7 and Ist quadrant

C. –7 and IVth quadrant

D. 1/7 and Ist quadrant

9. A line, with the slope greater than one, passes through the point $A(4, 3)$ and intersects the line $x - y - 2 = 0$ at the point B. If the length of the line segment AB is $\dfrac{\sqrt{29}}{3}$, then B also lies on the line

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A. $2x + y = 9$ B. $3x - 2y = 7$
C. $x + 2y = 6$ D. $2x - 3y = 3$

10. If the length of the latus rectum of a parabola, whose focus is (a, a) and the tangent at its vertex is $x + y = a$, is 16, then $|a|$ is equal to

A. $2\sqrt{2}$ B. $2\sqrt{3}$ C. $4\sqrt{2}$ D. 4

11. The letters of the word 'MANKIND' are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word 'MANKIND' is.

A. 1491 B. 1492
C. 1490 D. None of these

12. The remainder when $(2021)^{2022} + (2022)^{2021}$ is divided by 7 is

A. 0 B. 1 C. 2 D. 6

13. The mean of a set of 30 observations is 75. If each observation is multiplied by a non-zero number λ and then each of them is decreased by 25, their mean remains the same. Then λ is equal to

A. 2/3 B. 10/3 C. 1/3 D. 4/3

14. Let a be an integer such that $\lim\limits_{x \to 7} \dfrac{18-[1-x]}{[x-3a]}$ exists, where $[t]$ is greatest integer $\le t$. Then a is equal to

A. -6 B. -2 C. 2 D. 6

15. Two digits are selected randomly from the digits 1 to 9. If their sum is even, the probability that both the numbers are odd, is

A. 4/9 B. 5/9 C. 3/4 D. 5/8

ACHIEVERS SECTION

16. Consider all possible permutations of the letters of the word INDIANOIL. Then match the following:

	Column I		Column II
(P)	The number of permutations which contain the word 'INDIA' is	(1)	28866
(Q)	The number of permutations which contain the word 'OIL' is	(2)	1374
(R)	The number of permutations which contain neither word 'OIL' nor 'INDIA' is	(3)	1260
(S)	The number of permutations which contain at least one of the words 'OIL' or 'INDIA' is	(4)	120

	P	Q	R	S
A.	4	3	1	2
B.	4	3	2	1
C.	3	4	2	1
D.	1	2	3	4

17. Which of the following statements is incorrect.

A. $f(x) = \{x\}$ is a periodic function.

B. The fundamental period of $f(x) = 3\cos(6\pi x) + 4\sin(6\pi x)$ is 1/3.

C. The number of non-empty subsets of the set $\{a, b, c, d\}$ are 15.

D. The range of $y = \log_3(x^2 - 4x - 5)$ is R^+.

18. **Statement-1 :** The sum of n terms of two arithmetic progressions are in the ratio $(7n + 1) : (4n + 17)$, then the ratio of their n^{th} terms is 7 : 4.

Statement-2 : If $S_n = ax^2 + bx + c$, then $T_n = S_n - S_{n-1}$.

A. Both Statement-I and Statement-II are true.

B. Statement-I is true but Statement-II is false.

C. Statement-I is false but Statement-II is true.

D. Both Statement-I and Statement-II are false.

19. If a coin is tossed three times, consider the following events.

P : no head appear.

Q : exactly one head appear.

R : atleast two heads appear. Then,

Which of these statements are true ?

(i) P, Q and R are mutually exclusive events

(ii) P, Q and R are exhaustive events

(iii) P, Q, R are independent events

A. (i), (iii) B. (i), (ii)
C. (i), (ii), (iii) D. Only (iii)

20. Let the abscissae of the two points P and Q on a circle be the roots of $x^2 - 4x - 6 = 0$ and the ordinates of P and Q be the roots of $y^2 + 2y - 7 = 0$. If PQ is a diameter of the circle $x^2 + y^2 + 2ax + 2by + c = 0$, then the value of $(a + b - c)$ is

A. 12 B. 13 C. 14 D. 16

Darken your choice with HB Pencil

1.	Ⓐ Ⓑ Ⓒ Ⓓ	**5.**	Ⓐ Ⓑ Ⓒ Ⓓ	**9.**	Ⓐ Ⓑ Ⓒ Ⓓ	**13.**	Ⓐ Ⓑ Ⓒ Ⓓ	**17.**	Ⓐ Ⓑ Ⓒ Ⓓ
2.	Ⓐ Ⓑ Ⓒ Ⓓ	**6.**	Ⓐ Ⓑ Ⓒ Ⓓ	**10.**	Ⓐ Ⓑ Ⓒ Ⓓ	**14.**	Ⓐ Ⓑ Ⓒ Ⓓ	**18.**	Ⓐ Ⓑ Ⓒ Ⓓ
3.	Ⓐ Ⓑ Ⓒ Ⓓ	**7.**	Ⓐ Ⓑ Ⓒ Ⓓ	**11.**	Ⓐ Ⓑ Ⓒ Ⓓ	**15.**	Ⓐ Ⓑ Ⓒ Ⓓ	**19.**	Ⓐ Ⓑ Ⓒ Ⓓ
4.	Ⓐ Ⓑ Ⓒ Ⓓ	**8.**	Ⓐ Ⓑ Ⓒ Ⓓ	**12.**	Ⓐ Ⓑ Ⓒ Ⓓ	**16.**	Ⓐ Ⓑ Ⓒ Ⓓ	**20.**	Ⓐ Ⓑ Ⓒ Ⓓ

SOLUTIONS

1. (B) : We have

$$X=\{x\in R:|x+1|<2\}=\{x\in R:-3<x<1\}$$

$$y=\{x\in R:|x-1|\geq 2\}=\{x\in R:x\leq -1 \text{ or } x\geq 3\}$$

Now,

A. $X - Y = (-1, 1)$

B. $Y - X = (-\infty, -3] \cup [3, \infty) = R - (-3, 3)$

C. $X \cap Y = (-3, -1]$

D. $X \cup Y = (-\infty, 1) \cup [3, \infty) = R - [1, 3)$

2. (C) : Given, $R=\{(x,y):y=\left[x+\frac{6}{x}\right]$, where $x, y \in N$ and $x < 6\}$

For, $x = 1$, $y = 1 + \frac{6}{1} = 7$

$x = 2$, $y = 2 + \frac{6}{2} = 5$

$x = 3$, $y = 3 + \frac{6}{3} = 5$

$x = 4$, $y = 4 + \frac{6}{4}=\frac{11}{2}\notin N$

$x = 5$, $y = 5 + \frac{6}{5}=\frac{31}{5}\notin N$

$\therefore$ $R = \{(1, 7), (2, 5), (3, 5)\}$

Hence, Range of $R = \{5, 7\}$

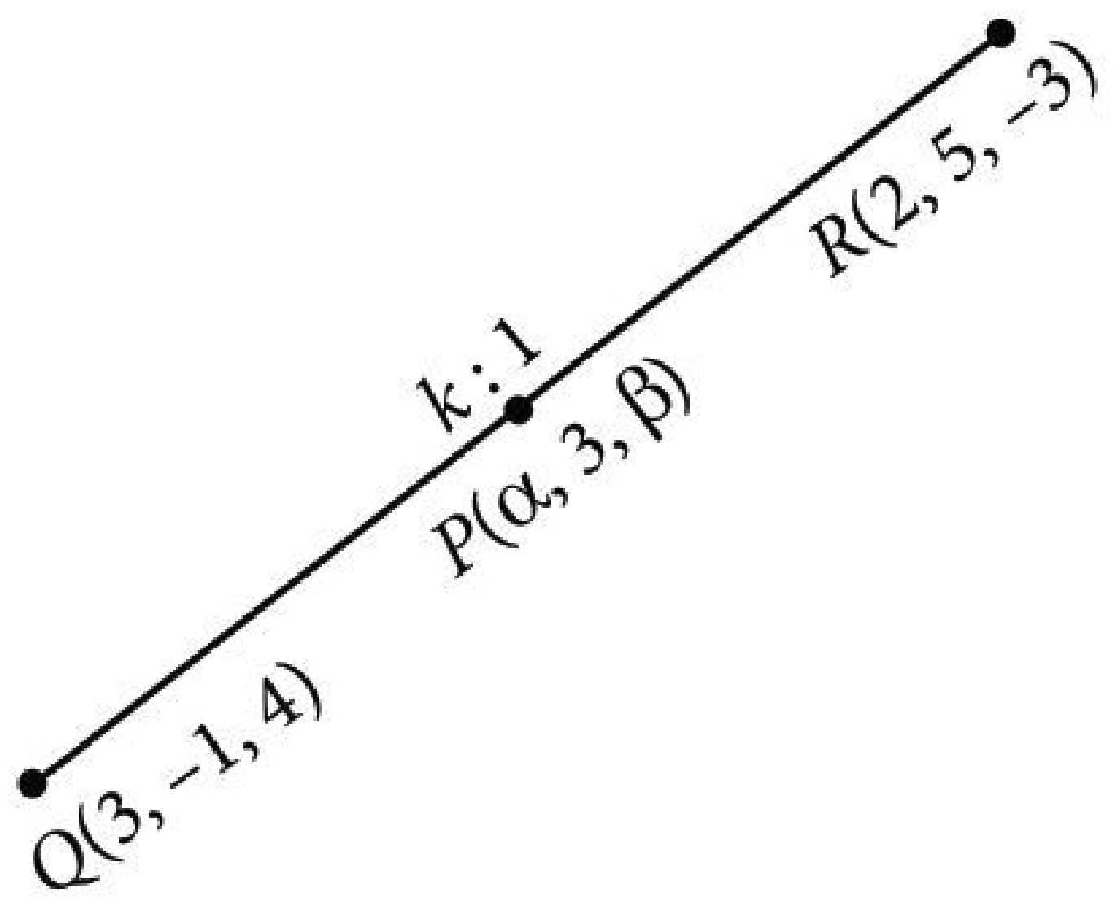
3. (C) : Here, $f(x)$ is an odd function and $f(0) = 0$

Now, when $x > 0$, $f(x) = \dfrac{1}{x+\frac{1}{x}} \in \left(0,\frac{1}{2}\right]$

When $x < 0$, $f(x) = \dfrac{1}{x+\frac{1}{x}} \in \left[-\frac{1}{2},0\right)$

$\therefore$ Range of $f(x)$ is $\left[-\frac{1}{2},\frac{1}{2}\right]$.

4. (D) : Let the point $P(\alpha, 3, \beta)$ divides the join of $Q(3, -1, 4)$ and $R(2, 5, -3)$ in the ratio $k : 1$.



$$\therefore\ (\alpha, 3, \beta)=\left(\frac{2k+3}{k+1},\frac{5k-1}{k+1},\frac{-3k+4}{k+1}\right)$$

Now, $3=\dfrac{5k-1}{k+1}$

$\Rightarrow\ 3k + 3 = 5k - 1 \ \Rightarrow\ k = 2$

Thus, point P divides line segment QR internally in the ratio 2 : 1.

5. (D) : Given, $\left(-2-\frac{1}{3}i\right)^3 = \dfrac{x+iy}{27}$

$\Rightarrow\ \dfrac{-(6+i)^3}{27}=\dfrac{x+iy}{27} \ \Rightarrow\ -(198 + 107i) = x + iy$

$\Rightarrow\ x = -198$ and $y = -107$ $\therefore$ $y - x = 198 - 107 = 91$

6. (D) : We have, $\dfrac{2x+3}{2x-9} < 0$

Two cases arises, either

$2x + 3 < 0$ and $2x - 9 > 0 \Rightarrow x < -3/2$ and $x > 9/2$ which is impossible.

Or $2x + 3 > 0$ and $2x - 9 < 0$

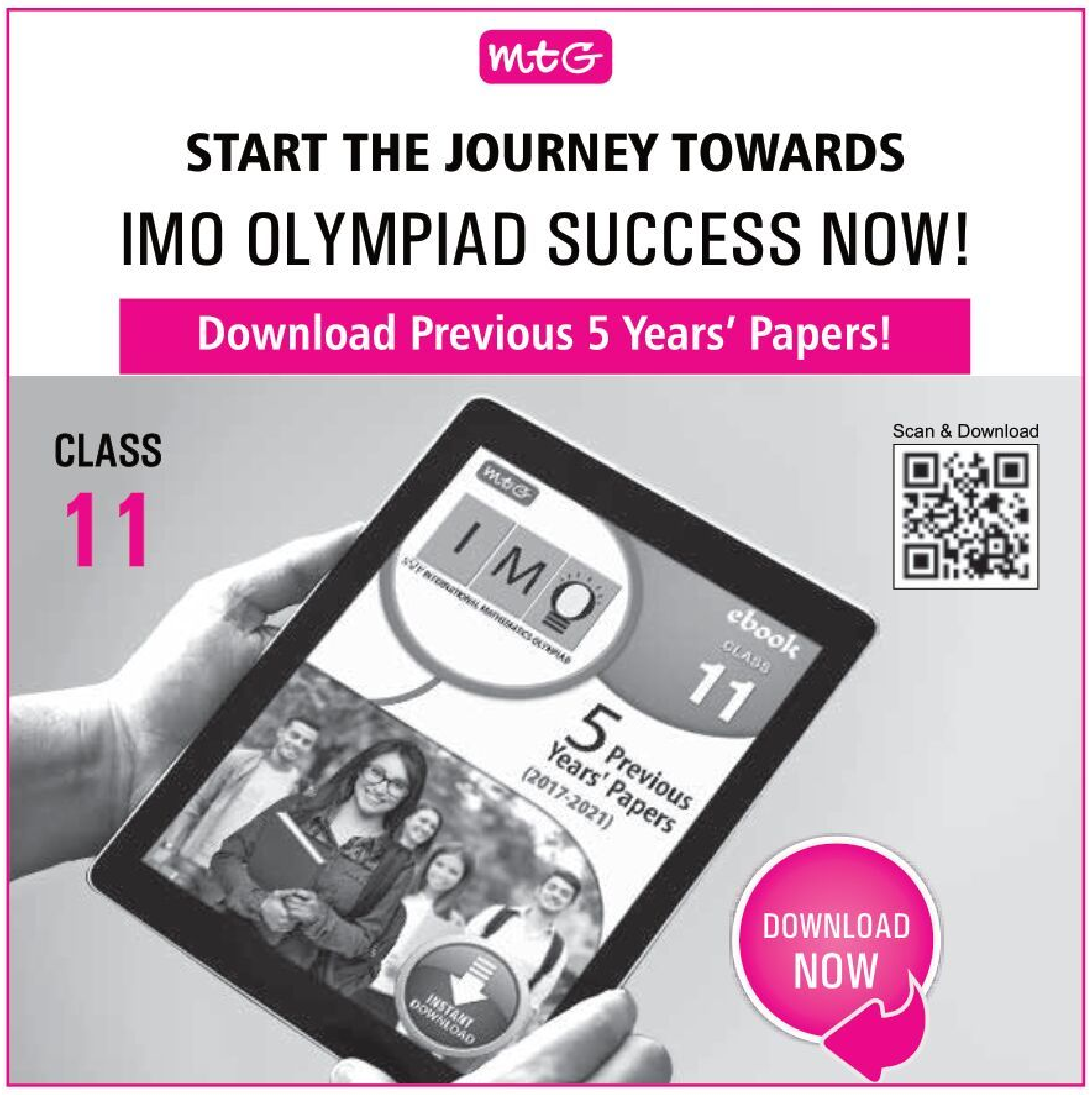
$\Rightarrow\ x > -3/2$ and $x < 9/2 \Rightarrow x \in (-3/2, 9/2)$

7. (C) : We have, $f(x)=\dfrac{5x^2}{2}+\dfrac{\alpha}{x^5}, x>0$

$$\frac{\frac{x^2}{2}+\frac{x^2}{2}+\frac{x^2}{2}+\frac{x^2}{2}+\frac{x^2}{2}+\frac{\alpha}{2x^5}+\frac{\alpha}{2x^5}}{7}$$

$$\geq\left[\left(\frac{x^2}{2}\right)^5\times\left(\frac{\alpha}{2x^5}\right)^2\right]^{\frac{1}{7}}$$

$$\Rightarrow\ \frac{f(x)}{7}\geq\left(\frac{\alpha^2}{2^7}\right)^{1/7} \ \Rightarrow\ f(x)\geq\frac{7\cdot\alpha^{2/7}}{2}$$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I M O

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LEVEL 2

12th February

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Since, $f_{\min}(x) = 14$

$\therefore\ \frac{7}{2}\alpha^{2/7} = 14 \ \Rightarrow\ \alpha^{2/7} = 4 \ \Rightarrow\ \alpha = 128$

8. (A) : We know that, $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$

It is given that, $\cot\alpha = 1$; $\alpha \in (\pi, 3\pi/2)$

$\tan\alpha = 1$

$\sec\beta = -\frac{5}{3} \qquad \beta \in \left(\frac{\pi}{2}, \pi\right);\ \tan\beta = -\frac{4}{3}$

$$\therefore\ \tan(\alpha+\beta) = \frac{1 - \frac{4}{3}}{1 + \frac{4}{3}} = -\frac{1}{7}$$

and $\alpha + \beta$ belongs to $\left(\frac{3\pi}{2}, 2\pi\right)$ that is IVth quadrant.

9. (C) : Let $B(\alpha, \alpha - 2)$, $A \equiv (4, 3)$

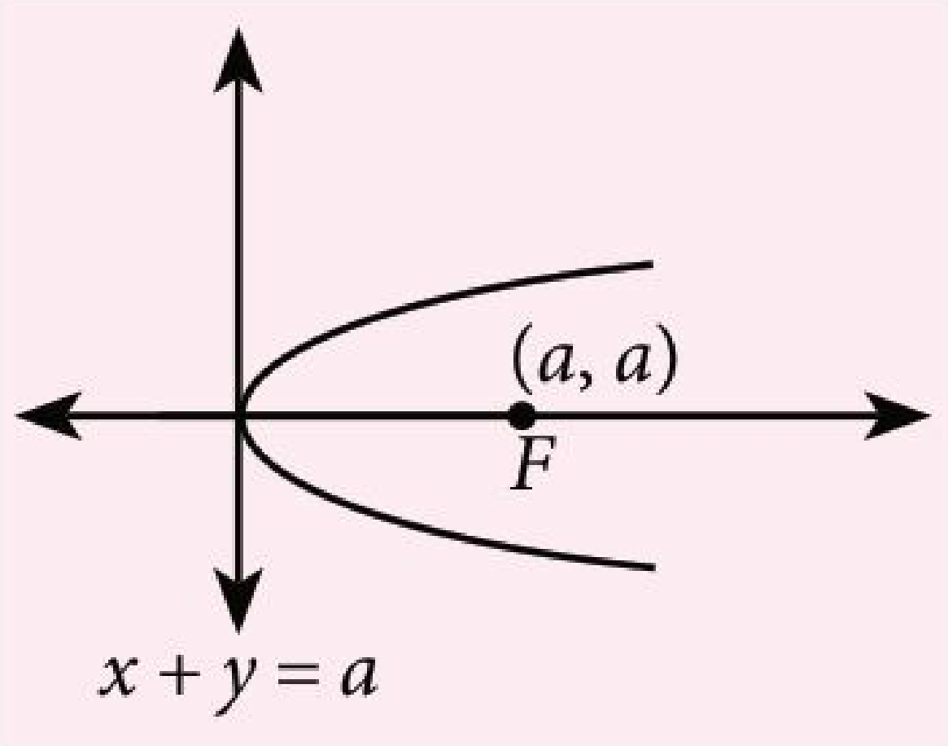
$AB = \frac{\sqrt{29}}{3}$

$\therefore\ \sqrt{(\alpha-4)^2 + (\alpha-2-3)^2} = \frac{\sqrt{29}}{3} \ \Rightarrow\ \alpha = \frac{17}{3}, \frac{10}{3}$

$\therefore\ B \equiv \left(\frac{17}{3}, \frac{11}{3}\right)$ or $\left(\frac{10}{3}, \frac{4}{3}\right)$

From the given option, the correct coordinates of B are $\left(\frac{10}{3}, \frac{4}{3}\right)$, which lies on $x + 2y = 6$.

10. (C) : Length of latus rectum of parabola is; 4 times the $\perp r$ distance from focus to tangent at vertex.



$\frac{|a + a - a|}{\sqrt{2}} = \frac{16}{4}$

$\Rightarrow\ |a| = 4\sqrt{2}$

11. (B) : A D I K M N

A ------------------ $\frac{6!}{2}$

D ------------------ $\frac{6!}{2}$

I ------------------ $\frac{6!}{2}$

K ------------------ $\frac{6!}{2}$

M A D ------------- $\frac{4!}{2}$

M A I -------------- $\frac{4!}{2}$

M A K ------------- $\frac{4!}{2}$

M A N D ---------- 3!

M A N I ----------- 3!

M A N K D -------- 2!

M A N K I D ------- 1!

M A N K I N D ---- 1

$\therefore$ Serial number of the word MANKIND = 1492

12. (A) : We have, $(2021)^{2022} + (2022)^{2021}$

We can write this as $(2023 - 2)^{2022} + (2023 - 1)^{2021}$

= Multiple of 2023 + 2^{2022} + Multiple of 2023 – 1

$= 7\lambda + 2^{2022} - 1 = 7\lambda + 8^{674} - 1 = 7\lambda + (7+1)^{674} - 1$

$= 7\lambda$ + (Multiple of 7) + 1 – 1 $= 7\lambda + 7\beta = 7(\lambda + \beta)$

Since, number is multiple of 7 so, remainder is 0.

13. (D) : Let $x_1, x_2, \ldots., x_{30}$ be 30 observations.

Given, $\frac{x_1 + x_2 + \ldots. + x_{30}}{30} = 75$

Each observation is multiplied by λ and decreased by 25.

$\therefore$ New observations are $\lambda x_1 - 25, \lambda x_2 - 25, \ldots, \lambda x_{30} - 25$.

$$\text{Now, new mean} = \frac{(\lambda x_1 - 25) + (\lambda x_2 - 25) + \ldots + (\lambda x_{30} - 25)}{30}$$

$$= \frac{\lambda(x_1 + x_2 + \ldots. + x_{30})}{30} - \left(\frac{25 \times 30}{30}\right) = \lambda(75) - 25$$

According to question, $\lambda(75) - 25 = 75$

$\Rightarrow \lambda(75) = 100 \ \Rightarrow\ \lambda = \frac{100}{75}\ \frac{4}{3}.$

14. (A) : $\lim\limits_{x\to 7} \frac{18 - [1-x]}{[x - 3a]}$

Limit exists when L.H.L. = R.H.L.

$\text{L.H.L.} = \lim\limits_{x\to 7^-} \frac{18 - [1-x]}{[x-3a]}$

$(x \to 7^- \Rightarrow x = 7 - h, h \to 0^+)$

$$= \lim_{h\to 0} \frac{18 - [-6+h]}{[7 - h - 3a]} = \frac{24}{7 - 3a + [-h]} = \frac{24}{6 - 3a}$$

$\text{R.H.L.} = \lim\limits_{x\to 7^+} \frac{18 - [1-x]}{[x-3a]} \qquad \left(\begin{array}{c} x \to 7^+ \\ \Rightarrow\ x = 7 + h, h \to 0 \end{array}\right)$

$$= \lim_{h\to 0} \frac{18 - [-6-h]}{[7 + h - 3a]} = \frac{25}{7 - 3a}$$

So, $\frac{24}{6 - 3a} = \frac{25}{7 - 3a} \ \Rightarrow\ a = -6$

15. (D) : Sum is even provided both are even or both are odd.

Let A denotes even, B denotes odd.

$n(S) = n(A \text{ or } B) = n(A) + n(B)$

$= n(\text{both odd}) + n(\text{both even}) = {}^5C_2 + {}^4C_2$

$\Rightarrow\ n(S) = 16$

$\therefore$ Required probability $= \dfrac{n(\text{both odd})}{n(S)} = \dfrac{10}{16} = \dfrac{5}{8}$

16. (A) : P. To find the number of permutations which contain the word 'INDIA' , we need to arrange (INDIA), N, O, I and L.

Number of ways = 5! = 120

Q. To find the number of permutations which contain the word 'OIL' , we need to arrange (OIL), I, N, D, I, A and N.

Number of ways = 7! / (2! 2!) = 1260.

R. We need to find the number of permutations which contain neither word 'OIL' nor 'INDIA'.

Total number of permutation of all letters (III NN D A O L) = 9!/(3! 2!) = 30240.

Number of words which contain both words 'INDIA' and 'OIL' = 3!

$\therefore$ Required number of ways $= 9!/3!\,2! - 5! - 7!/(2!\,2!) + 3!$
$= 28866$

S. We need to find the number of permutations which contain at least one of the words 'OIL' or 'INDIA'.

Required number of ways = 5! + 7!/2! 2! – 3! = 1374.

17. (D) : A. $f(x) = x - [x]$ is a periodic function.

B. $f(x) = 5\sin(6\pi x + \alpha)$

$\therefore$ The fundamental period is $\dfrac{2\pi}{6\pi} = \dfrac{1}{3}$.

C. Let $A = \{a, b, c, d\}$ $\therefore$ $n(A) = 4$

$\therefore$ Number of subsets of $A = 2^4 = 16$, out of which only one set is empty set because empty set is subset of every set.

$\therefore$ Number of non-empty subsets of $A = 2^4 - 1 = 15$.

D. $f(x) = \log_3(x^2 - 4x - 5)$
$= \log_3((x - 2)^2 - 9)$

For $f(x)$ to get defined, $0 < (x - 2)^2 - 9 < \infty$

$\Rightarrow\ -\infty < \log_3\left((x-2)^2 - 9\right) < \infty \Rightarrow -\infty < f(x) < \infty$

Hence, the range is R.

18. (C) : $\because \dfrac{S_n}{S'_n} = \dfrac{(7n+1)}{(4n+17)} = \dfrac{n(7n+1)}{n(4n+17)}$

$\therefore$ $S_n = (7n^2 + n)\lambda$, $S'_n = (4n^2 + 17n)\lambda$

Then, $\dfrac{T_n}{T'_n} = \dfrac{S_n - S_{n-1}}{S'_n - S'_{n-1}} = \dfrac{7(2n-1)+1}{4(2n-1)+17} = \dfrac{14n-6}{8n+13}$

$\Rightarrow$ $T_n : T'_n = (14n - 6) : (8n + 13)$

19. (B) : The sample space of the experiment is

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

Now, $A = \{TTT\}$, $B = \{HTT, THT, TTH\}$ and

$C = \{HHT, HTH, THH, HHH\}$

$A \cup B \cup C = \{TTT, HTT, THT, TTH, HHT, HTH, THH, HHH\} = S$

Therefore, A, B and C are mutually exhaustive events.

Also $A \cap B = \phi$, $A \cap C = \phi$ and $B \cap C = \phi$

Therefore, the events are pair-wise disjoint, *i.e.*, they are mutually exclusive.

Hence, A, B and C form a set of mutually exclusive and exhaustive events.

$\therefore$ A, B, C are not independent events.

20. (A) : Required equation of circle is

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

where (x_1, y_1) and (x_2, y_2) are end points of diameter.

$\Rightarrow$ $2a = -4$, $2b = 2$ and $c = -6 - 7$

i.e., $a = -2$, $b = 1$, and $c = -13$

$\Rightarrow$ $a + b - c = -2 + 1 + 13 = 12$

For other sections/subjects please refer to Physics For You, Chemistry Today and Biology Today

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PUBLIC NOTICE

WBJEE 2023

The West Bengal Joint Entrance Examinations Board (WBJEEB) will conduct OMR based Common Entrance Examination WBJEE-2023 for admission in the academic session 2023-24 for Undergraduate Courses in Engineering/Technology, Pharmacy and Architecture of different Universities, Government Colleges as well as Self-Financing Engineering/Technological Institutes in the State of West Bengal.

Schedule of WBJEE-2023

Date of Examination	Paper/Subject	Time schedule
30/04/2023 (Sunday) (Tentative and may be changed in extra ordinary circumstances)	Paper-I (Mathematics)	11:00 a.m. to 1.00 p.m.
	Paper-II (Physics & Chemistry)	2:00 p.m. to 4:00 p.m.

The test will be held once in a year and there shall be no further examination under any circumstances for those who are unable to appear on the date and time mentioned above.

Pattern of Question Papers in WBJEE-2023

All questions will be **Multiple-Choice Questions (MCQ)**, with four options against each question. There will be three categories of questions in each subject. The number of questions as well as maximum marks for each are given in the following table:

Subject	**Category-1** Each Q carries 1 mark (–ve marks = –1/4)	**Category-2** Each Q carries 2 marks (–ve marks = –1/2)	**Category-3** Each Q carries 2 marks (No –ve marks)	Total Number of Questions	**Total Marks**
	No. of Q	No. of Q	No. of Q		
Mathematics	50	15	10	75	100
Physics	30	5	5	40	50
Chemistry	30	5	5	40	50

For more information, please visit the official website www.wbjeeb.nic.in

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LEVEL 2

Exam on 12th February 2023

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CLASS XII

SYLLABUS*

Section – 1 : Verbal and Non-Verbal Reasoning.

Section – 2 : Relations and Functions, Inverse Trigonometric Functions, Matrices and Determinants, Continuity and Differentiability, Application of Derivatives, Integrals, Application of Integrals, Differential Equations, Vector Algebra, Three Dimensional Geometry, Probability, Linear Programming.

OR

Section – 2 : Numbers, Quantification, Numerical Applications, Solutions of Simultaneous Linear Equations, Matrices, Determinants, Application of Derivatives, Integration, Application of Integrations, Differential Equations, Probability, Inferential Statistics, Index numbers, Time-based data, Financial Mathematics, Linear Programming.

Section – 3 : The syllabus of this section will be based on the syllabus of Quantitative Aptitude.

Section – 4 : Matrices, Determinants, Application of Derivatives, Integration, Application of Integrations, Differential Equations, Linear Programming, Probability.

Total Questions : 50 Time : 1 hr.

PATTERN & MARKING SCHEME

Section	(1) Logical Reasoning	(2) Mathematical Reasoning or Applied Mathematics	(3) Everyday Mathematics	(4) Achievers Section
No. of Questions	15	20	10	5
Marks per Ques.	1	1	1	3

Practice Questions

1. Let R_1 and R_2 be two relations defined on R by $aR_1b \Leftrightarrow ab \geq 0$ and $aR_2b \Leftrightarrow a \geq b$. Then,

A. R_1 is an equivalence relation but not R_2

B. R_2 is an equivalence relation but not R_1

C. both R_1 and R_2 are equivalence relations

D. neither R_1 nor R_2 is an equivalence relation

2. Let $f(x)=\dfrac{\sin^{101} x}{\left[\dfrac{x}{\pi}\right]+\dfrac{1}{2}}$, where $[x]$ denotes the integral part of x, is

A. an odd function

B. an even function

C. neither odd nor even function

D. none of these

3. The domain of the function

$f(x) = \sin^{-1}\left(\dfrac{x^2-3x+2}{x^2+2x+7}\right)$ is

A. $[1, \infty)$ B. $[-1, 2]$

C. $[-1, \infty)$ D. $(-\infty, 2]$

4. Let A and B be any two 3×3 symmetric and skew symmetric matrices respectively. Then which of the following is not true?

A. $A^4 - B^4$ is a symmetric matrix

B. $AB - BA$ is a symmetric matrix

C. $B^5 - A^5$ is a skew-symmetric matrix

D. $AB + BA$ is a skew-symmetric matrix

5. If $A=\begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$ and $\det\left(A^2-\dfrac{1}{2}I\right)=0$, then a possible value of α is

A. $\dfrac{\pi}{3}$ B. $\dfrac{\pi}{6}$

C. $\dfrac{\pi}{2}$ D. $\dfrac{\pi}{4}$

6. The number of points, where the function $f: R \rightarrow R$, $f(x) = |x-1| \cos|x-2| \sin|x-1| + (x-3)\,|x^2-5x+4|$, is not differentiable, is

A. 1 B. 2 C. 3 D. 4

7. The function $f(x) = xe^{x(1-x)}$, $x \in R$, is

A. increasing in $\left(-\dfrac{1}{2}, 1\right)$

B. decreasing in $\left(\dfrac{1}{2}, 2\right)$

C. increasing in $\left(-1, -\dfrac{1}{2}\right)$

D. decreasing in $\left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$

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8. The value of integral $\int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x}dx$ is

A. $\pi\sqrt{2}$ B. $2\pi(\sqrt{2}-1)$

C. $\frac{\pi}{2}(\sqrt{2}+1)$ D. $\pi(\sqrt{2}-1)$

9. The area (in sq. units) of the region bounded by $y - x = 2$ and $x^2 = y$ is equal to

A. 2/3 B. 4/3 C. 9/2 D. 16/3

10. If $\frac{dy}{dx} + 2y\tan x = \sin x, 0 < x < \frac{\pi}{2}$ and $y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of $y(x)$ is

A. 1/8 B. 3/4 C. 1/4 D. 3/8

11. Let $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and let $\vec{b}$ be a vector such that $\vec{a}\times\vec{b} = 2\hat{i} - \hat{k}$ and $\vec{a}\cdot\vec{b} = 3$. Then the projection of $\vec{b}$ on the vector $\vec{a} - \vec{b}$ is

A. $\frac{2}{\sqrt{21}}$ B. $2\sqrt{\frac{3}{7}}$ C. $\frac{2}{3}\sqrt{\frac{7}{3}}$ D. $\frac{2}{3}$

12. If the shortest distance between the lines $\vec{r} = (-\hat{i} + 3\hat{k}) + \lambda(\hat{i} - a\hat{j})$ and $\vec{r} = (-\hat{j} + 2\hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$ is $\sqrt{\frac{2}{3}}$, then the integral value of a is equal to

A. 2 B. 1 C. 3 D. 0

13. A fair die is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability $P(X \geq 5 \mid X > 2)$ is

A. $\frac{25}{36}$ B. $\frac{5}{6}$

C. $\frac{11}{36}$ D. $\frac{125}{216}$

14. The objective function $z = 4x_1 + 5x_2$, subject to $2x_1 + x_2 \geq 7$, $2x_1 + 3x_2 \leq 15$, $x_2 \leq 3$, $x_1, x_2 \geq 0$ has minimum value at the point

A. On x-axis B. On y-axis

C. At the origin

D. On the line parallel to x-axis

15. Find the domain of the continuity of the function $f(x) = \sin^{-1}x - [x]$, where $[x]$ denotes the greatest integer less than or equal to x.

A. $(-1, 1)$ B. $[-1, 1]$

C. $(-1, 0) \cup (0, 1)$ D. $(0, 1)$

ACHIEVERS SECTION

16. Read the given statement carefully and state 'T' for true and 'F' for false.

(i) If for a matrix A, $A^2 = A$ and $B = I - A$, then $AB + BA + I - (I - A)^2 = A$.

(ii) If $f''(x) < 0$ and $g(x) = f(x) + f(1 - x)$ for $0 < x < 1$, then $g(x)$ decreases in $\left(\frac{1}{2}, 1\right)$.

(iii) AB is skew symmetric matrix if matrix multiplication of A and B is commutative.

	(i)	(ii)	(iii)
A.	T	F	T
B.	T	T	F
C.	F	F	T
D.	F	T	F

17. Fill in the blanks and select the correct option.

(i) General solution of differential equation $dy/dx = xe^{x-y}$ is __**P**__.

(ii) The solution of differential equation $dy/dx = (x + y)^2$ is __**Q**__.

(iii) The degree of the differential equation $\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = 2x^2\log\left(\frac{d^2y}{dx^2}\right)$ is __**R**__.

	P	**Q**	**R**
A.	$e^y + e^x(x - 1) = C$	$\tan^{-1}(x - y) = x + c$	-1
B.	$e^y - e^x(x - 1) = C$	$\tan^{-1}(x + y) = x + c$	not defined
C.	$e^y - e^x(x + 1) = C$	$\tan^{-1}(x + y) = y + c$	not defined
D.	$e^y + e^x(x + 1) = C$	$\tan^{-1}(x - y) = y + c$	2

18. Read the following statements carefully and select and correct option.

Statement-I : Given E_i $(i = 1, 2, ..., n)$ are n independent events such that, $P(\overline{E}_i) = i/i{+}1, 1 \leq i \leq n$, the probability that none of the n events occur is $1/n$.

Statement-II : Probability of simultaneous occurrence independent events is equal to the product of the probabilities of these events.

A. Statement-1 is true, Statement-2 is true.

B. Statement-1 is false, Statement-2 is false.

C. Statement-1 is true, Statement-2 is false.

D. Statement-1 is false, Statement-2 is true.

19. Match Column I with Column II and select the correct option.

Column I		Column II	
(P)	If $\int \frac{2^x}{\sqrt{1-4^x}}dx = \frac{k}{\log 2}\sin^{-1}2^x$ $+C$, then k is equal to	(1)	2
(Q)	If $\int \frac{(\sqrt{x})^5}{(\sqrt{x})^7+x^6}dx = a\log\frac{x^k}{x^k+1}+C$, then $5a+2k$ is equal to	(2)	1
(R)	If $\int \frac{x^4+1}{x(x^2+1)^2}dx = k\log\lvert x\rvert+\frac{m}{x^2+1}$ then $(k+m)$ is	(3)	3
(S)	If $\int \frac{dx}{5+4\cos x} = k\tan^{-1}\left(m\tan\frac{x}{2}\right)+C$, then $3(k+m)=$	(4)	7

A. P → 1, Q → 3, R → 2, S → 4

B. P → 2, Q → 4, R → 1, S → 3

C. P → 3, Q → 2, R → 4, S → 1

D. P → 4, Q → 1, R → 3, S → 2

20. Find the intervals in which the function given by $f(x) = \sin x + \cos x, 0 \le x \le 2\pi$ is increasing.

A. $\left[0, \frac{\pi}{4}\right)$

B. $\left(\frac{5\pi}{4}, 2\pi\right)$

C. $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

D. Both A and B

Darken your choice with HB Pencil

1.	Ⓐ Ⓑ Ⓒ Ⓓ	**5.**	Ⓐ Ⓑ Ⓒ Ⓓ	**9.**	Ⓐ Ⓑ Ⓒ Ⓓ	**13.**	Ⓐ Ⓑ Ⓒ Ⓓ	**17.**	Ⓐ Ⓑ Ⓒ Ⓓ
2.	Ⓐ Ⓑ Ⓒ Ⓓ	**6.**	Ⓐ Ⓑ Ⓒ Ⓓ	**10.**	Ⓐ Ⓑ Ⓒ Ⓓ	**14.**	Ⓐ Ⓑ Ⓒ Ⓓ	**18.**	Ⓐ Ⓑ Ⓒ Ⓓ
3.	Ⓐ Ⓑ Ⓒ Ⓓ	**7.**	Ⓐ Ⓑ Ⓒ Ⓓ	**11.**	Ⓐ Ⓑ Ⓒ Ⓓ	**15.**	Ⓐ Ⓑ Ⓒ Ⓓ	**19.**	Ⓐ Ⓑ Ⓒ Ⓓ
4.	Ⓐ Ⓑ Ⓒ Ⓓ	**8.**	Ⓐ Ⓑ Ⓒ Ⓓ	**12.**	Ⓐ Ⓑ Ⓒ Ⓓ	**16.**	Ⓐ Ⓑ Ⓒ Ⓓ	**20.**	Ⓐ Ⓑ Ⓒ Ⓓ

SOLUTIONS

1. (D) : $R_1 = \{(a, b) : ab \ge 0\}$

Case I : $(a, a) \in R_1$ because $a^2 \ge 0$

So, reflexive.

Case II : If $(a, b) \in R_1 \Rightarrow ab \ge 0$

So, $(b, a) \in R_1$ because $ba \ge 0$

So, R_1 is symmetric.

Case III : If $(a, b) \in R_1$ and $(b, c) \in R_1$ then $ab \ge 0$, $bc \ge 0$ but $ac \ngeq 0$

Let us consider an example $a = 2, b = 0, c = -2$

$a \cdot b \ge 0, b \cdot c \ge 0$ but $a \cdot c = -4 \ngeq 0]$

$\therefore \quad (a, c) \notin R_1$

So, R_1 is not transitive. Hence, R_1 is not equivalence.

In $R_2 = \{(a, b) : b \le a\} \Rightarrow (5, 4) \in R_2$ but $(4, 5) \notin R_2$

So, R_2 is not symmetric. Hence, R_2 cannot be equivalence.

2. (B) : When $x = n\pi$, $f(x) = 0$ and $f(-x) = 0$

$\therefore \quad f(-x) = f(x)$

When $x \ne n\pi$, $n \in I$, $\sin x \ne 0$

and $\left[\frac{x}{\pi}\right]+\frac{1}{2} \ne 0$

$\therefore \; f(x) \ne 0$

Also, $\left[\frac{x}{\pi}\right]+\left[-\frac{x}{\pi}\right] = -1 \;\therefore\; \left[-\frac{x}{\pi}\right] = -1-\left[\frac{x}{\pi}\right]$

$$\Rightarrow \left[-\frac{x}{\pi}\right]+\frac{1}{2} = -\left[\frac{x}{\pi}\right]-\frac{1}{2} = -\left(\left[\frac{x}{\pi}\right]+\frac{1}{2}\right)$$

Now, $f(-x) = \dfrac{\sin^{101}(-x)}{\left[-\frac{x}{\pi}\right]+\frac{1}{2}} = \dfrac{-\sin^{101}x}{-\left(\left[\frac{x}{\pi}\right]+\frac{1}{2}\right)} = \dfrac{\sin^{101}x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} = f(x)$

Hence, in both the cases, $f(-x) = f(x)$.

3. (C) : Consider $f(x) = \sin^{-1}\left(\frac{x^2-3x+2}{x^2+2x+7}\right)$,

where $-1 \le \frac{x^2-3x+2}{x^2+2x+7} \le 1$

$\Rightarrow \quad -x^2 - 2x - 7 \le x^2 - 3x + 2 \le x^2 + 2x + 7$

$\Rightarrow \quad 0 \le 2x^2 - x + 9$ and $5x + 5 \ge 0$

$\Rightarrow \quad 2x^2 - x + 9 \ge 0 \Rightarrow x \in R$ and $x + 1 \ge 0$

$\therefore \quad$ Domain of $f(x) = [-1, \infty)$

4. (C) : We have, $A^T = A$ and $B^T = -B$

A. $(A^4-B^4)^T=(A^4)^T-(B^4)^T=(A^T)^4-(B^T)^4=A^4-(-B)^4=A^4-B^4$

Thus given matrix is symmetric.

B. $(AB-BA)^T=(AB)^T-(BA)^T=B^TA^T-A^TB^T=-BA+AB$

Thus, given matrix is symmetric.

C. $(B^5 - A^5)^T = (B^5)^T-(A^5)^T=(B^T)^5-(A^T)^5=(-B)^5-(A)^5$
$= -B^5 - A^5$

Thus, given matrix is neither symmetric nor skew symmetric.

D. $(AB+BA)^T=(AB)^T+(BA)^T=B^TA^T+A^TB^T$
$=-BA-AB=-(AB+BA)$

Thus, given matrix is skew symmetric.

5. (D) : We have, $A=\begin{pmatrix}0 & \sin\alpha\\ \sin\alpha & 0\end{pmatrix}$

$$\therefore\quad A^2=\begin{pmatrix}0 & \sin\alpha\\ \sin\alpha & 0\end{pmatrix}\cdot\begin{pmatrix}0 & \sin\alpha\\ \sin\alpha & 0\end{pmatrix}$$

$$\Rightarrow\quad A^2=\begin{pmatrix}\sin^2\alpha & 0\\ 0 & \sin^2\alpha\end{pmatrix}$$

Now, $\left|A^2-\frac{1}{2}I\right|=\begin{vmatrix}\sin^2\alpha-\frac{1}{2} & 0\\ 0 & \sin^2\alpha-\frac{1}{2}\end{vmatrix}$

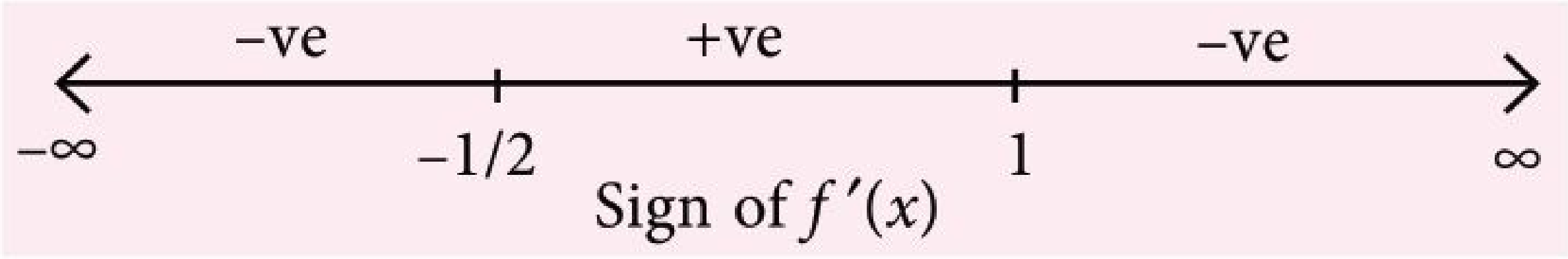
$$=\left(\sin^2\alpha-\frac{1}{2}\right)^2=0\Rightarrow\ \sin\alpha=\pm\frac{1}{\sqrt{2}}$$

$\Rightarrow\ \alpha=\frac{\pi}{4}$ is the only possible value.

6. (B): $f(x)=|x-1|\cos|x-2|\sin|x-1|+(x-3)|x^2-5x+4|$
$=|x-1|\cos|x-2|\sin|x-1|+(x-3)\,|x-1|\,|x-4|$
$=|x-1|\,[\cos|x-2|\sin|x-1|+(x-3)|x-4|]$,
which is not differentiable at $x=1$ and $x=4$.

7. (A) : We have, $f(x)=xe^{x(1-x)}$

$$\therefore\quad f'(x)=e^{x(1-x)}+xe^{x(1-x)}\cdot(1-2x)$$
$$=e^{x(1-x)}\,[1+x-2x^2]=e^{x(1-x)}\,(1-x)(2x+1)$$



Sign of $f'(x)$

Thus, $f(x)$ is increasing in $\left(-\frac{1}{2},\ 1\right)$ and decreasing in $\left(-\infty,\frac{-1}{2}\right)\cup(1,\infty)$.

8. (D) : $\displaystyle\int_{\pi/4}^{3\pi/4}\frac{x}{1+\sin x}dx$

$$=\int_{\pi/4}^{3\pi/4}\left(\frac{x}{1+\sin x}\times\frac{1-\sin x}{1-\sin x}\right)dx=\int_{\pi/4}^{3\pi/4}\frac{x(1-\sin x)}{\cos^2 x}dx$$

$$=\int_{\pi/4}^{3\pi/4}x\cdot\sec^2 x\,dx-\int_{\pi/4}^{3\pi/4}x\cdot\frac{\sin x}{\cos^2 x}dx$$

$$\Rightarrow I=\int_{\pi/4}^{3\pi/4}x\cdot\sec^2 x dx-\int_{\pi/4}^{3\pi/4}x(\sec x\tan x)dx$$

$$=|x\tan x|_{\pi/4}^{3\pi/4}-|\log|\sec x||_{\pi/4}^{3\pi/4}-\left|\frac{x}{\cos x}\right|_{\pi/4}^{3\pi/4}+\int_{\pi/4}^{3\pi/4}\frac{1}{\cos x}dx$$

$$=\left|x\tan x\right|_{\pi/4}^{3\pi/4}-\left|\frac{x}{\cos x}\right|_{\pi/4}^{3\pi/4}+\log\left|\sec x+\tan x\right|_{\pi/4}^{3\pi/4}$$

$$=-\frac{3\pi}{4}-\frac{\pi}{4}+\frac{3\pi}{4}(\sqrt{2})+\frac{\pi}{4}\sqrt{2}+\log(\sqrt{2}+1)-\log(\sqrt{2}+1)$$

$$=-\pi+\sqrt{2}\pi\ =\pi\,(\sqrt{2}-1)$$

9. (C) : We have,

$y-x=2$ ...(i)

and $y=x^2$ ...(ii)

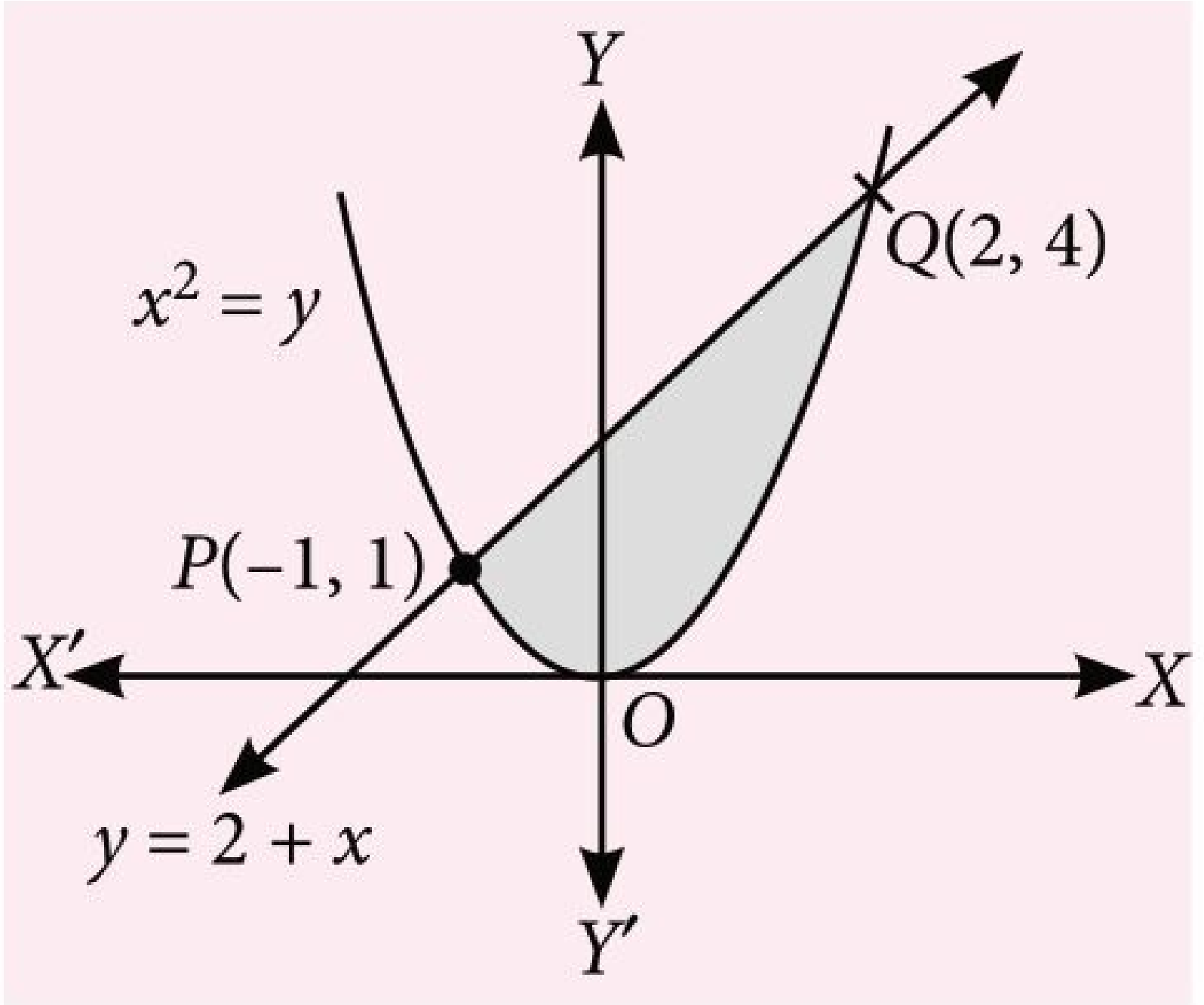
From (i) and (ii), we get

$x^2-x-2=0$

$\Rightarrow\ (x+1)(x-2)=0$

$\Rightarrow\ x=-1,\ x=2$

$\therefore\ y=1,\ y=4$



So, points of intersection are $P(-1,1)$ and $Q(2,4)$.

$$\therefore\quad \text{Required area}=\int_{-1}^{2}(2+x-x^2)dx=\left[2x+\frac{x^2}{2}-\frac{x^3}{3}\right]_{-1}^{2}$$

$$=\left(4+2-\frac{8}{3}\right)-\left(-2+\frac{1}{2}+\frac{1}{3}\right)=\frac{10}{3}+\frac{7}{6}=\frac{9}{2}$$

10. (A) : $\frac{dy}{dx}+2y\tan x=\sin x$

I.F. $=e^{\int 2\tan x\,dx}=e^{2\log|\sec x|}=\sec^2 x$

$$\therefore\quad y(\sec^2 x)=\int\sin x\cdot\sec^2 x\,dx+c=\int\frac{\sin x}{\cos^2 x}dx+c$$

Let $\cos x=t\Rightarrow\sin x\,dx=-dt\Rightarrow\ \frac{y}{\cos^2 x}=\int\frac{-dt}{t^2}+c$

$\Rightarrow\quad\frac{y}{\cos^2 x}=\frac{1}{\cos x}+c$. Now, $y\left(\frac{\pi}{3}\right)=0$

$\Rightarrow\ 0=2+c\ \Rightarrow\ c=-2\quad\therefore\ y=\cos x-2\cos^2 x$

$$=-2\left(\cos^2 x-\frac{1}{2}\cos x+\frac{1}{16}-\frac{1}{16}\right)=\frac{1}{8}-2\left(\cos x-\frac{1}{4}\right)^2$$

$\Rightarrow\quad y_{\max}=\frac{1}{8}$ at $\cos x=\frac{1}{4}$

11. (A) : Given, $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$, $\vec{a}\times\vec{b}=2\hat{i}-\hat{k}$ and $\vec{a}\cdot\vec{b}=3$

Projection of $\vec{b}$ on $(\vec{a}-\vec{b})$ is $=\frac{\vec{b}\cdot(\vec{a}-\vec{b})}{|\vec{a}-\vec{b}|}$

Now, $|\vec{a}\times\vec{b}|^2+(\vec{a}\cdot\vec{b})^2=|\vec{a}|^2|\vec{b}|^2$

$$\Rightarrow\ 5+9=6|\vec{b}|^2\ \Rightarrow\ \frac{7}{3}=|\vec{b}|^2\ \Rightarrow\ |\vec{b}|=\sqrt{\frac{7}{3}}$$

$$\therefore\quad |\vec{a}-\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2-2(\vec{a}\cdot\vec{b})\ =6+\frac{7}{3}-2\times3=\frac{7}{3}$$

$$\Rightarrow\quad |\vec{a}-\vec{b}|=\sqrt{\frac{7}{3}}$$

Projection $\vec{b}$ on $(\vec{a}-\vec{b})=\dfrac{\vec{b}\cdot\vec{a}-|\vec{b}|^2}{|\vec{a}-\vec{b}|}$

$$=\frac{3-\frac{7}{3}}{\sqrt{\frac{7}{3}}}=\frac{2}{3}\times\sqrt{\frac{3}{7}}=\frac{2}{\sqrt{21}}$$

12. (A) : We have, $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -a & 0\\ 1 & -1 & 1\end{vmatrix}$

$=-a\hat{i}-\hat{j}+(a-1)\hat{k}$

Also, $\vec{a}_1-\vec{a}_2=-\hat{i}+\hat{j}+\hat{k}$

$\therefore$ Shortest distance $=\left|\dfrac{(\vec{a}_1-\vec{a}_2)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

$$\Rightarrow\quad \sqrt{\frac{2}{3}}=\left|\frac{a-1+a-1}{\sqrt{a^2+1+(a-1)^2}}\right|$$

$$\Rightarrow\quad \frac{2}{3}=\frac{(2a-2)^2}{a^2+1+a^2+1-2a}\ \Rightarrow\ \frac{1}{3}=\frac{2(a^2+1-2a)}{2a^2+2-2a}$$

$\Rightarrow\ 2a^2-5a+2=0\Rightarrow(a-2)(2a-1)=0\ \Rightarrow\ a=2,\frac{1}{2}$

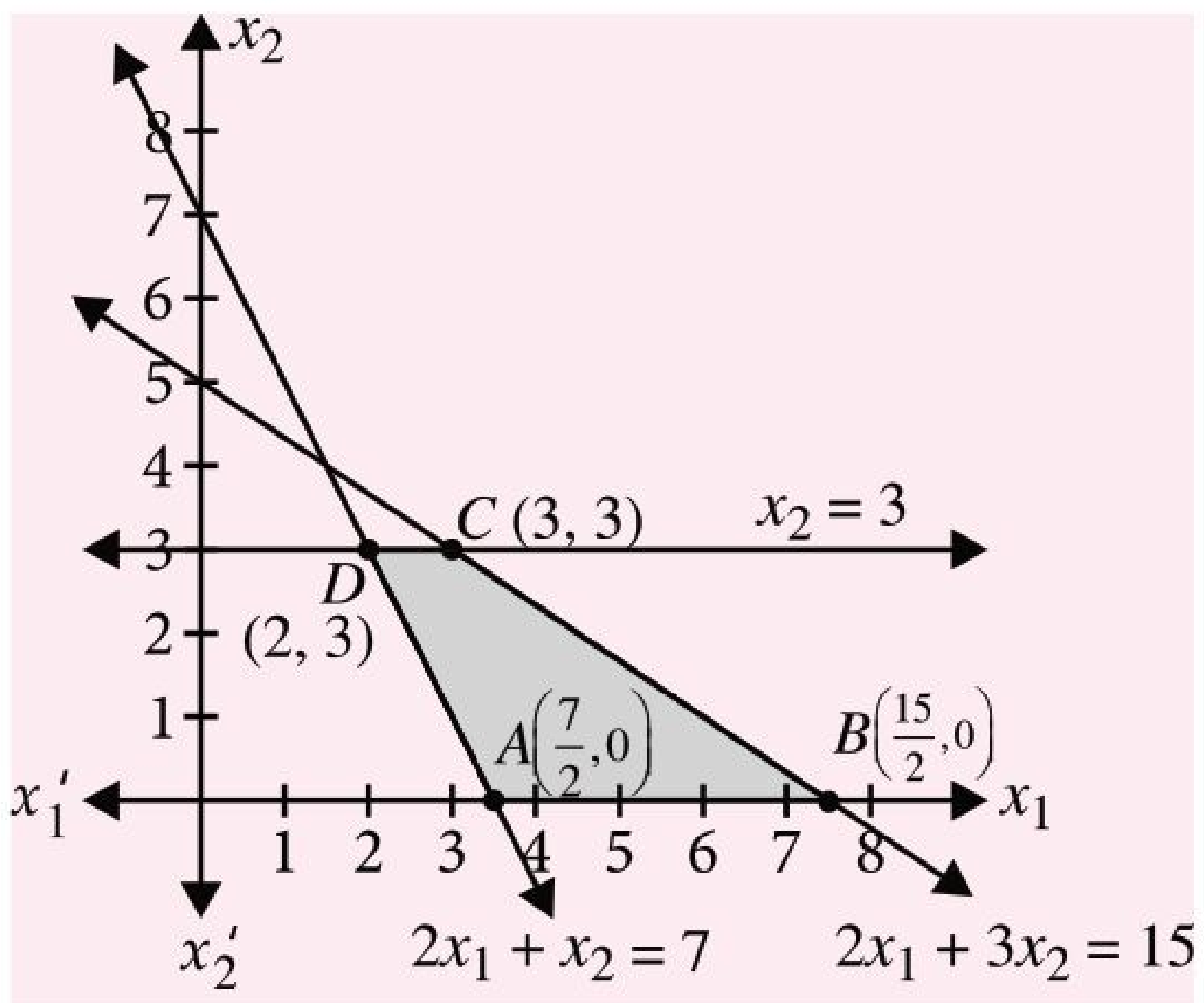
13. (A) : $P(X = 1) = 1/6$

$P(X=2)=\dfrac{5}{36}$; $P(X>2)=\dfrac{25}{36}$

$P(X\geq 5)=\dfrac{625}{1296}$

$\therefore\ P(X\geq 5|X>2)=\dfrac{P(X\geq 5)}{P(X>2)}=\dfrac{25}{36}$

14. (A) : Converting the given inequalities into equations, we get $2x_1 + x_2 = 7$, $2x_1 + 3x_2 = 15$, $x_2 = 3$, $x_1 = x_2 = 0$



The feasible region is $ABCD$ which is shaded in the figure. The vertices of the feasible region are $A\left(\frac{7}{2},0\right)$, $B\left(\frac{15}{2},0\right)$, C (3, 3) and D(2, 3).

The values of $z = 4x_1 + 5x_2$ at these vertices are

Corner Points	Value of $z = 4x_1 + 5x_2$
$A\left(\frac{7}{2},0\right)$	14
$B\left(\frac{15}{2},0\right)$	30
C (3, 3)	27
D (2, 3)	23

From table, we can observe that minimum value of z is 14, which is attained at $A\left(\frac{7}{2},0\right)$.

Hence, minimum value occurs on x-axis.

15. (C) : Let $f(x) = \sin^{-1} x$ and $g(x) = [x]$.

Since f is continuous for all $x \in [-1, 1]$ and g is continuous for all $x \in (R - I)$.

Therefore, domain of continuity of $(f - g)(x)$ $= \sin^{-1} x - [x]$ is $[-1, 1] \cap (R - I) = (-1, 0) \cup (0, 1)$.

16. (B)

17. (B) : (i) We have, $\dfrac{dy}{dx}=xe^{x-y}=xe^{x}\cdot e^{-y}$

$$\Rightarrow\quad \int e^{y}dy=\int xe^{x}dx+C\Rightarrow\quad e^{y}=xe^{x}-\int 1\,e^{x}dx+C$$

$$\Rightarrow\quad e^y = xe^x - e^x + C \Rightarrow e^y - e^x\,(x-1) = C$$

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(ii) We have, $\frac{dy}{dx}=(x+y)^2$

Let $x+y=t \Rightarrow 1+\frac{dy}{dx}=\frac{dt}{dx}$

∴ Given equation can be written as

$$\frac{dt}{dx}-1=t^2 \Rightarrow \frac{dt}{dx}=t^2+1 \Rightarrow \int\frac{dt}{t^2+1}=\int dx$$

⇒ $\tan^{-1}t = x + c \Rightarrow \tan^{-1}(x+y) = x + c$

(iii) We have, $\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2=2x^2\log\left(\frac{d^2y}{dx^2}\right)$

$$\Rightarrow \frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-2x^2\log\left(\frac{d^2y}{dx^2}\right)=0$$

Since, L.H.S of the differential equation cannot be expressed as polynomial in $\frac{dy}{dx}$. So, its degree is not defined.

18. (D) : $P(\bar{E}_i)=\frac{i}{1+i}\ (i=1, 2, ..., n)$

∴ $P(\bar{E}_1)=\frac{1}{2},\ P(\bar{E}_2)=\frac{2}{3},\ P(\bar{E}_3)=\frac{3}{4},\ ...,\ P(\bar{E}_n)=\frac{n}{n+1}$

∴ $P(\bar{E}_1\,\bar{E}_2\,\bar{E}_3 ...\ \bar{E}_n)=\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\ ...\ \frac{n-1}{n}\cdot\frac{n}{n+1}=\frac{1}{n+1}$

∴ Statement-I is false but Statement-II is true.

19. (B) : P. Let $I=\int\frac{2^x}{\sqrt{1-4^x}}dx$

Putting $2^x = t \Rightarrow 2^x \log 2\, dx = dt$, we get

$$I=\frac{1}{\log 2}\int\frac{1}{\sqrt{1-t^2}}dt$$

$$\Rightarrow I=\frac{1}{\log 2}\sin^{-1}t+C=\frac{1}{\log 2}\sin^{-1}(2^x)+C$$

∴ $k = 1$

Q. Let $I=\int\frac{dx}{(\sqrt{x})^2+(\sqrt{x})^7}=\int\frac{dx}{(\sqrt{x})^7\left(1+\frac{1}{(\sqrt{x})^5}\right)}$

Putting $\frac{1}{(\sqrt{x})^5}=y \Rightarrow \frac{-5}{2(\sqrt{x})^7}=\frac{dy}{dx}$

$$\therefore I=\int\frac{-2dy}{5(1+y)}=\frac{-2}{5}\log(1+y)+C=\frac{2}{5}\log\left(\frac{1}{1+\frac{1}{(\sqrt{x})^5}}\right)+C$$

$\Rightarrow a=\frac{2}{5},\ k=\frac{5}{2}$

∴ $5a + 2k = 7$

R. Let $I=\int\frac{x^4+1}{x(x^2+1)^2}dx=\int\frac{x^4+1+2x^2-2x^2}{x(x^2+1)^2}dx$

$$=\int\frac{(x^2+1)^2-2x^2}{x(x^2+1)^2}dx=\int\frac{1}{x}dx-\int\frac{2x}{(x^2+1)^2}dx$$

$$=\log|x|-\int\frac{2x}{(x^2+1)^2}dx$$

Putting $x^2 + 1 = t \Rightarrow 2x\, dx = dt$

$$\therefore\ I=\log|x|-\int\frac{dt}{t^2}$$

$$\Rightarrow I=\log|x|+\frac{1}{t}+C=\log|x|+\frac{1}{x^2+1}+C$$

∴ $k = 1, m = 1 \Rightarrow m + k = 2$

S. Let $I=\int\frac{dx}{5+4\cos x}$

$$=\int\frac{dx}{5\left(\sin^2\frac{x}{2}+\cos^2\frac{x}{2}\right)+4\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)}$$

$$=\int\frac{dx}{9\cos^2\frac{x}{2}+\sin^2\frac{x}{2}}=\int\frac{\sec^2\frac{x}{2}}{9+\tan^2\frac{x}{2}}dx$$

Let $\tan\frac{x}{2}=t \Rightarrow \sec^2\frac{x}{2}dx=2dt$

$$\therefore I=\int\frac{2dt}{9+t^2}=\frac{2}{3}\tan^{-1}\left(\frac{t}{3}\right)+C$$

$$\Rightarrow I=\frac{2}{3}\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{3}\right)+C$$

$\therefore k=\frac{2}{3}, m=\frac{1}{3}$ ∴ $3(k + m) = 3$

20. (D) : The given function is

$f(x) = \sin x + \cos x;\ 0 \le x \le 2\pi$

⇒ $f'(x) = \cos x - \sin x$

Now $f'(x) = 0 \Rightarrow \cos x - \sin x = 0$

⇒ $\tan x=1 \Rightarrow x=\frac{\pi}{4},\frac{5\pi}{4}$. Thus $f'(x) > 0$ in $\left[0,\frac{\pi}{4}\right)$,

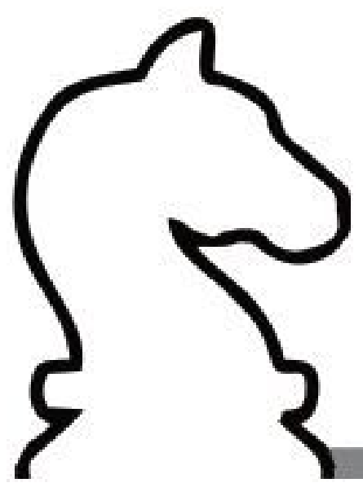
$f'(x)<0$ in $\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$ and $f'(x)>0$ in $\left(\frac{5\pi}{4},2\pi\right]$

⇒ The function f is increasing in $\left[0,\frac{\pi}{4}\right)\cup\left(\frac{5\pi}{4},2\pi\right]$.

For other sections/subjects please refer to Physics For You, Chemistry Today and Biology Today

⁂

Challenging PROBLEMS For JEE



ON Vector Algebra

SINGLE OPTION CORRECT TYPE

1. The length of the perpendicular from the origin to the plane passing through the point $\vec{a}$ and containing the line $\vec{r}=\vec{b}+\lambda\vec{c}$ is

(a) $\dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$

(b) $\dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}|}$

(c) $\dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$ (d) $\dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{c}\times\vec{a}+\vec{a}\times\vec{b}|}$

2. If $\vec{a}$ and $\vec{b}$ are unit vectors and $\vec{c}$ is a vector such that $\vec{c}=\vec{a}\times\vec{c}+\vec{b}$, then

(a) $[\vec{a}\ \vec{b}\ \vec{c}]=\vec{b}\cdot\vec{c}+(\vec{a}\cdot\vec{b})^2$

(b) $[\vec{a}\ \vec{b}\ \vec{c}]=0$

(c) Maximum value of $[\vec{a}\ \vec{b}\ \vec{c}]$ is $\dfrac{1}{2}$

(d) Minimum value of $[\vec{a}\ \vec{b}\ \vec{c}]$ is $\dfrac{1}{2}$

3. If $\vec{a}'=\hat{i}+\hat{j}$, $\vec{b}'=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{c}'=2\hat{i}+\hat{j}-\hat{k}$, Then altitude of the parallelopiped formed by the vectors $\vec{a}, \vec{b}, \vec{c}$ having base formed by $\vec{b}$ and $\vec{c}$ is ($\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vectors)

(a) 1 (b) $\dfrac{3\sqrt{2}}{2}$ (c) $\dfrac{1}{\sqrt{6}}$ (d) $\dfrac{1}{\sqrt{2}}$

4. If $((\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}))\cdot(\vec{a}\times\vec{d})=0$, then which of the following is always true.

(a) $\vec{a}, b, \vec{c}, \vec{d}$ are necessarily coplaner.

(b) Either $\vec{a}$ or $\vec{d}$ must lie in the plane of $\vec{b}$ or $\vec{c}$.

(c) Either $\vec{b}$ or $\vec{c}$ must lie in place of $\vec{a}$ and $\vec{d}$.

(d) Either $\vec{a}$ or $\vec{b}$ must lie in plane of $\vec{c}$ and $\vec{d}$

5. Let $\vec{r}=(\vec{a}\times\vec{b})\sin x+(\vec{b}\times\vec{c})\cos y+2(\vec{c}\times\vec{a})$, where $\vec{a}\,\vec{b}\,\vec{c}$ are three noncoplanar vectors. If $\vec{r}$ is perpendicular to $\vec{a}+\vec{b}+\vec{c}$, then minimum value of x^2+y^2 is

(a) π^2 (b) $\pi^2/4$

(c) $5\pi^2/4$ (d) None of these

6. In a quadrilateral $ABCD$, $\overrightarrow{AC}$ is the bisector of $\overrightarrow{AB}$ and $\overrightarrow{AD}$, angle between $\overrightarrow{AB}$ and $\overrightarrow{AD}$ is $2\pi/3$, $15|\overrightarrow{AC}|=3|\overrightarrow{AB}|=5|\overrightarrow{AD}|$, then cosine of angle between $\overrightarrow{BA}$ and $\overrightarrow{CD}$ is

(a) $-\dfrac{\sqrt{14}}{7\sqrt{2}}$ (b) $-\dfrac{\sqrt{21}}{7\sqrt{3}}$ (c) $\dfrac{2}{\sqrt{7}}$ (d) $\dfrac{2\sqrt{7}}{14}$

7. The lengths of two opposite edges of a tetrahedron are a, b. Their shortest distance is d and the angle between them is θ. Then its volume is

(a) $\dfrac{1}{2}abd\sin\theta$ (b) $\dfrac{1}{3}abd\cos\theta$

(c) $\dfrac{1}{6}abd\sin\theta$ (d) $\dfrac{1}{6}abd\cos\theta$

8. The position vectors of the vertices A,B,C of a tetrahedron $ABCD$ are $\hat{i}+\hat{j}+\hat{k}, \hat{i}$ and $3\hat{i}$ respectively and the altitude from the vertex D to the opposite face ABC meets the face at E. If the length of the edge AD is 4 and the volume of the tetrahedron is $\dfrac{2\sqrt{2}}{3}$, then the length of DE is

(a) 1 (b) 2 (c) 3 (d) 4

9. A ray of light strikes a plane mirror and gets reflected as shown in the diagram. If $\hat{u}, \hat{v}, \hat{n}$ are unit vectors along the incident ray, reflected ray and normal respectively, then

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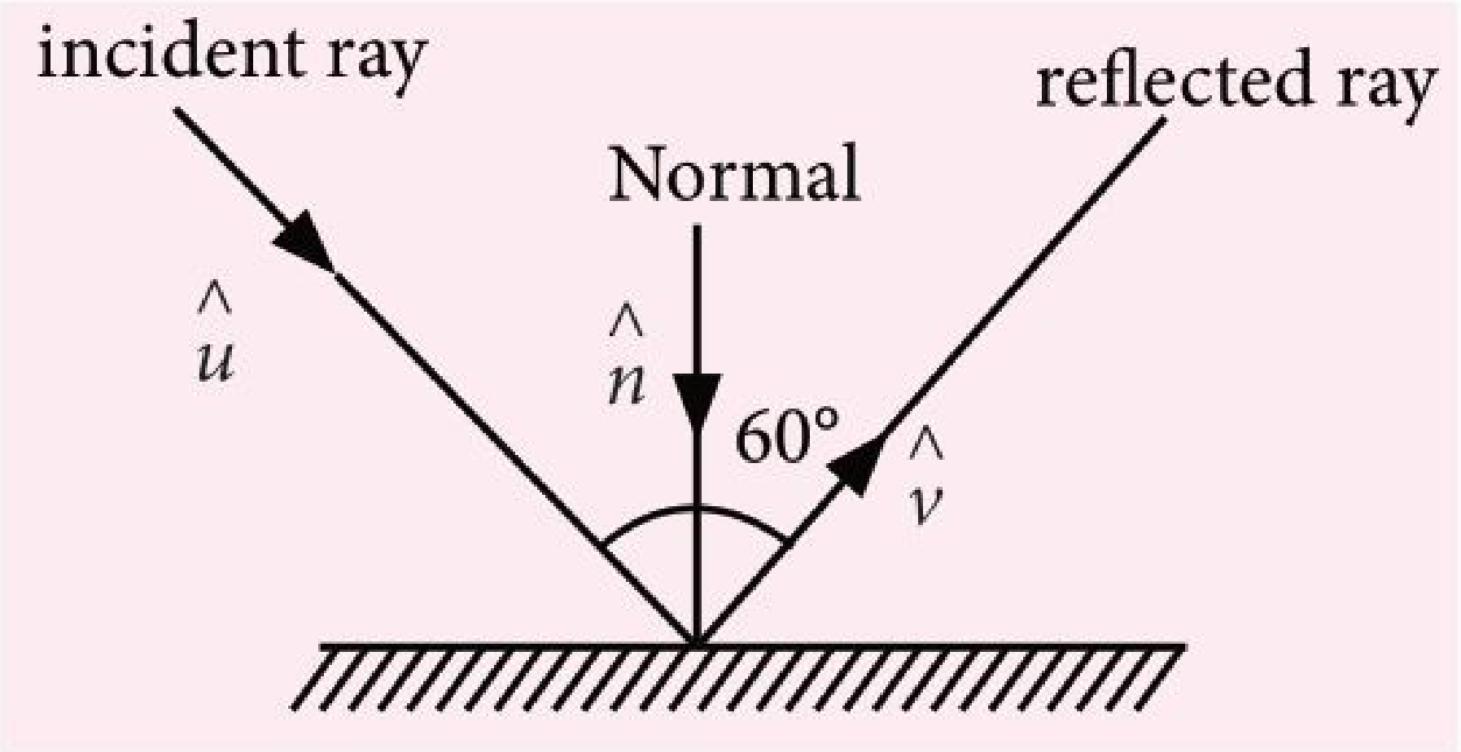


(a) $\hat{v}-\hat{u}+\hat{n}=\vec{0}$ (b) $\hat{v}+\hat{u}-\sqrt{3}\hat{n}=\vec{0}$

(c) $\hat{v}-\hat{u}+\sqrt{3}\hat{n}=\vec{0}$ (d) $\hat{v}+\hat{u}-3\hat{n}=0$

NUMERICAL ANSWER TYPE

10. If $\vec{a},\vec{b},\vec{c}$ be non-coplanar unit vectors equally inclined to one another at an acute angle θ, and if $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$, then $p-r=$ ________ $(p, q, r\in R)$

11. Let A be a point on the line $\overline{r}=(-3\hat{i}+6\hat{j}+3\hat{k})+t(2\hat{i}+3\hat{j}-2\hat{k})$ and B be a point on the line $\overline{r}=6\hat{j}+s(2\hat{i}+2\hat{j}-\hat{k})$. The least value of the distance AB is _______.

12. The projection length of a variable vector $x\hat{i}+y\hat{j}+z\hat{k}$ on the vector $\vec{p}=\hat{i}+2\hat{j}+3\hat{k}$ is 6. Let l be the minimum projection length of the vector $x^2\hat{i}+y^2\hat{j}+z^2\hat{k}$ on the vector $\vec{p}$, , then the value of $\sqrt[3]{l^2+15^2}$ is _______.

COMPREHENSION TYPE

Paragraph for Q. (13 to 15)

Let $\vec{x},\vec{y},\vec{z}$ be the vector, such that $|\vec{x}|=|\vec{y}|=|\vec{z}|=\sqrt{2}$ and $\vec{x}$, $\vec{y},\vec{z}$, make angles of 60° with each other also, $\vec{x}\times(\vec{y}\times\vec{z})=\vec{a}$, $\vec{y}\times(\vec{z}\times\vec{x})=\vec{b}$ and $\vec{x}\times\vec{y}=\vec{c}$. Then

13. The value of $\vec{x}$ is

(a) $\{(\vec{a}+\vec{b})\times\vec{c}-(\vec{a}+\vec{b})\}$ (b) $\{(\vec{a}+\vec{b})-(\vec{a}+\vec{b})\times\vec{c}\}$

(c) $\frac{1}{2}\{(\vec{a}+\vec{b})\times\vec{c}-(\vec{a}+\vec{b})\}$ (d) None of these

14. The value of $\vec{y}$ is

(a) $\frac{1}{2}[(\vec{a}+\vec{b})+(\vec{a}+\vec{b})\times\vec{c}]$

(b) $2[(\vec{a}+\vec{b})+(\vec{a}+\vec{b})\times\vec{c}]$

(c) $4[(\vec{a}+\vec{b})+(\vec{a}+\vec{b})\times\vec{c}]$

(d) None of these

15. The value of $\vec{z}$ is

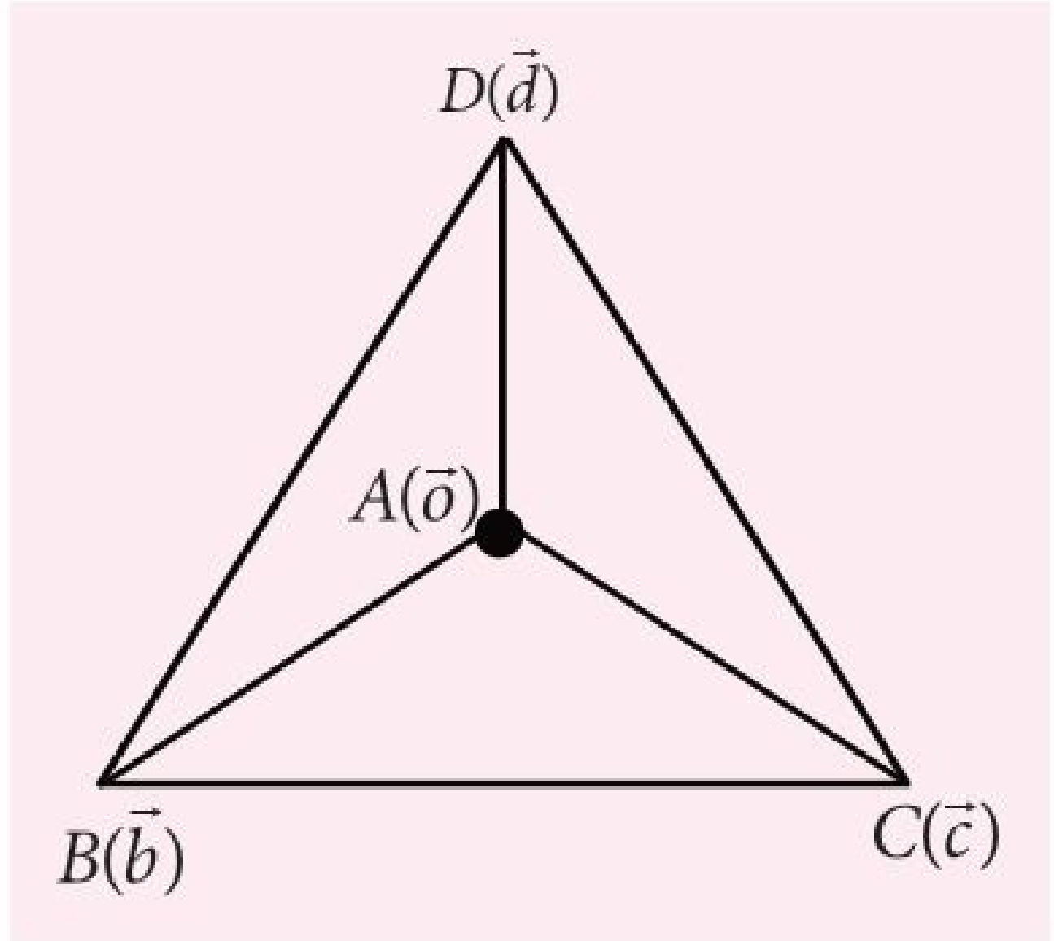
(a) $\frac{1}{2}[(\vec{b}-\vec{a})\times\vec{c}+(\vec{a}+\vec{b})]$

(b) $\frac{1}{2}[(\vec{b}-\vec{a})+(\vec{a}+\vec{b})\times\vec{c}]$

(c) $[(\vec{b}-\vec{a})\times\vec{c}+(\vec{a}+\vec{b})]$

(d) None of these

Paragraph for Q. (16 to 18)



In tetrahedron $ABCD$, the face ABC is a regular (Equilateral triangle) and the face BCD is perpendicular to it. $\angle DAC=\frac{\pi}{3}, |AD|=6$ units. Angle between the lines $\overrightarrow{AD}$ and $\overrightarrow{BC}$ is $\cos^{-1}\frac{1}{4}$ and if 'A' origin, $\overrightarrow{AD}=\vec{d}, \overrightarrow{AB}=\vec{b}, \overrightarrow{AC}=\vec{c}$.

16. Angle between $\vec{d}$ and $\vec{b}$ is

(a) $\cos^{-1}\frac{1}{4}$ (b) $\cos^{-1}\frac{1}{2}$

(c) $\cos^{-1}\frac{1}{\sqrt{2}}$ (d) $\cos^{-1}\frac{3}{4}$

17. $|\vec{b}|+|\vec{c}|$ is equal to

(a) 4 (b) 6 (c) 8 (d) 14

18. Volume of tetrahedron $ABCD$ (in Cu. units) is

(a) $\frac{27}{8}$ (b) $\frac{27}{4}$ (c) $\frac{9}{4}$ (d) $\frac{8}{3}$

Paragraph for Q. (19 to 21)

Let $\overline{r}$ is position vector of a point in Cartesian OXY plane such that $\overline{r}\cdot(10\hat{j}-8\hat{i}-\vec{r})=40$, $\max\{|\vec{r}+2\hat{i}-3\hat{j}|^2\}=l$ and $\min\{|\vec{r}+2\hat{i}-3\hat{j}|^2\}=m$. A tangent line is drawn to the curve $y=\frac{8}{x^2}$ at the point A with abscissa 2. The drawn tangent cuts the x-axis at B, then

19. Value of m is

(a) 9 (b) $2\sqrt{2}-1$

(c) $6\sqrt{2}+3$ (d) $9-4\sqrt{2}$

20. $l+m$ is equal to

(a) 2 (b) 10 (c) 18 (d) 5

21. The value of $\overrightarrow{AB}\cdot\overrightarrow{OB}$ (where O is origin) is

(a) 1 (b) 2 (c) 3 (d) 4

Paragraph for Q. (22 to 24)

For two vectors $\vec{x},\vec{y}$, we defined

$\vec{x}\cdot\vec{y}=|\vec{x}||\vec{y}|\cos\theta, \vec{x}\times\vec{y}=|\vec{x}||\vec{y}|\sin\theta\hat{n}$.

Let $*$ is a operation defined by $\vec{x}*\vec{y}=|\vec{x}||\vec{y}|\tan\frac{\theta}{2}$, where θ is angle between $\vec{x}$ and $\vec{y}$.

22. Projection of $\vec{x}$ on $\vec{y}$ will be

(a) $\dfrac{\vec{x} * \vec{y}}{|\vec{y}|}$ (b) $\dfrac{\vec{x} * \vec{y}}{|\vec{x}|}$

(c) $|\vec{x}|\left(\dfrac{|\vec{x}|^2|\vec{y}|^2-(\vec{x}*\vec{y})^2}{|\vec{x}||\vec{y}|^2+(\vec{x}*\vec{y})^2}\right)$

(d) $\left(\dfrac{|\vec{x}|^2|\vec{y}|^2-(\vec{x}*\vec{y})^2}{|\vec{x}|^2|\vec{y}|^2+(\vec{x}*\vec{y})^2}\right)$

23. If $\vec{x}$ and $\vec{y}$ represent the adjacent sides of a parallelogram, then its area is given by

(a) $|\vec{x}*\vec{y}|$ (b) $\dfrac{2(\vec{x}*\vec{y})|\vec{x}|^2|\vec{y}|^2}{|\vec{x}*\vec{y}|^2+|\vec{x}|^2|\vec{y}|^2}$

(c) $\dfrac{\vec{x}*\vec{y}}{1+(\vec{x}*\vec{y})^2}$ (d) None of these

24. If $\vec{x}$ and $\vec{y}$ are two non-zero linearly independent vectors such that $|\vec{x}\times\vec{y}|=|\vec{x}\times\vec{y}|$, then

(a) $\vec{x}$ and $\vec{y}$ are parallel

(b) $\vec{x}$ and $\vec{y}$ are perpendicular

(c) angle between $\vec{x}$ and $\vec{y}$ is $\dfrac{\pi}{4}$

(d) None of these

SOLUTIONS

1. (c) : Let $\vec{r}$ be any point in the plane. Then $\vec{r}-\vec{a}, \vec{b}-\vec{a}$, and $\vec{c}$ are coplanar.

Given that the plane through $\vec{a}$ and $\vec{b}$ containing the line is $[\overrightarrow{AP}\ \overrightarrow{AB}\ \vec{c}]=0$

$\therefore$ Equation of plane is $[\vec{r}-\vec{a}, \vec{b}-\vec{a}, \vec{c}]=0$

$\Rightarrow (\vec{r}-\vec{a})\cdot((\vec{b}-\vec{a})\times\vec{c})=0$

$\Rightarrow \vec{r}\cdot(\vec{b}\times\vec{c}+\vec{c}\times\vec{a})=[\vec{a}\,\vec{b}\,\vec{c}]$

$\therefore$ Length of $\perp^r$ from the origin $=\dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$

2. (c) : $\vec{c}\cdot\vec{a}=((\vec{a}\times\vec{c})+\vec{b})\cdot\vec{a}=\vec{b}\cdot\vec{a}$

$\vec{b}\times\vec{c}=(\vec{b}\cdot\vec{c})\cdot\vec{a}-(\vec{a}\cdot\vec{b})\cdot\vec{c}\ \therefore\ [\vec{a}\ \vec{b}\ \vec{c}]=\vec{b}\cdot\vec{c}-(\vec{a}\cdot\vec{b})\cdot(\vec{a}\cdot c)$

Also $\vec{c}\cdot\vec{b}=1-[\vec{a}\ \vec{b}\ \vec{c}]$

$\therefore\ 2[\vec{a}\ \vec{b}\,\vec{c}]=1-(\vec{a}\cdot\vec{b})^2\le 1 \quad \therefore\ [\vec{a}\,\vec{b}\,\vec{c}]\le\dfrac{1}{2}$

3. (d) : Volume of the parallelepiped formed by $\vec{a}', \vec{b}', \vec{c}'$ is 2

$\therefore$ Volume of the parallelepiped formed by

$\vec{a}, \vec{b}, \vec{c}$ is $\dfrac{1}{2}$ $(\vec{b}\times\vec{c})=\dfrac{(\vec{c}'\times\vec{a}')\times\vec{c}}{4}=\dfrac{1}{2}\vec{a}'$

$\vec{b}\times\vec{c}=a'[\vec{a}\,\vec{b}\,\vec{c}]\ =\dfrac{1}{2}a'$

$\therefore |\vec{b}\times\vec{c}|=\dfrac{1}{2}\sqrt{2}=\dfrac{\sqrt{2}}{2}=\dfrac{1}{\sqrt{2}}$

$\therefore$ length of altitude $=\dfrac{1/2}{1/\sqrt{2}}=\dfrac{1}{\sqrt{2}}$

4. (c) : We have, $((\vec{a}\times\vec{b}\times(\vec{c}\times\vec{d}))\cdot(\vec{a}\times\vec{d})=0$

$\Rightarrow ([\vec{a}\,\vec{c}\,\vec{d}]\,\vec{b}-[\vec{b}\,\vec{c}\,\vec{d}]\,\vec{a})\cdot(\vec{a}\times\vec{d})=0 \Rightarrow [\vec{a}\,\vec{c}\,\vec{d}]\,[\vec{b}\,\vec{a}\,\vec{d}]=0$

$\Rightarrow$ Either $\vec{c}$ or $\vec{b}$ must lie in the plane of $\vec{a}$ and $\vec{d}$.

5. (c) : $\vec{r}=(\vec{a}\times\vec{b})\sin x+(\vec{b}\times\vec{c})\cos y+2(\vec{c}\times\vec{a})$

$\therefore\ \vec{r}\cdot(\vec{a}+\vec{b}+\vec{c})=0 \Rightarrow [\vec{a}\,\vec{b}\,\vec{c}](\sin x+\cos y+2)=0$

If $[\vec{a}\,\vec{b}\,\vec{c}]\ne 0$, then $\sin x+\cos y=-2$

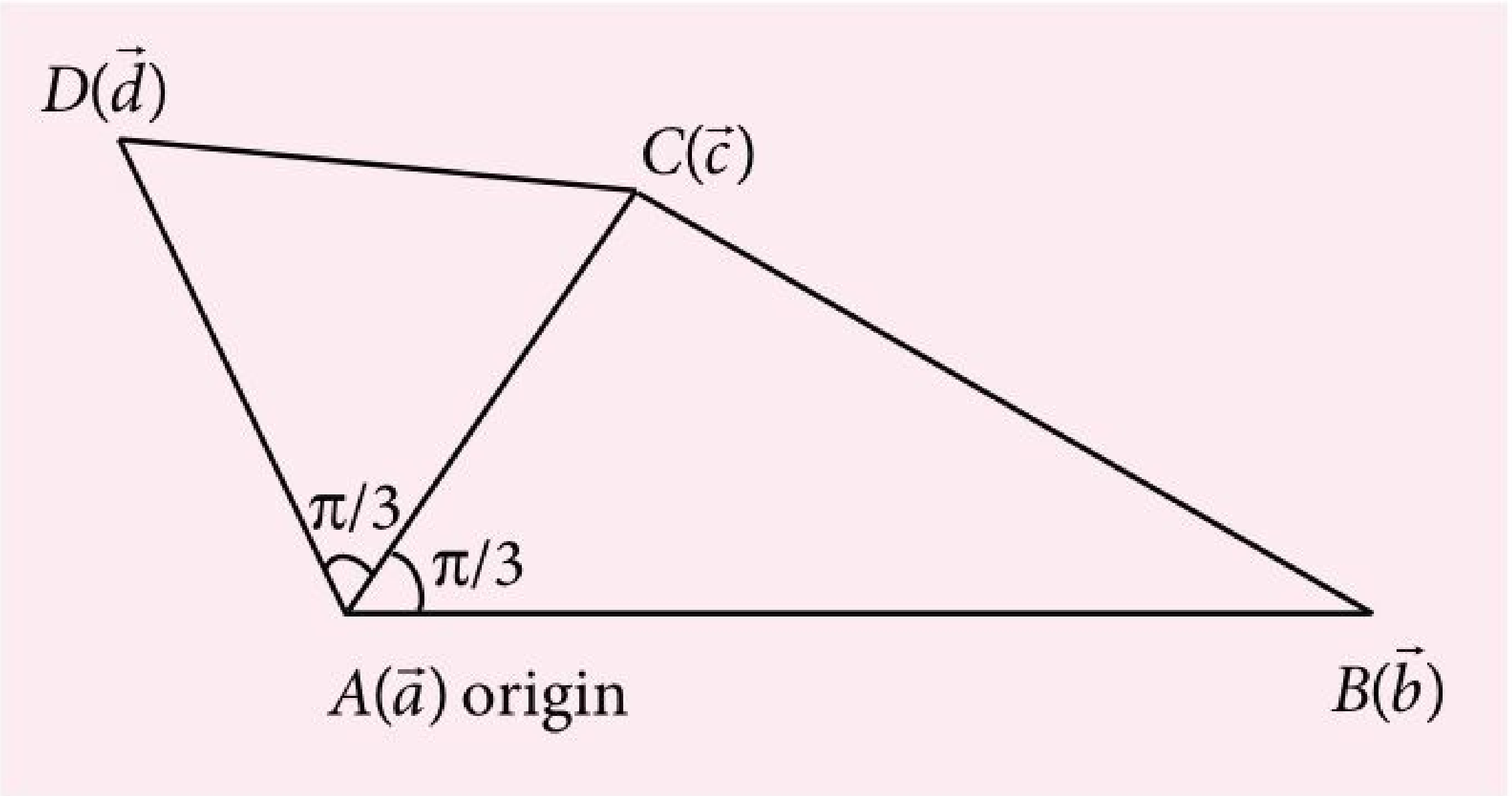
This is possible only when $\sin x=-1$ and $\cos y=-1$

For x^2+y^2 to be minimum, $x=-\dfrac{\pi}{2}$ and $y=\pi$

$\Rightarrow$ Minimum value of (x^2+y^2) is $=\dfrac{\pi^2}{4}+\pi^2=\dfrac{5\pi^2}{4}$

6. (c) : Given, $15|\overrightarrow{AC}|=3|\overrightarrow{AB}|=5|\overrightarrow{AD}|$

Let $|\overrightarrow{AC}|=\lambda>0$



COMIC CAPSULE

Why did the student wear glasses in math class?

To improve di-vision.

$\therefore \ |\overrightarrow{AB}| = 5\lambda, \ |\overrightarrow{AD}| = 3\lambda$

Now, cosine of angle between $\overrightarrow{BA}$ and $\overrightarrow{CD}$

$$= \frac{\overrightarrow{BA}\cdot\overrightarrow{CD}}{|\overrightarrow{BA}||\overrightarrow{CD}|} = \frac{-\vec{b}\cdot(\vec{d}-\vec{c})}{|\vec{b}||\vec{d}-\vec{c}|} \quad \text{...(i)}$$

Now, numerator of (i) = $\vec{b}.\vec{c} - \vec{b}.\vec{d}$

$$= |\vec{b}||\vec{c}|\cos\frac{\pi}{3} - |\vec{b}||\vec{d}|\cos\frac{2\pi}{3}$$

$$= (5\lambda)(\lambda)\cdot\frac{1}{2} + 5\lambda(3\lambda)\frac{1}{2} = \frac{5\lambda^2+15\lambda^2}{2} = 10\lambda^2$$

Denominator of (i) = $|\vec{b}|\,|\vec{d}-\vec{c}|$

Now, $|\vec{d}-\vec{c}|^2 = |\vec{d}|^2 + |\vec{c}|^2 - 2\vec{c}\cdot\vec{d} =$
$9\lambda^2 + \lambda^2 - 2(\lambda)\,(3\lambda)1/2 = 10\lambda^2 - 3\lambda^2 = 7\lambda^2$
$\Rightarrow |\vec{d}-\vec{c}| = \sqrt{7}\,\lambda$

Denominator of (i) = $(5\lambda)(\sqrt{7}\,\lambda) = 5\sqrt{7}\,\lambda^2$

$\therefore$ Cosine of angle between $\overrightarrow{BA}$ and $\overrightarrow{CD} = \dfrac{10\lambda^2}{5\sqrt{7}\,\lambda^2} = \dfrac{2}{\sqrt{7}}$

7. (c) : Considers, $OABC$ where $\overrightarrow{OA} = \vec{\alpha}, \overrightarrow{OB} = \vec{\beta}, \overrightarrow{OC} = \vec{\gamma}$

And OA, BC as a pair of opposite edges.
$|\overrightarrow{OA}| = a, \ |\overrightarrow{BC}| = b$

Equation of $OA \Rightarrow \vec{r} = \vec{O} + t\vec{\alpha}$
Equation of BC is $\vec{r} = \vec{\beta} + s(\vec{\beta} - \vec{\gamma})$

Thus, $d = \dfrac{-[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]}{|\vec{\alpha}||\vec{\beta}-\vec{\gamma}|\sin\theta} \Rightarrow V = \dfrac{abd\sin\theta}{6}$

8. (b) : $\overleftrightarrow{BC}$ is the x- axis and $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

Area of $\Delta ABC = \dfrac{1}{2}\times\sqrt{2}\times 2 = \sqrt{2}$

Also, volume $= \dfrac{1}{3}\times\sqrt{2}\times DE = \dfrac{2\sqrt{2}}{3} \Rightarrow DE = 2$

9. (c) : $\hat{n} = \dfrac{\hat{u}-\hat{v}}{|\hat{u}-\hat{v}|}; \ |\hat{u}-\hat{v}| = \sqrt{1+1-2(1)(1)\left(-\dfrac{1}{2}\right)} = \sqrt{3}$

10. (0) : Taking the dot product with $\vec{a}$,

$[\vec{a}\,\vec{b}\,\vec{c}] = p + q\cos\theta + r\cos\theta$ ...(i)

Taking dot product with $\vec{c}$,

$[\vec{a}\,\vec{b}\,\vec{c}] = p\cos\theta + q\cos\theta + r$ ...(ii)
From (i) and (ii), $p = r$

11. (3) : Let $A_0 = (-3, 6, 3)$, $B_0 = (0, 6, 0)$,
$\vec{c} = (2, 3, -2), \vec{d} = (2, 2, -1)$

Then $AB_{\text{min}} = |\text{projection of } A_0B_0 \text{ on } \vec{c}\times\vec{d}\,|$

$$= \frac{|(3, 0, -3)\cdot(1, -2, -2)|}{3} = 3$$

12. (9) : Projection length = $\dfrac{|\vec{a}\cdot\vec{p}|}{|\vec{p}|}$

So, $\dfrac{|x+2y+3z|}{\sqrt{14}} = 6 \Rightarrow |x+2y+3z| = 6\sqrt{14}$

$\Rightarrow \left|(x\hat{i}+\sqrt{2}y\hat{j})+\sqrt{3}z\hat{k})\cdot(i+\sqrt{2}\hat{j}+\sqrt{3}\hat{k})\right| = 6\sqrt{14}$

$\Rightarrow (x^2 + 2y^2 + 3z^2)(1 + 2 + 3)\cos^2\theta = \left(6\sqrt{14}\right)^2$

$\Rightarrow \dfrac{x^2+2y^2+3z^2}{\sqrt{14}} \geq 6\sqrt{14} \Rightarrow l = 6\sqrt{14}$

So, $(l^2 + 15^2)^{1/3} = (504 + 225)^{1/3} = (729)^{1/3} = 9$

13. (c) **14. (a)** **15. (b)**

We have $|\vec{x}| = |\vec{y}| = |\vec{z}| = \sqrt{2}$ and $\vec{x}, \vec{y}, \vec{z}$ make angle of 60° with each other.

$\therefore \ \vec{x}\cdot\vec{y} = |\vec{x}||\vec{y}|\cos 60° = \sqrt{2}\,(\sqrt{2})\cdot\dfrac{1}{2} = 1$

and $\vec{y}\cdot\vec{z} = |\vec{y}||\vec{z}|\cos 60° = \sqrt{2}\,(\sqrt{2})\cdot\dfrac{1}{2} = 1$

and $\vec{x}\cdot\vec{z} = |\vec{x}||\vec{z}|\cos 60° = \sqrt{2}(\sqrt{2})\left(\dfrac{1}{2}\right) = 1$

Also, $\vec{x}\cdot\vec{x} = |\vec{x}|^2 = 2$, $\vec{y}\cdot\vec{y} = |\vec{y}|^2 = 2$ and $\vec{z}\cdot\vec{z} = |\vec{z}|^2 = 2$

Now, $\vec{x}\times(\vec{y}\times\vec{z}) = \vec{a}$ and $\vec{y}\times(\vec{z}\times\vec{x}) = \vec{b}$ {given}
$\Rightarrow \ (\vec{x}\cdot\vec{z})\,\vec{y} - (\vec{x}\cdot\vec{y})\vec{z} = \vec{a}$ and $(\vec{y}\cdot\vec{x})\vec{z} - (\vec{y}\cdot\vec{z})\vec{x} = \vec{b}$

$\Rightarrow \ \vec{y} - \vec{z} = \vec{a}$ and $\vec{z} - \vec{x} = \vec{b} \ \Rightarrow \ \vec{y} - \vec{x} = \vec{a} + \vec{b}$

Thus, we have
$\vec{y} - \vec{z} = \vec{a}$ ...(i) ; $\vec{z} - \vec{x} = \vec{b}$ ...(ii) ; $\vec{y} - \vec{x} = \vec{a} + \vec{b}$ ...(iii) (given)
Now, $\vec{x}\times\vec{y} = \vec{c}$
$\Rightarrow \ \vec{x}\times(\vec{x}\times\vec{y}) = \vec{x}\times\vec{c}$
{taking cross-product with $\vec{x}$ }

$\Rightarrow \ (\vec{x}\cdot\vec{y})\,\vec{x} - (\vec{x}\cdot\vec{x})\vec{y} = \vec{x}\times\vec{c}$
$\Rightarrow \ \vec{x} - 2\vec{y} = \vec{x}\times\vec{c}$ ...(iv)
Again, $\vec{x}\times\vec{y} = \vec{c}$

$\Rightarrow \ \vec{y}\times(\vec{x}\times\vec{y}) = \vec{y}\times\vec{c}$ {taking cross product with $\vec{y}$ }
$\Rightarrow \ (\vec{y}\cdot\vec{y})\vec{x} - (\vec{y}\cdot\vec{x})\vec{y} = \vec{y}\times\vec{c}$
$\Rightarrow \ 2\vec{x} - \vec{y} = \vec{y}\times\vec{c}$ ...(v)

On subtracting (iv) from (v), we get
$\vec{x} + \vec{y} = (\vec{y}\times\vec{c}) - (\vec{x}\times\vec{c}) \Rightarrow \vec{x} + \vec{y} = (\vec{y} - \vec{x})\times\vec{c}$
$\Rightarrow \vec{x} + \vec{y} = (\vec{a} + \vec{b})\times\vec{c}$ ...(vi)
Adding (iii) and (vi), we get

$$2\vec{y}=(\vec{a}+\vec{b})+(\vec{a}+\vec{b})\times\vec{c}\ \Rightarrow\ \vec{y}=\frac{1}{2}[(\vec{a}+\vec{b})+(\vec{a}+\vec{b})\times\vec{c}]$$

Substituting the value of $\vec{y}$ in (iii) and (i), we get

$$\vec{x}=\frac{1}{2}[(\vec{a}+\vec{b})+(\vec{a}+\vec{b})\times\vec{c}]-(\vec{a}+\vec{b})$$

$$\Rightarrow\quad \vec{x}=\frac{1}{2}[(\vec{a}+\vec{b})\times\vec{c}-(\vec{a}+\vec{b})]$$

$$\vec{z}=\frac{1}{2}[(\vec{a}+\vec{b})+(\vec{a}+\vec{b})\times\vec{c}]-\vec{a}$$

16. (a) : Let $\alpha=\cos^{-1}\frac{1}{4}\Rightarrow\cos\alpha=\frac{1}{4}$

$$\frac{\overrightarrow{AD}\cdot\overrightarrow{BC}}{|\overrightarrow{AD}||\overrightarrow{BC}|}=\frac{\vec{d}\cdot(\vec{c}-\vec{b})}{|\vec{d}||\vec{c}-\vec{b}|}$$

$$\Rightarrow\ 4\vec{d}\cdot(\vec{c}-\vec{b})=|\vec{d}||\vec{c}-\vec{b}|=|\vec{b}||\vec{d}|$$

Since $|\vec{b}-\vec{c}|=|\vec{b}|=|\vec{c}|$

Let angle between $\vec{d}$ and $\vec{b}$ is θ.

and the angle between $\vec{d}$ and $\vec{c}$ is $\pi/3$.

$$\Rightarrow\ 4\left(|\vec{d}||\vec{c}|\frac{1}{2}-|\vec{d}||\vec{b}|\cos\theta\right)=|\vec{d}||\vec{b}|$$

$$\Rightarrow\ 4\left(\frac{1}{2}-\cos\theta\right)=1\Rightarrow\cos\theta=\frac{1}{4}$$

17. (b) : $ABC\perp DBC$

$$\Rightarrow(\overrightarrow{AB}\times\overrightarrow{AC})\cdot(\overrightarrow{BD}\times\overrightarrow{DC})=0$$

$$\Rightarrow\ (\vec{b}\times\vec{c})\cdot((\vec{b}-\vec{d})\times(\vec{c}-\vec{d}))=0$$

$$\Rightarrow\ (\vec{b}\times\vec{c})\cdot(\vec{b}\times\vec{c}-\vec{b}\times\vec{d}-\vec{d}\times\vec{c})=0$$

$$\Rightarrow\ |\vec{b}\times\vec{c}|^2-(\vec{b}\times\vec{c})\cdot(\vec{b}\times\vec{d})-(\vec{b}\times\vec{c})\cdot(\vec{d}\times\vec{c})=0$$

$$\Rightarrow\ |\vec{b}|^2|\vec{c}|^2\frac{3}{4}-\begin{vmatrix}\vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{d}\\ \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{d}\end{vmatrix}-\begin{vmatrix}\vec{b}\cdot\vec{d} & \vec{b}\cdot\vec{c}\\ \vec{c}\cdot\vec{d} & \vec{c}\cdot\vec{c}\end{vmatrix}=0$$

$$\Rightarrow\ \frac{3|\vec{b}|}{4}-6\left(\frac{1}{2}-\frac{1}{8}\right)-6\left(\frac{1}{4}-\frac{1}{4}\right)=0$$

$$\Rightarrow\ |\vec{b}|=3\text{ and }|\vec{c}|=3\Rightarrow|\vec{b}|+|\vec{c}|=6$$

18. (b) : $[\vec{d}\ \vec{b}\ \vec{c}]^2=\begin{vmatrix}\vec{d}\cdot\vec{d} & \vec{d}\cdot\vec{b} & \vec{d}\cdot\vec{c}\\ \vec{b}\cdot\vec{d} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c}\\ \vec{c}\cdot\vec{d} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c}\end{vmatrix}$

$$=|\vec{d}|^2|\vec{b}|^2|\vec{c}|^2\begin{vmatrix}1 & 1/4 & 1/2\\ 1/4 & 1 & 1/2\\ 1/2 & 1/2 & 1\end{vmatrix}$$

∴ Volume of tetrahedron $=\frac{1}{6}[\vec{d}\ \vec{b}\ \vec{c}]=\frac{27}{4}$ Cu.units

19. (d) : Let $\vec{r}=x\hat{i}+y\hat{j}$; $\vec{r}\cdot(10\hat{j}-8\hat{i}-\vec{r})=40$

$\Rightarrow x^2+y^2+8x-10y+40=0$

$\Rightarrow$ $\vec{r}$ lie on a circle in XOY plane.

$l=$ (max. distance of $\vec{r}$ from $-2\hat{i}+3\hat{j})^2=9+4\sqrt{2}$

$m=$ (min. distance of $\vec{r}$ from $-2\hat{i}+3\hat{j})^2=9-4\sqrt{2}$

20. (c) : $l=9+4\sqrt{2},\ m=9-4\sqrt{2}$; $l+m=18$

21. (c) : Clearly, point A (2, 2)

Equation of tangent at $A(2, 2)$ is $2x+y-6=0$

Co-ordinate of B (3, 0)

$\overrightarrow{AB}=\hat{i}-2\hat{j}$, $\overrightarrow{OB}=3\hat{i}$, $\overrightarrow{AB}\cdot\overrightarrow{OB}=3$

22. (c) : Projection of $\vec{x}$ on $\vec{y}$ is $\frac{\vec{x}\cdot\vec{y}}{|\vec{y}|}=|\vec{x}|\cos\theta$

$$=|\vec{x}|\left(\frac{1-\tan^2\theta/2}{1+\tan^2\theta/2}\right)$$

But $\vec{x}*\vec{y}=|\vec{x}|\cdot|\vec{y}|\tan\frac{\theta}{2}$

So, projection of $\vec{x}$ on $\vec{y}$ is $|\vec{x}|\left(\frac{|\vec{x}|^2|\vec{y}|^2-(\vec{x}*\vec{y})^2}{|\vec{x}|^2|\vec{y}|^2+(\vec{x}*\vec{y})^2}\right)$

23. (b) : Area of parallelogram $=|\vec{x}||\vec{y}|\sin\theta$

$$=|\vec{x}||\vec{y}|\left(\frac{2\tan\theta/2}{1+\tan^2\theta/2}\right)$$

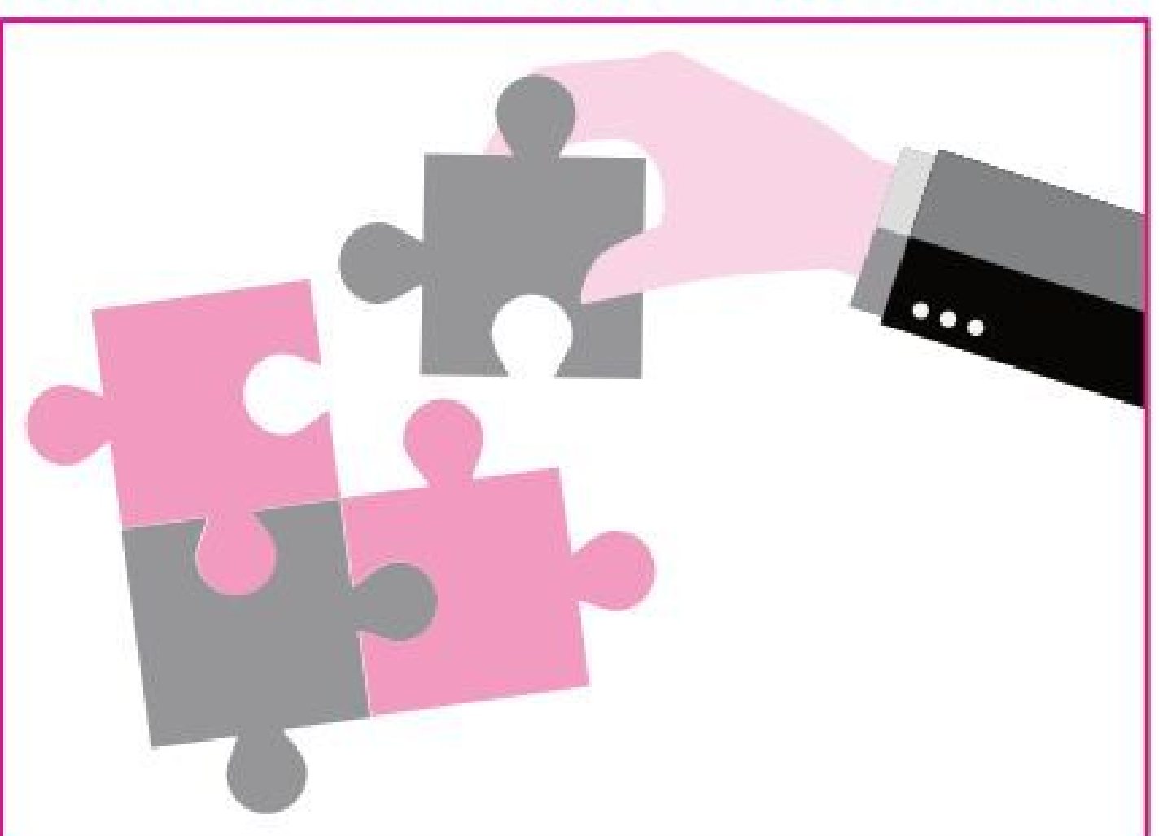
24. (b) : $|\vec{x}\times\vec{y}|=|\vec{x}*\vec{y}|$

$$\Rightarrow\ |\vec{x}||\vec{y}|\sin\theta=|\vec{x}||\vec{y}|\tan\frac{\theta}{2}\ \Rightarrow\ \theta=\frac{\pi}{2}$$

◆ ◆

PUZZLE CORNER

ANSWER - DECEMBER 2022



3 3	4– 5	1	2÷ 6	10+ 4	2
3÷ 6	10+ 2	5	3	5× 1	4
2	3	2– 6	4	5	1
3– 4	24× 6	24× 3	3+ 1	2	2– 5
1	4	2	3– 5	2÷ 6	3
4– 5	1	4	2	3	6 6

GK CORNER

Enhance Your General Knowledge with Current Updates!

INDIA AND THE WORLD

IDRA

- The International Drought Resilience Alliance (IDRA) was launched on November 7, 2022 during the 27th Conference of Parties (COP27) to the United Nations Framework Convention on Climate Change (UNFCC).

NATO

- India will assume the chairmanship of Wassenaar Arrangement on January 1, 2023. Recently, on 26th annual plenary of the Wassenaar Arrangement in Vienna, Ireland handed over the chairmanship to India. Wassenaar Arrangement's Secretariat is in Vienna, Austria. It has 42 member states comprising mostly **NATO (North Atlantic Treaty Organisation)** and EU states.
- India became a member of the Arrangement in 2017.

UNSC

- On 1st December, India assumed the monthly rotating presidency of the **UN Security Council (UNSC)** for the second time in its two-year tenure as an elected member of the Council in 2021-22.
- The most important function of the UNSC is maintaining peace and security at the international level.
- The five permanent members of UNSC (also known as permanent five or P5) include China, France, Russia, the United Kingdom, and the United States.

GCC

- India and the **Gulf Cooperation Council (GCC)** have agreed to pursue a **Free Trade Agreement (FTA)** between the two regions and resume the negotiations.
- GCC is a union of six countries in the Gulf region — Saudi Arabia, the UAE, Qatar, Kuwait, Oman and Bahrain. The council is the largest trading bloc of India.

CITES

- The 19th Meeting of the Conference of the Parties (CoP19) to the **Convention on International Trade in Endangered Species of Wild Fauna and Flora (CITES)** is held at Panama City between 14-25 November 2022.
- CITES is an international agreement between governments of 184 countries at present to ensure that international trade in wild animals and plants does not threaten the survival of the species.
- The convention entered into force in 1975 and India joined in 1976 as a state that voluntarily agrees to be bound by the Convention.

G20

- The G20 was formed in 1999 after the Asian Financial Crisis as a forum for the Finance Ministers and Central Bank Governors to discuss global economic and financial issues.

- The group of twenty (G20) comprises 19 countries: Argentina, Australia, Brazil, Canada, China, France, Germany, India, Indonesia, Italy, Japan, Republic of Korea, Mexico, Russia, Saudi Arabia, South Africa, Turkey, the United Kingdom, the United States and the European Union.
- The 17th annual summit of G-20 was hosted by the Indonesian G20 presidency in Bali under the theme 'Recover Together, Recover Stronger'.
- Now, India has assumed the charge of the G20 presidency and the 18th summit will be held in India in 2023.
- The G20 members represent around 85% of the global GDP, over 75% of the global trade and about two-thirds of the world population.

ASEAN

- ASEAN is a regional grouping that promotes economic, political and security cooperation.
- It was established in August 8, 1967 in Bangkok, Thailand by five original members countries: Indonesia, Malaysia, Philippines, Singapore and Thailand.
- Its chairmanship rotates annually, based on the alphabetical order of the English names of Member Countries.
- ASEAN brings together eleven Southeast Asian countries – Brunei, Cambodia, Indonesia, Laos, Malaysia, Myanmar, Philippines, Singapore, Thailand, Timor-leste and Vietnam.
- The headquarter of ASEAN is in Jakarta, Indonesia.

BIMSTEC

- The Bay of Bengal Initiative for Multi-Sectoral Technical and Economic Cooperation (BIMSTEC) is a regional organisation that was established on 6th June, 1997. BIMSTEC comprising Seven member states: Bangladesh, Bhutan, India, Nepal, Sri Lanka, Myanmar and Thailand.
- The BIMSTEC region is home to around 1.5 billion people which constitute around 22% of the global population with a combined Gross Domestic Product (GDP) of 2.7 trillion economy.
- The BIMSTEC Secretariat was established in Dhaka, Bangladesh on 13th September 2014.

OECD

- The Organisation for Economic Co-operation and Development (OECD) recently slashed India's GDP forecast for the current financial year (2023) from 6.9 percent to 6.6 percent in its latest economic outlook report.
- OECD currently has 38 members and the Member Countries are represented by a delegation which is led by their ambassadors.
- The most recent countries to join the OECD were Colombia, in April 2020 and Costa Rica, in May 2021.

MILITARY EXERCISES

Exercise	Countries Participated
Agni Warrior	Singapore and India
SITMEX	India, Thailand and Singapore
Varuna	India and France
Naseem Al Bahr-2022	India and Oman
Garuda Shakti	India and Indonesia

MALAYSIA

- On November 24, veteran opposition leader Anwar Ibrahim sworn as the 10th Prime Minister of Malaysia.
- Currency of Malaysia is Malaysian ringgit.

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Test Yourself!

1. Who is the current Prime Minister of Malaysia ?
   (a) Ismail Sabri yaakob (b) Muhyiddin Yassin
   (c) Anwar Ibrahim (d) Abdullah Ahmed
2. At present, how many members countries are associated with CITES.
   (a) 183 (b) 182 (c) 184 (d) 185
3. Which is not the permanent member of UNSC?
   (a) China (b) France
   (c) India (d) Russia
4. Which of the following country is not the member of G20 group?
   (a) Japan (b) Republic of Korea
   (c) Republic of Iraq (d) European Union
5. The International Drought Resilience Alliance was launched on
   (a) November 25, 2020 (b) November 7, 2022
   (c) September 8, 2021 (d) December 7, 2022
6. In which year, India Join OECD?
   (a) 1993 (b) 1995 (c) 1998 (d) 2000
7. Where is the headquarters of ASEAN?
   (a) Hague (b) Jakarta (c) Paris (d) Brussels
8. SITMEX is a series of naval exercises conducted by the navies of
   (a) India, Thailand, Malaysia
   (b) India, Signapore, Thailand
   (c) Malaysia, Thailand, Switzerland
   (d) Indonesia, Sweden, Malaysia
9. The Bay of Bengal Initiative for Multi-Sectoral Technical and Economic Cooperation (BIMSTEC) was established in
   (a) 1998 (b) 1999 (c) 2001 (d) 1997
10. Secretariat of BIMSTEC is located in
    (a) Thailand (b) Bangladesh
    (c) Malaysia (d) Sweden

Answer Key

1. (c) **2.** (c) **3.** (c) **4.** (c) **5.** (b)
6. (b) **7.** (b) **8.** (b) **9.** (d) **10.** (b)

◆ ◆

UNSCRAMBLE ME

Unscramble the words given in column I and match them with their explanations in column II.

Column I	Column II
1. MAAXIM	(a) A polygon that consists of three arms and three vertices.
2. BHOLAYREP	(b) Two dimensional surface that extends indefinitely, in all directions.
3. CEQUSENE	(c) The exponent of the highest order derivative.
4. GMENTDERANE	(d) The operation of resolving a quantity into factors.
5. HOCRD	(e) A group of numbers and variables with no equal sign.
6. PRIXEOSNSE	(f) The line segment joining any two points on the circumference of the circle.
7. SACTFOARINIOT	(g) The peaks in the curve of a function.
8. REEDGE	(h) The part of a conic, whose eccentricity is greater than one.
9. ANELP	(i) An arrangement of numbers in a particular order.
10. RIATNELG	(j) The permutation of the elements of a set such that no element appear in its original position.

Readers can send their responses at editor@mtg.in or post us with complete address by 10th of every month. Winners' names and answers will be published in next issue.

Quiz CLUB

Unlock Your Knowledge!

1. If $P = \{0, 1, 2, 3, 4\}$, $Q = \{4, 5, 6, 7\}$, $R = \{3, 8, 9\}$ and $S = \{6, 12, 18\}$. Then what is $(P \cup Q) \cap (S \cup R)$.
2. Write the number of bijective functions from set A to itself, when A contains 207 elements.
3. If A and B are two events, where $P(A) = 0.2$, $P(B) = 0.4$ and $P(A \cup B) = 0.5$, then find the value of $P(A/B)$.
4. Find the range of the function $\frac{1}{1 - \sin x}$.
5. Let $f : R \to R$ be defined by $f(x) = 1 - |x|$. Find the range of f.
6. Find the value of $(1 - i)^3 (1 - i^3)$.
7. Find 6th term in the expansion of $(1 + x)^{10}$.
8. Write the number of constant functions from the set containing m elements to a set containing n elements.
9. If $f(x) = x \sin x$, then find $f'(\pi/2)$.
10. Find the geometric mean of series having arithmetic mean = 25 and harmonic mean = 16.
11. What is total number of elements in sample space when a coin is tossed and a die is thrown?
12. If $A = \{(x, y) : x^2 + y^2 = 25\}$ and $\{(x, y) : x^2 + 9y^2 = 144\}$, then find the number of points that contain $A \cap B$.
13. Write the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4, respectively and $\vec{a} \cdot \vec{b} = 2\sqrt{3}$.
14. Find the integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$.
15. Find the interval on which the $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing.
16. Find the slope of the normal to the curve $y = 2x^2 + 3 \sin x$ at $x = 0$.
17. Find the value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in the interval $[0, \sqrt{3}]$.
18. Which matrix is both symmetric and skew symmetric.
19. Name the matrix which is not a square matrix .
20. Write the principal value of $\cos^{-1}(-1/2)$.

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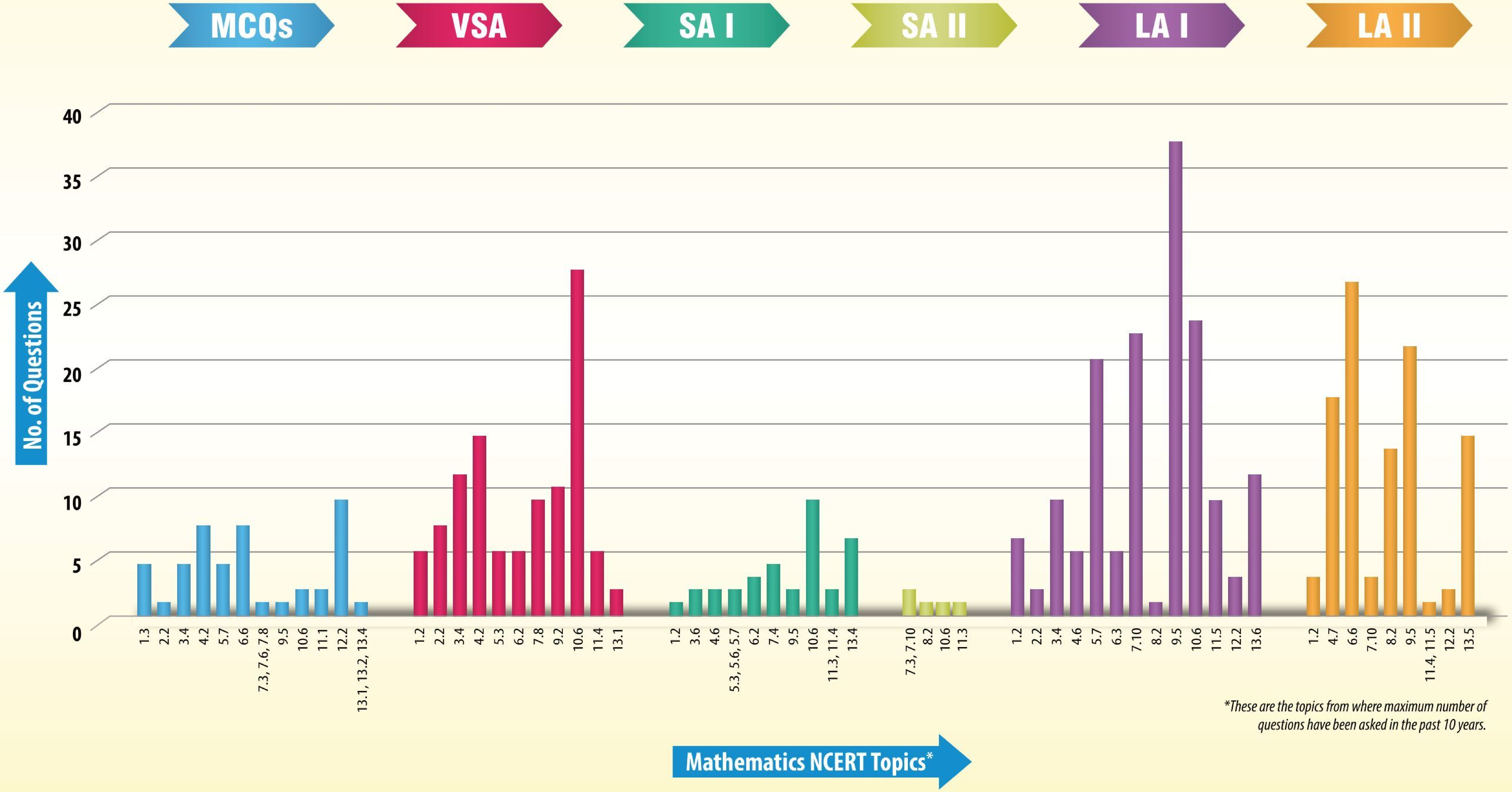
Are You Ready for CBSE Board Exams?

Past 10 Years (2013-2022) Chapterwise and Topicwise Trend Analysis of CBSE Board Questions



*These are the topics from where maximum number of questions have been asked in the past 10 years.

Some of the important topics extracted from graphical analysis are explained here.

Relations and Functions

Types of Relations

Reflexive : A relation R on a set A is reflexive if $(a, a) \in R \ \forall \ a \in A$.

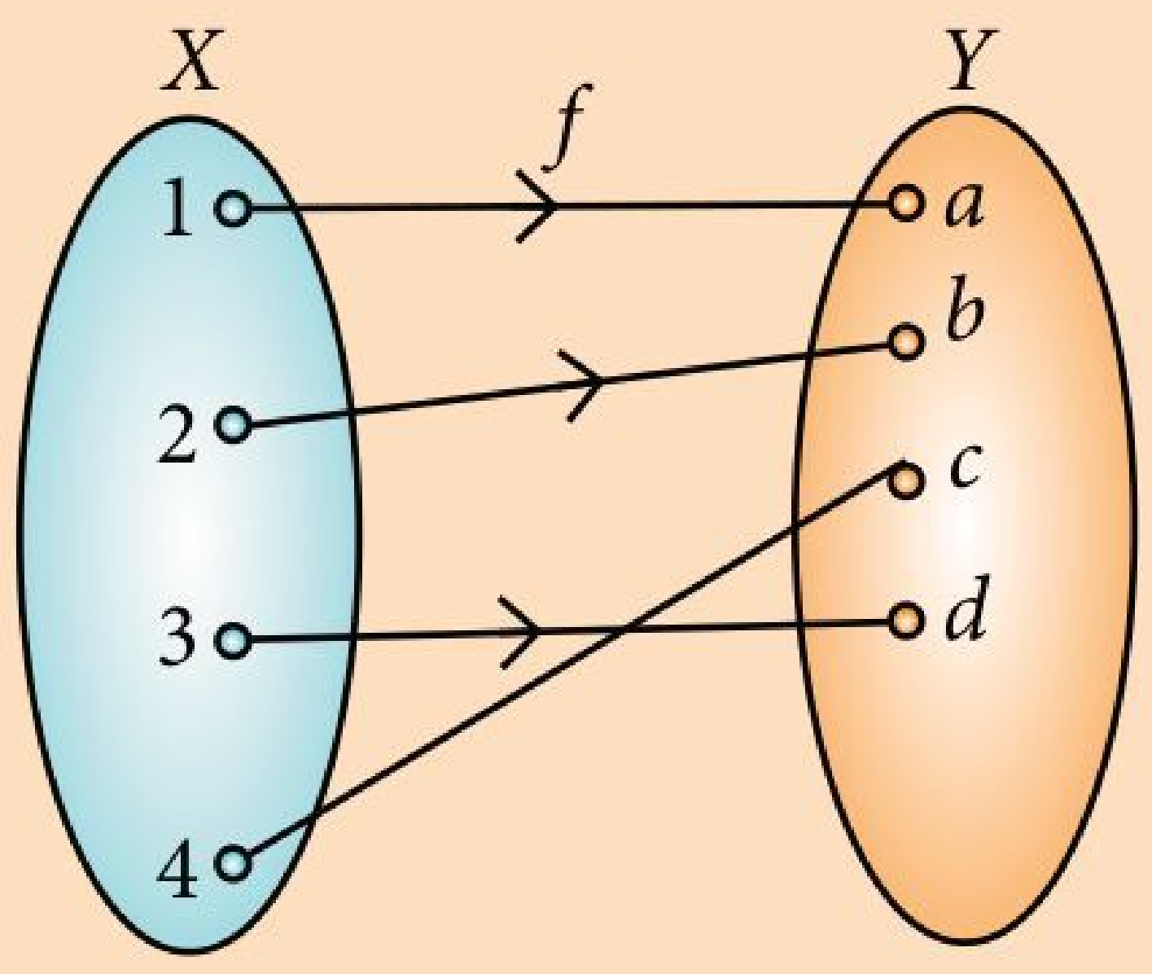
Symmetric : A relation R on a set A is symmetric if $(a_1, a_2) \in R \Rightarrow (a_2, a_1) \in R \ \forall \ a_1, a_2 \in A$.

Transitive : A relation R on a set A is transitive if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R \Rightarrow (a_1, a_3) \in R \ \forall \ a_1, a_2, a_3 \in A$.

Equivalence : A relation R is an equivalence relation if R is reflexive, symmetric and transitive.

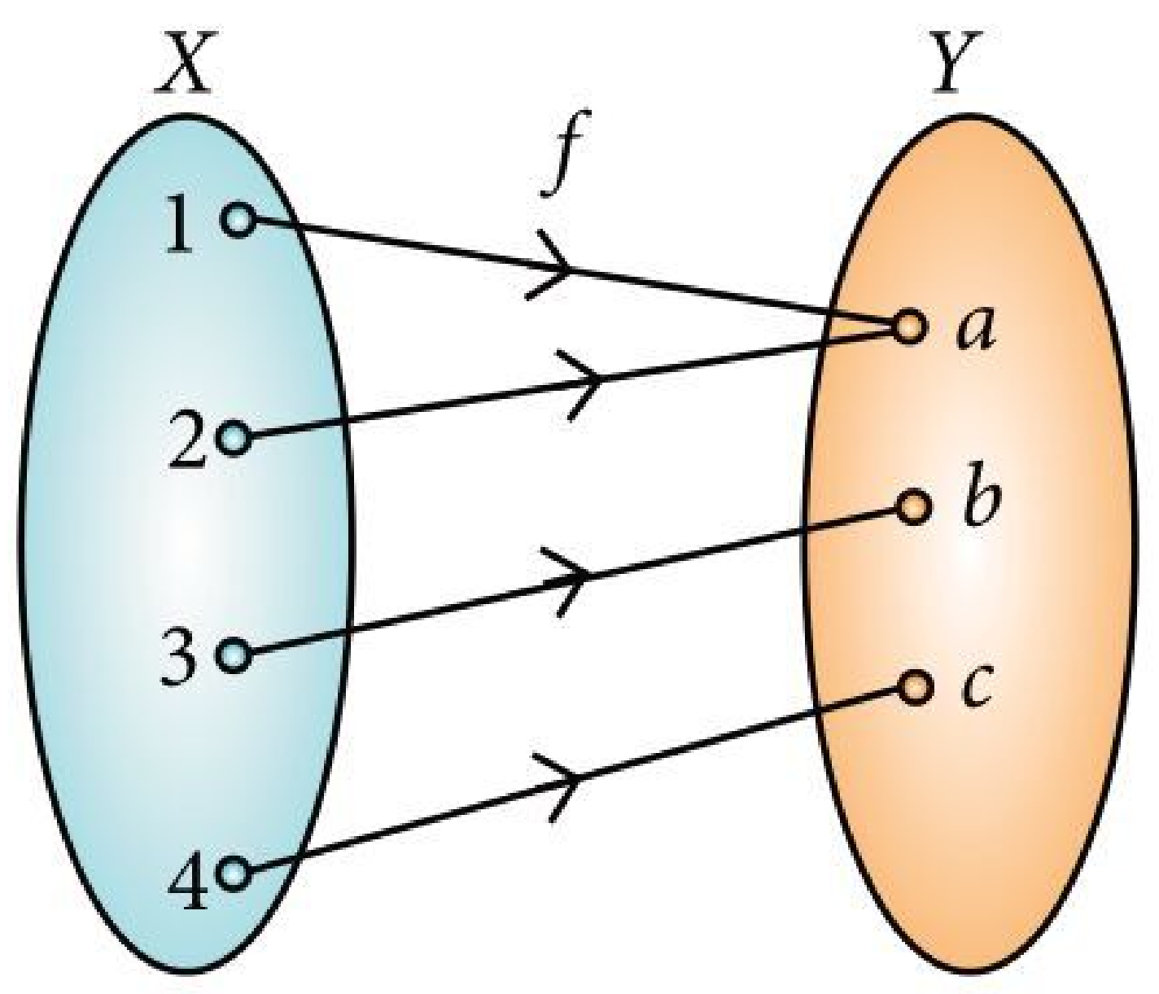
Types of Functions

One-one function



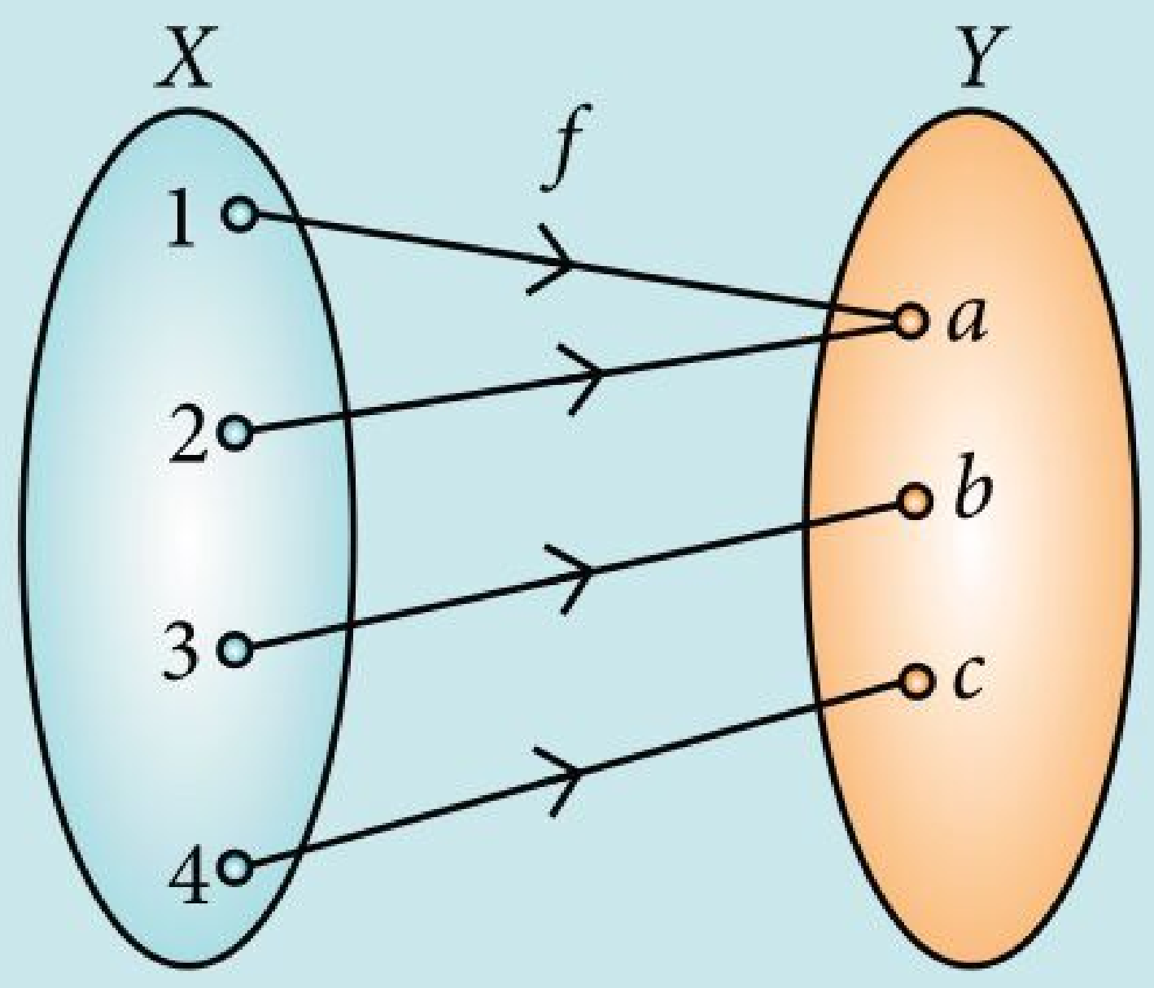
Distinct elements of X have distinct images in Y.

Many one function



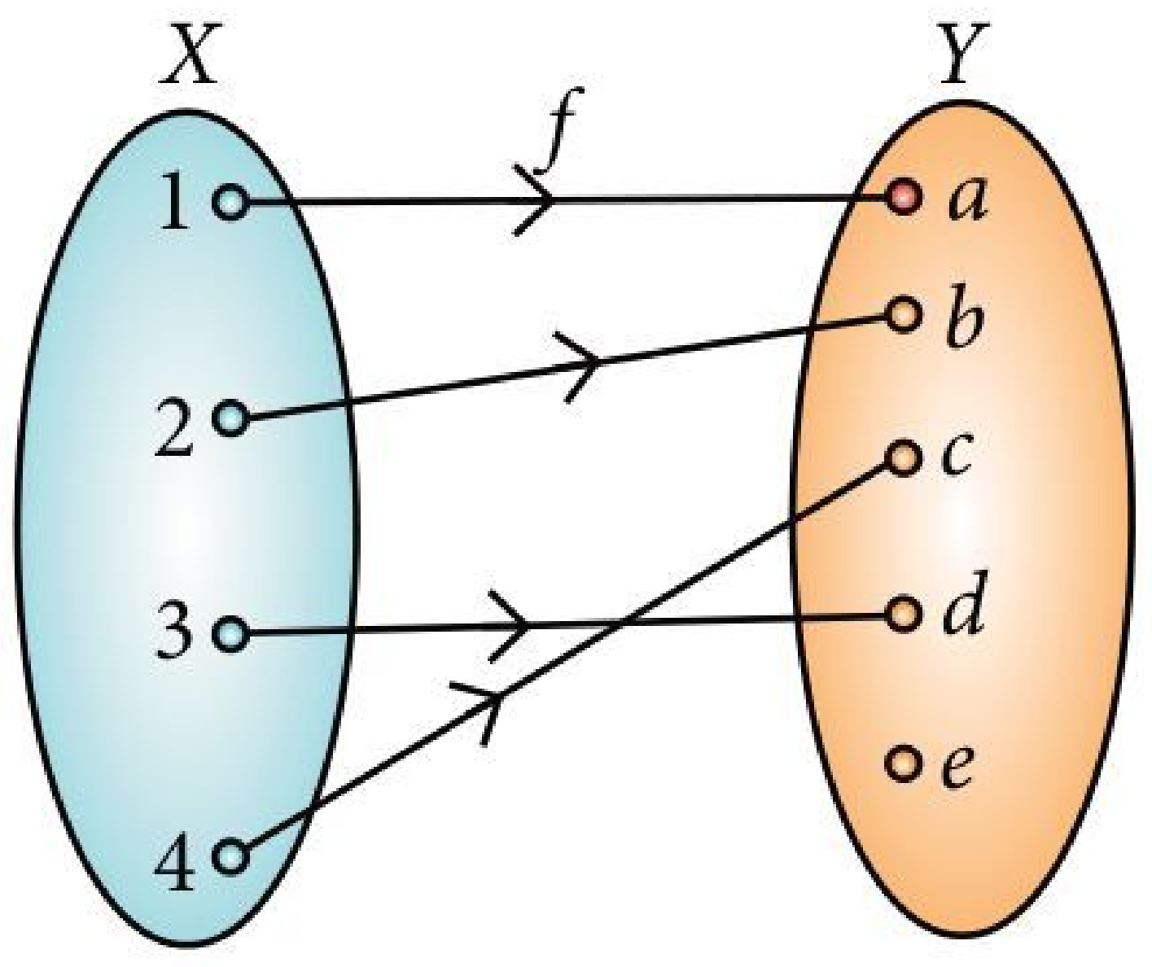
Two or more elements of X have same image in Y.

Onto function



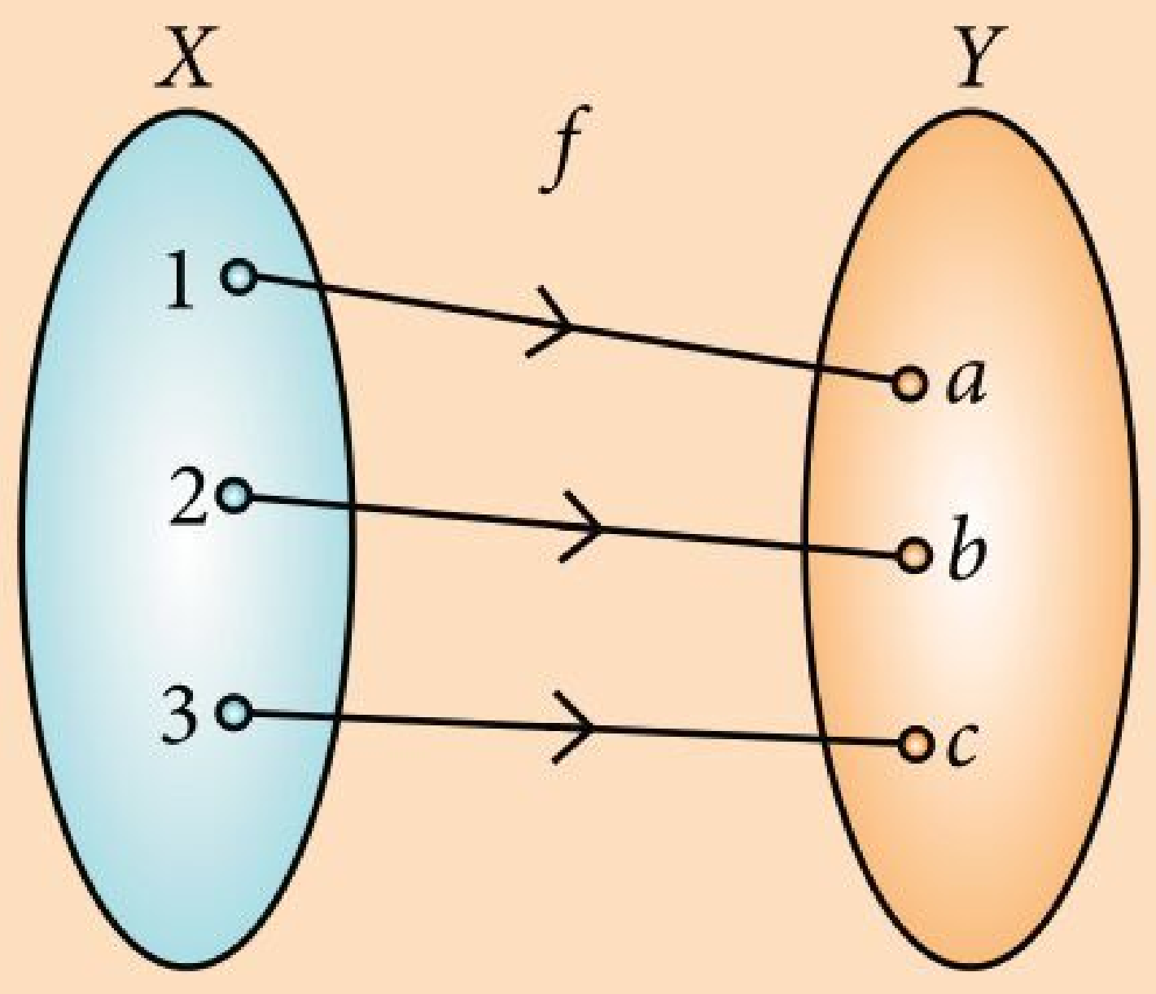
Range of f = co-domain (Every element in Y has a pre-image in X)

Into function



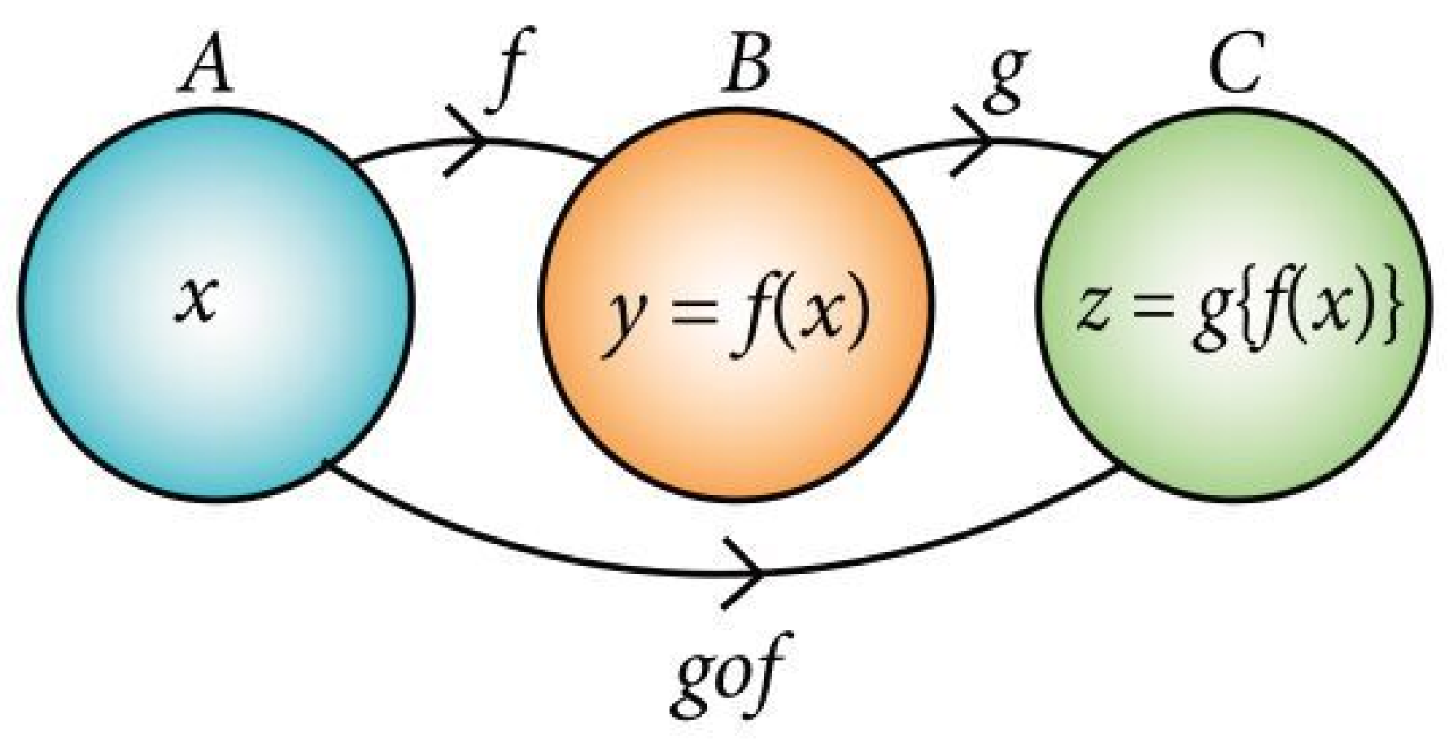
Range of f is a proper subset of co-domain (There exists at least one element in Y which is not the image of any element of X)

Bijective function



Range of f = co-domain

Composition of function



$$gof(x) = g\{f(x)\} \ \forall \ x \in A$$

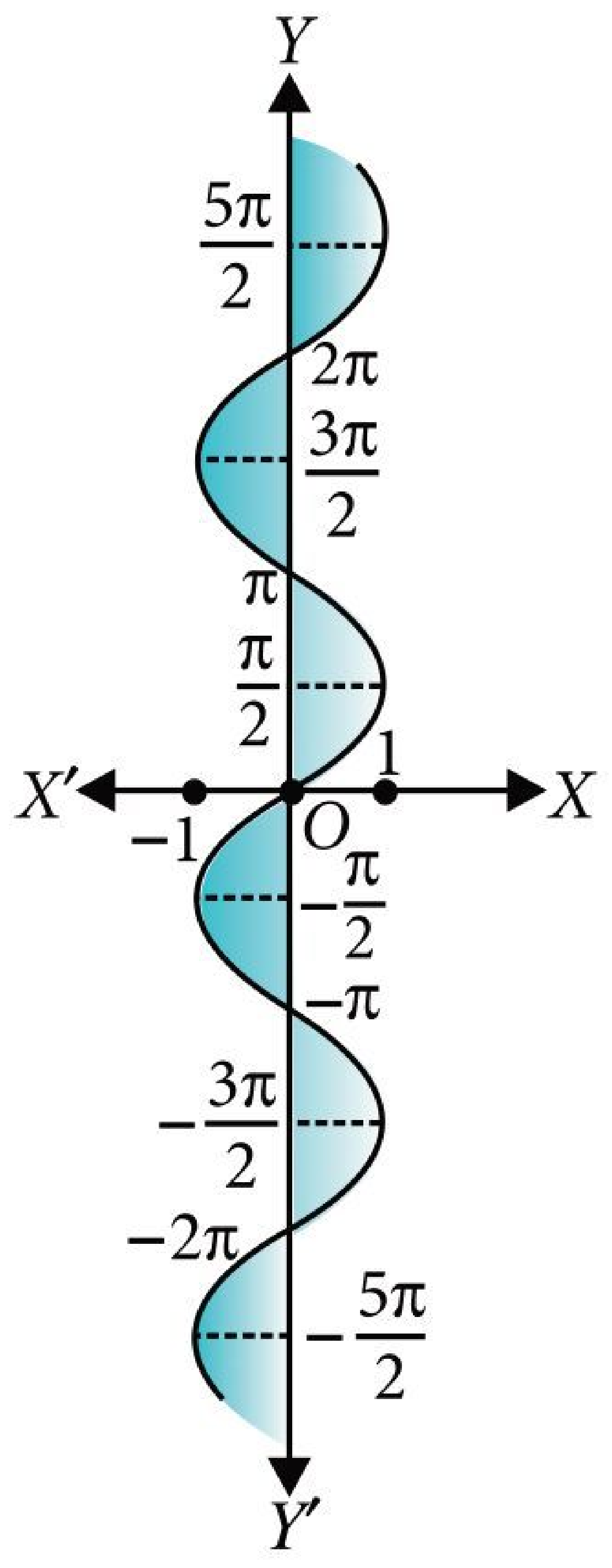
$$fo(goh) = (fog)oh$$

$$fog \neq gof$$

Inverse Trigonometric Functions

Graph of Inverse Trigonometric Functions

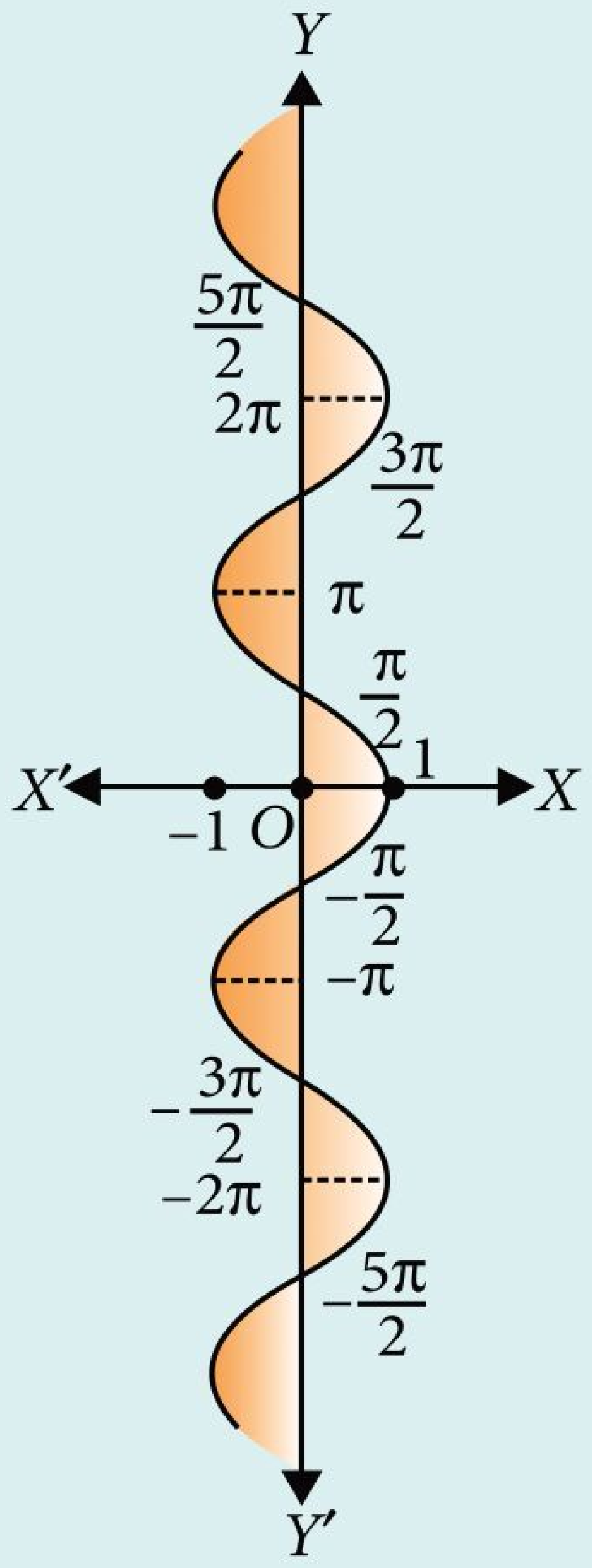
$y = \sin^{-1}x$



Domain : [−1, 1]

Range : $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

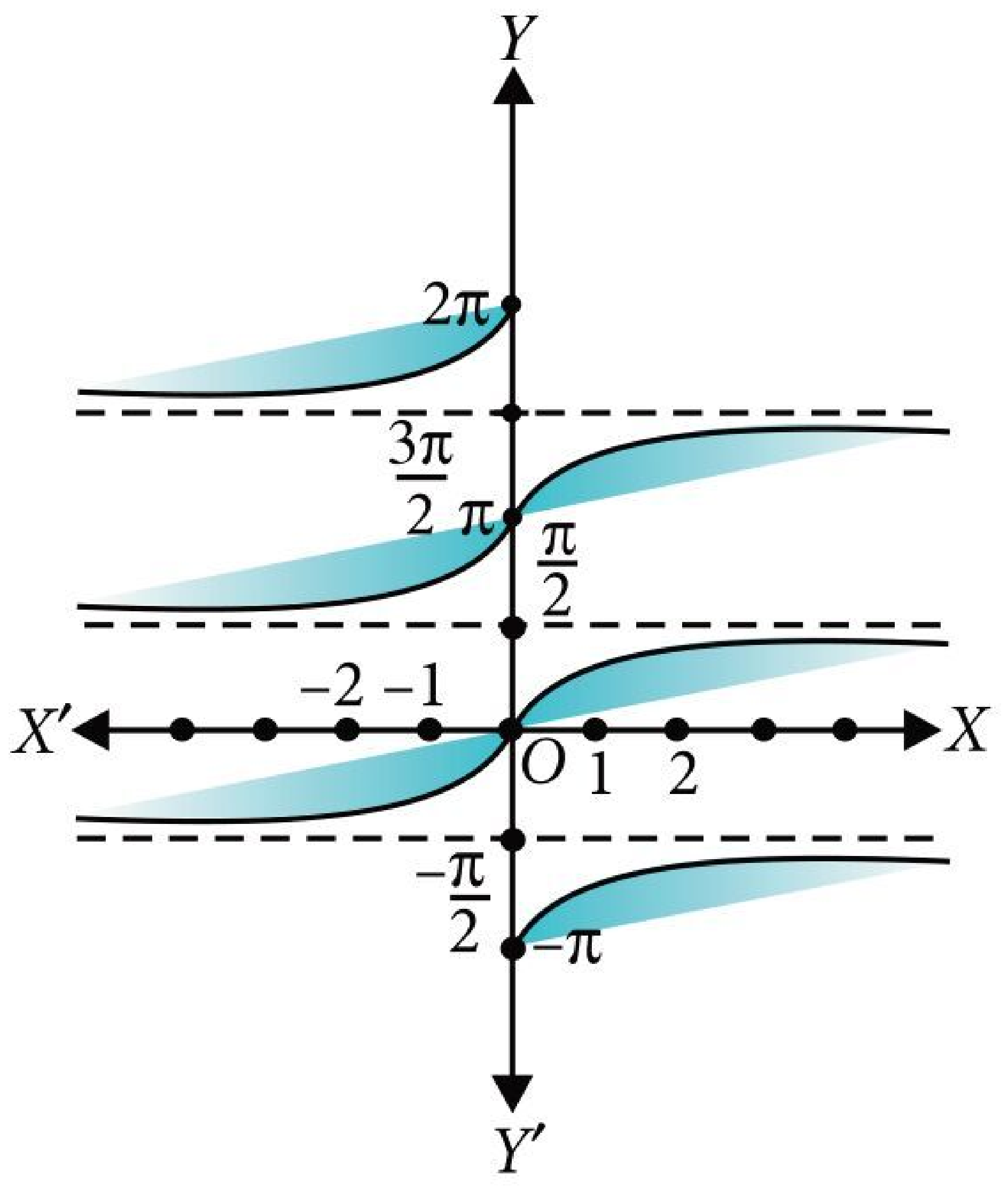
$y = \cos^{-1}x$



Domain : [−1, 1]

Range : [0, π]

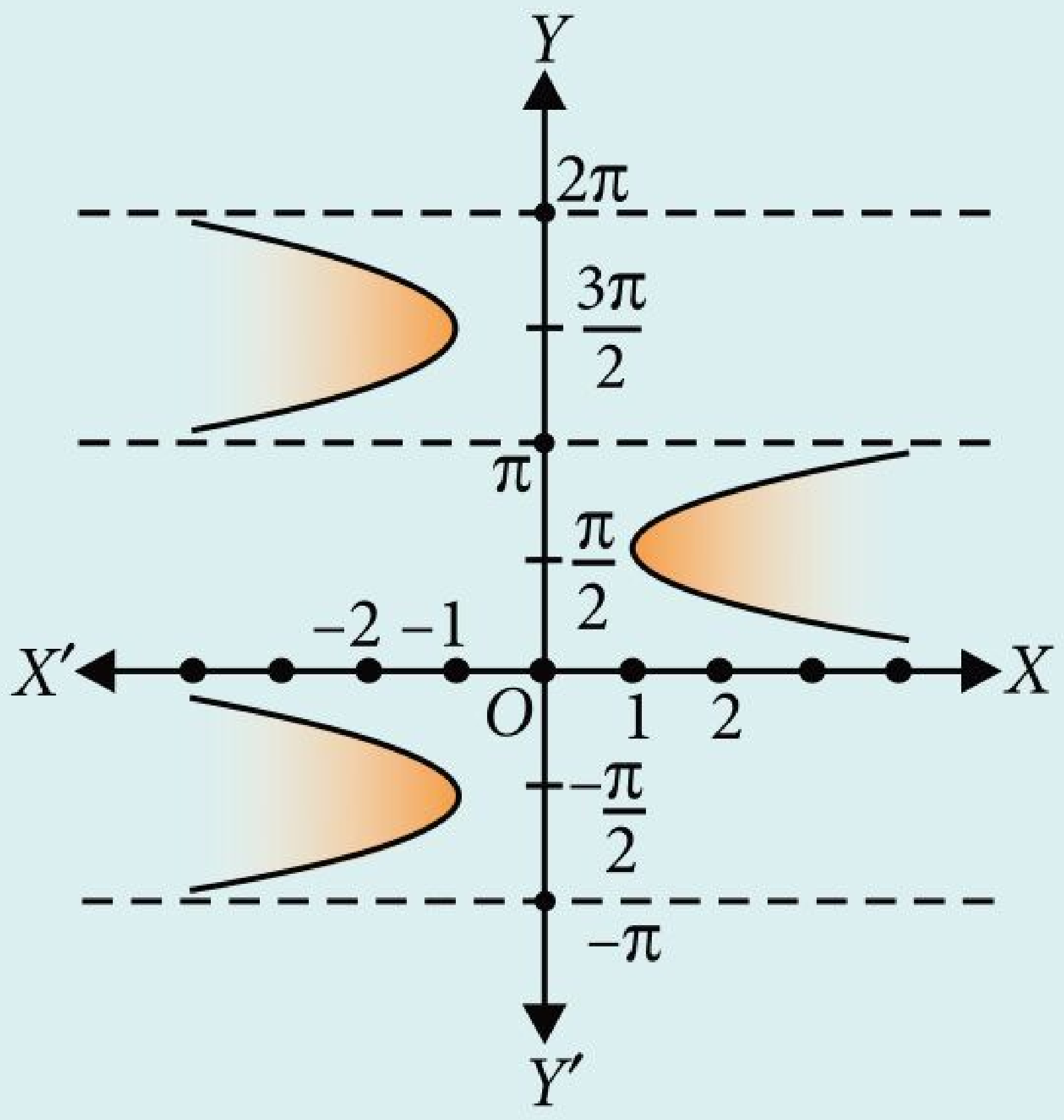
$y = \tan^{-1}x$



Domain : R

Range : $\left]-\frac{\pi}{2},\frac{\pi}{2}\right[$

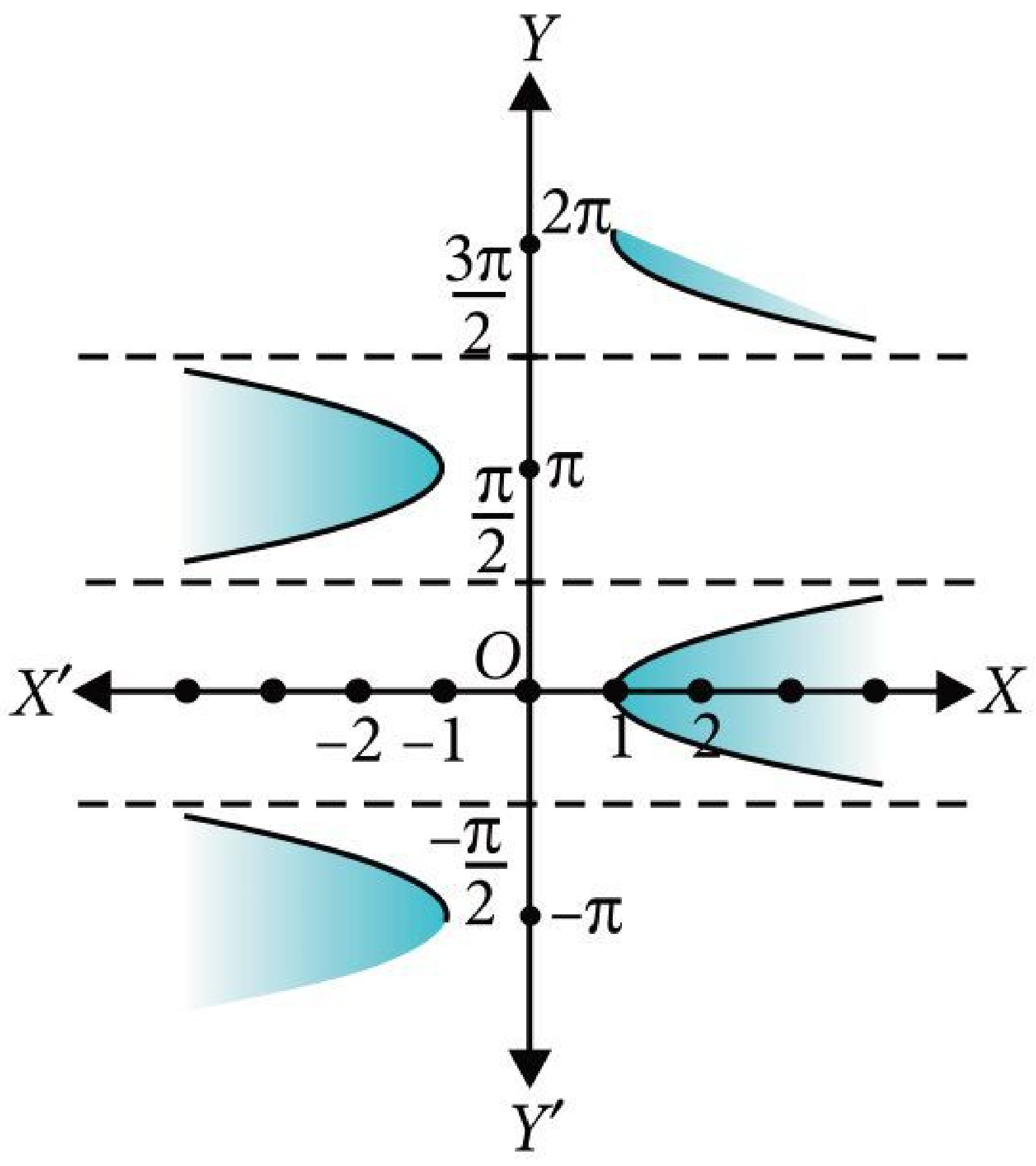
$y = \text{cosec}^{-1}x$



Domain : R −]−1, 1[

Range : $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$

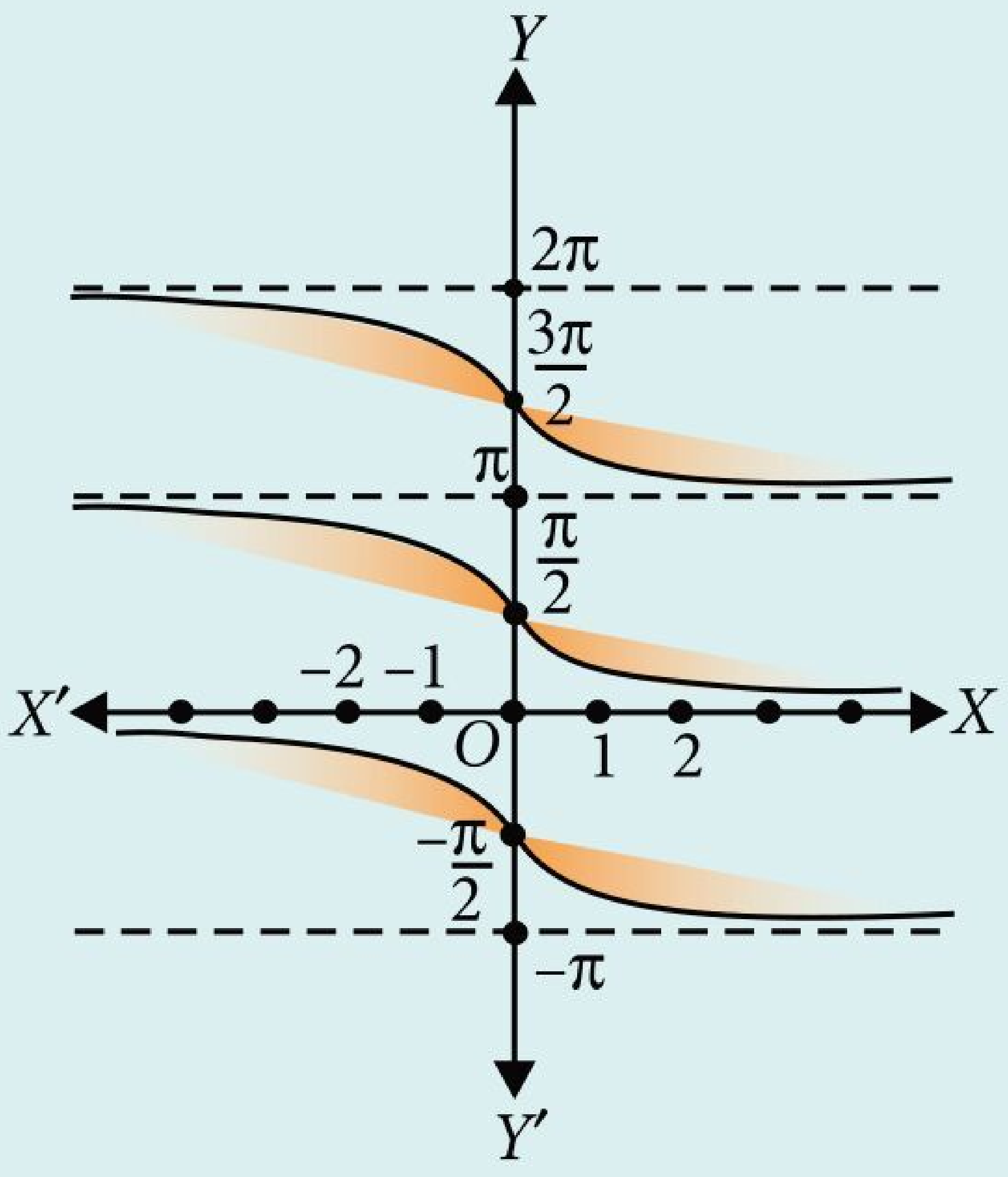
$y = \sec^{-1}x$



Domain : R −] − 1, 1[

Range : $[0,\pi]-\left\{\frac{\pi}{2}\right\}$

$y = \cot^{-1}x$



Domain : R

Range :]0, π[

Matrices and Determinants

Types of Matrices

Row matrix

$$\begin{bmatrix} a & b & c \end{bmatrix}$$

Column matrix

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Zero matrix

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Unit matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Lower triangular matrix

$$\begin{bmatrix} a & 0 & 0 \\ d & b & 0 \\ e & f & c \end{bmatrix}$$

Upper triangular matrix

$$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

Diagonal matrix

$$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

Scalar matrix

$$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

Operations on Matrices

Addition and subtraction

$A \pm B = C$

i.e., $[a_{ij}]_{m \times n} \pm [b_{ij}]_{m \times n} = [c_{ij}]_{m \times n}$

Multiplication

$AB = C$

i.e., $[a_{ij}]_{m \times n} \times [b_{jk}]_{n \times p} = [c_{ik}]_{m \times p}$

Scalar multiplication

$kA = B$

$k[a_{ij}]_{m \times n} = [ka_{ij}]_{m \times n} = [b_{ij}]_{m \times n}$

Inverse of Matrix

Adjoint relation with cofactor : Adj A is the transpose of the cofactor matrix.

Properties of inverse :

- $(A^{-1})^{-1} = A$
- $(A^{-1})^{T} = (A^{\mathrm{T}})^{-1}$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

Adjoins relation with inverse :

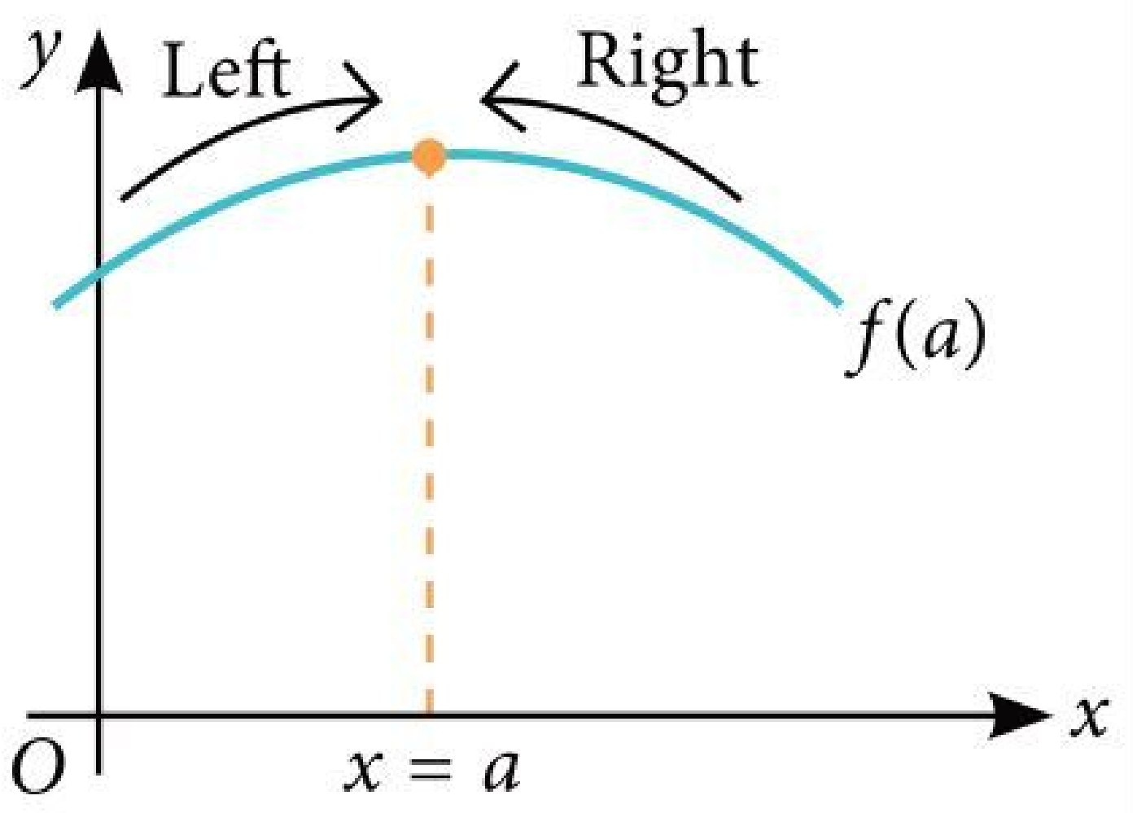
$$A^{-1} = \frac{1}{|A|}(\mathrm{adj}A)$$

Continuity and Differentiability

Continuity

Continuity at a point

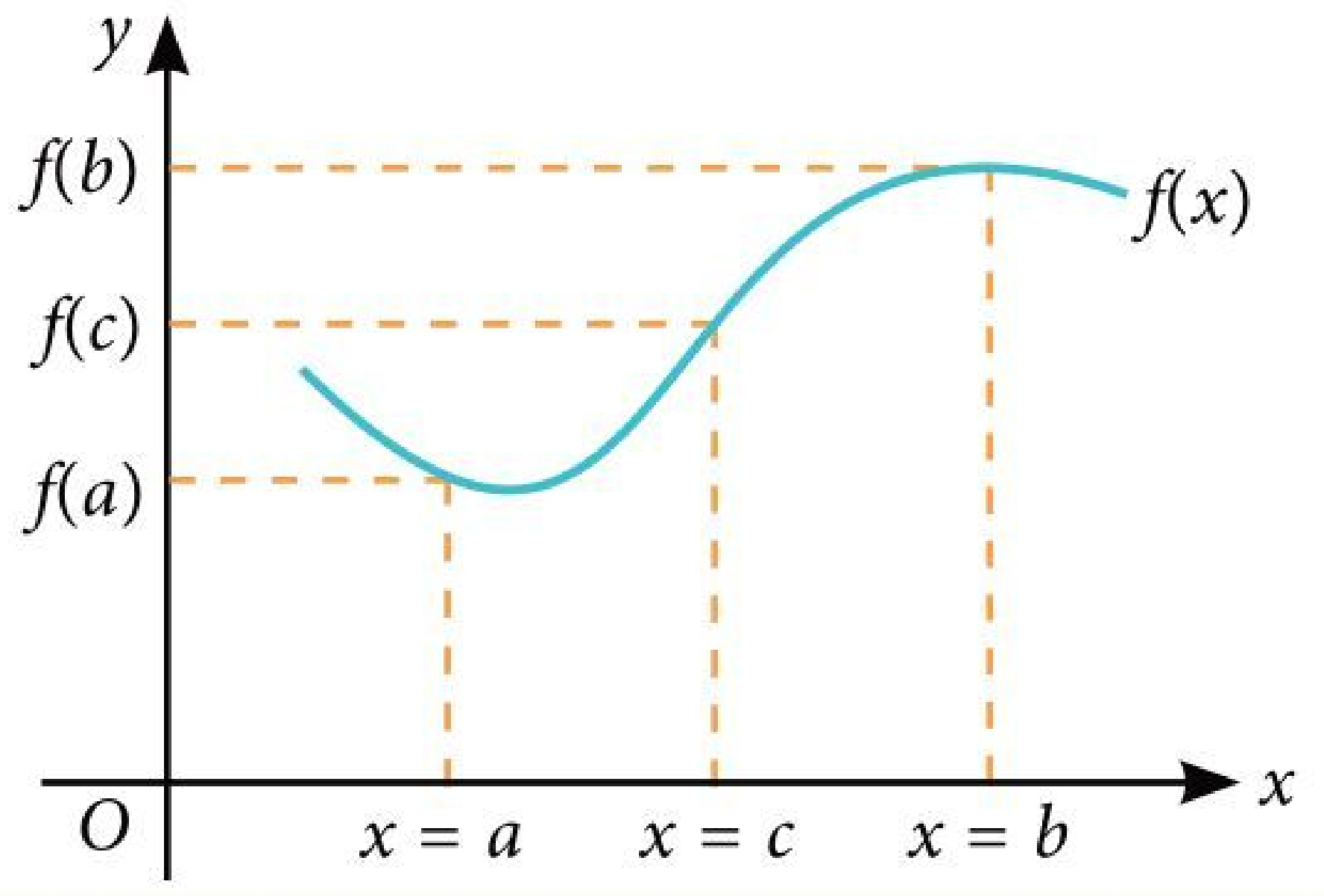
- $f(a)$ exists
- continuous at $x = a$ from left if $\lim\limits_{x \to a^-} f(x) = f(a)$
- continuous at $x = a$ from right if $\lim\limits_{x \to a^+} f(x) = f(a)$
- $\lim\limits_{x \to a^+} f(x) = \lim\limits_{x \to a^-} f(x) = f(a)$


y
Left
Right
f(a)
O
x = a
x

Continuity in an Interval

Continuity of a function, $f(x)$ in an interval $[a, b]$

- $f(x)$ should be continuous in (a, b)
- $\lim\limits_{x \to a^+} f(x) = f(a)$
- $\lim\limits_{x \to b^-} f(x) = f(b)$


y
f(b)
f(c)
f(a)
f(x)
O
x = a
x = c
x = b
x

Algebra of Derivative

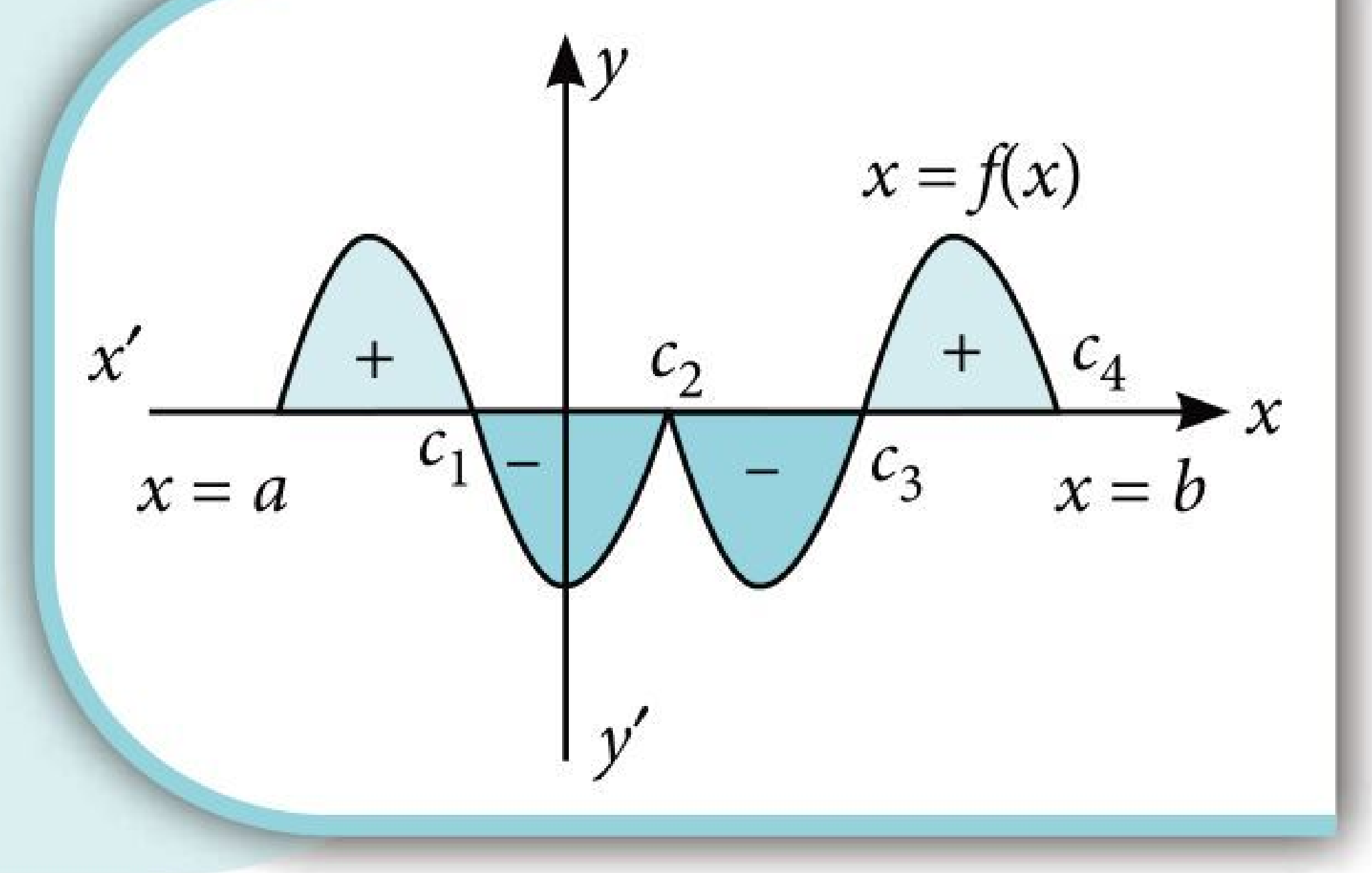
- $(u \pm v)' = u' \pm v'$
- $(uv)' = uv' + u'v$
- $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}, v \neq 0$

Integrals and Application of Integrals

Definite Integral

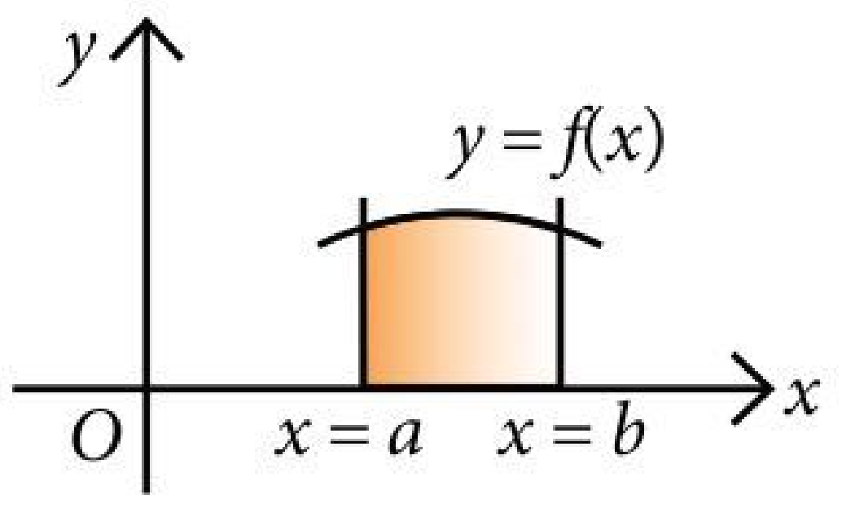
- For curve $y = f(x)$, x-axis and lines $x = a$ and $x = b$
- $\int_a^{c_1} f(x)dx = A_1 > 0$
- $\int_{c_2}^{c_1} f(x)dx = A_2 < 0$
- $\int_{c_2}^{c_3} f(x)dx = A_3 < 0$
- $\int_{c_3}^{b} f(x)dx = A_4 > 0$

$\therefore \int_a^b f(x)dx = A_1 + A_2 + A_3 + A_4$

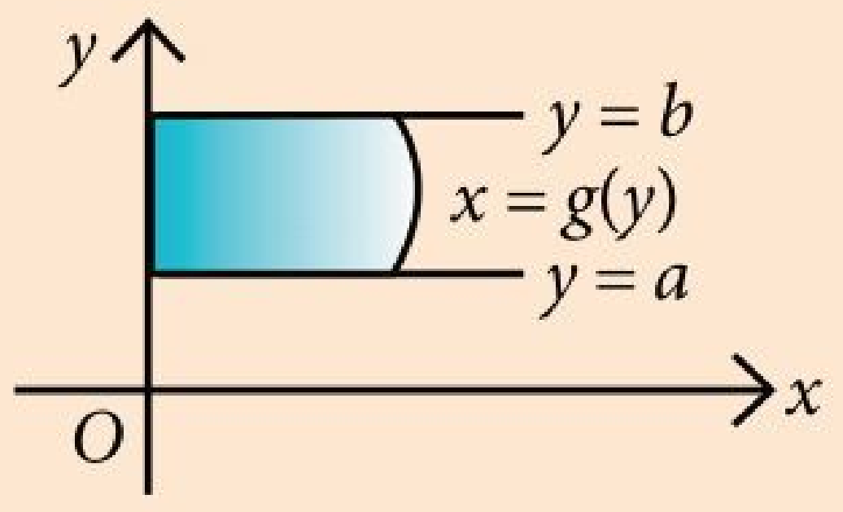

y
x = f(x)
x′
+
c₂
+
c₄
x
x = a
c₁
−
−
c₃
x = b
y′

Area Under Simple Curves

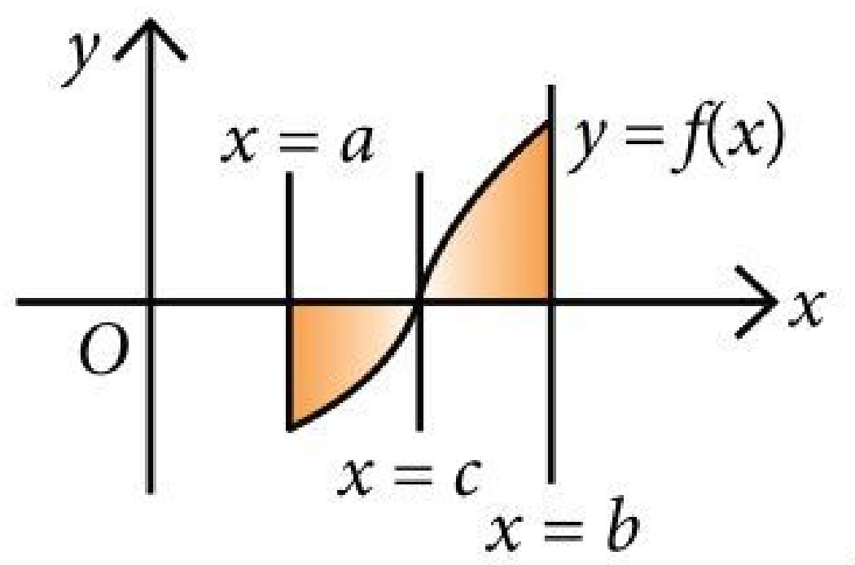
Area $= \int_a^b y dx = \int_a^b f(x)dx$ (where $b > a$)


y
y = f(x)
O
x = a
x = b
x

Area $= \int_a^b x\, dy = \int_a^b g(y)dy$ (where $b > a$)

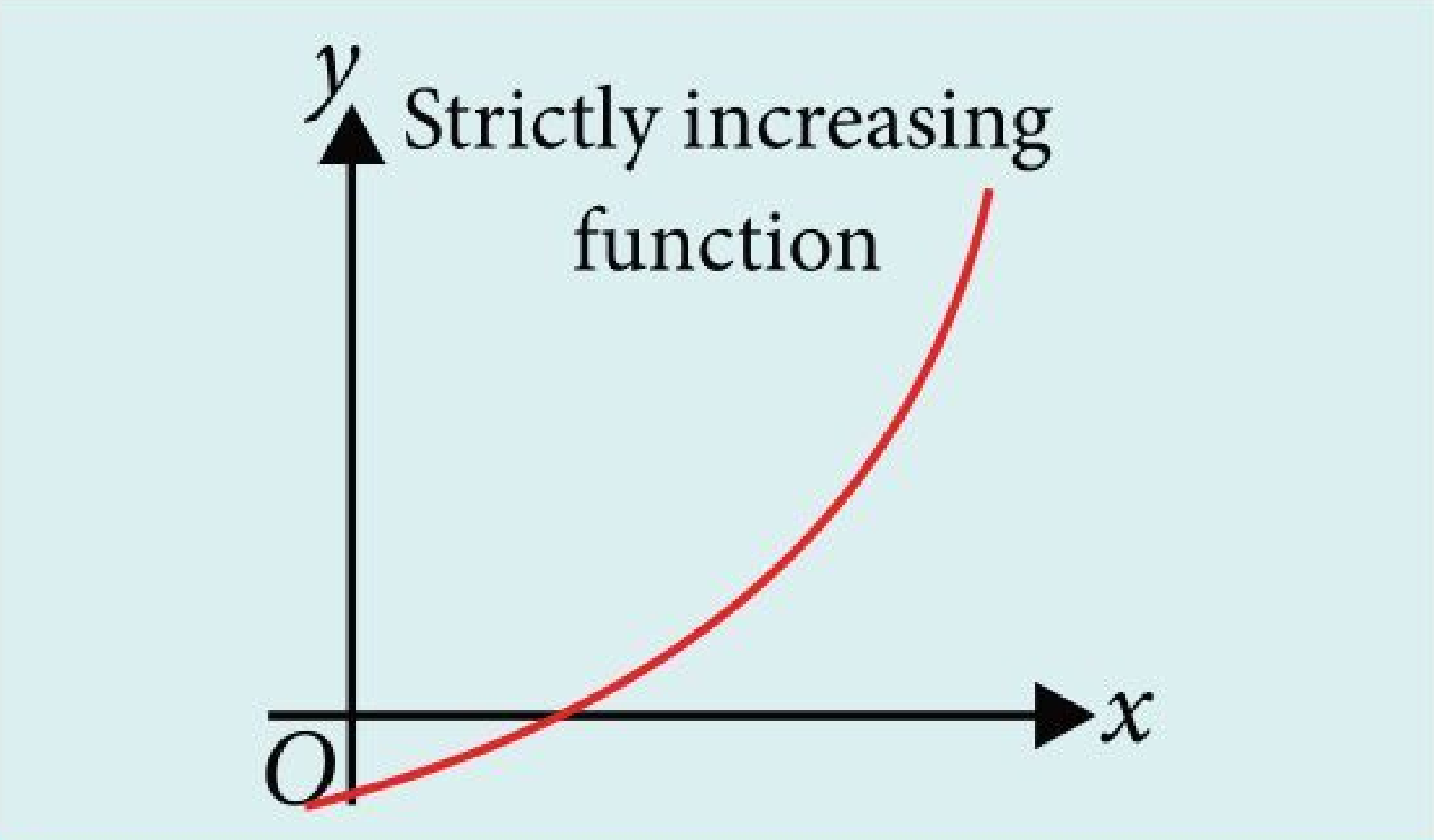

y
y = b
x = g(y)
y = a
O
x

Area $= \left|\int_a^c f(x)dx\right| + \int_c^b f(x)dx$

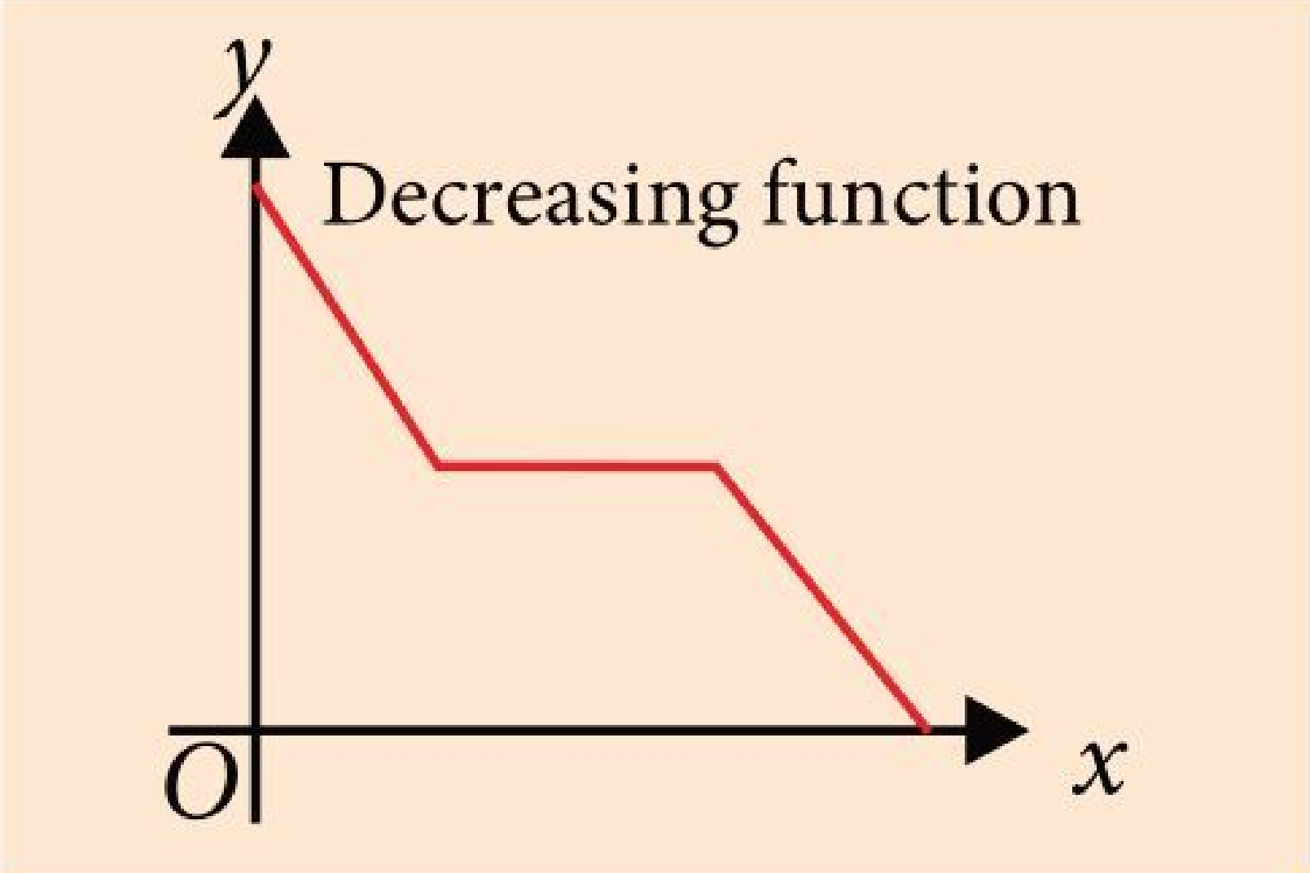

y
x = a
y = f(x)
x
O
x = c
x = b

Application of Derivatives

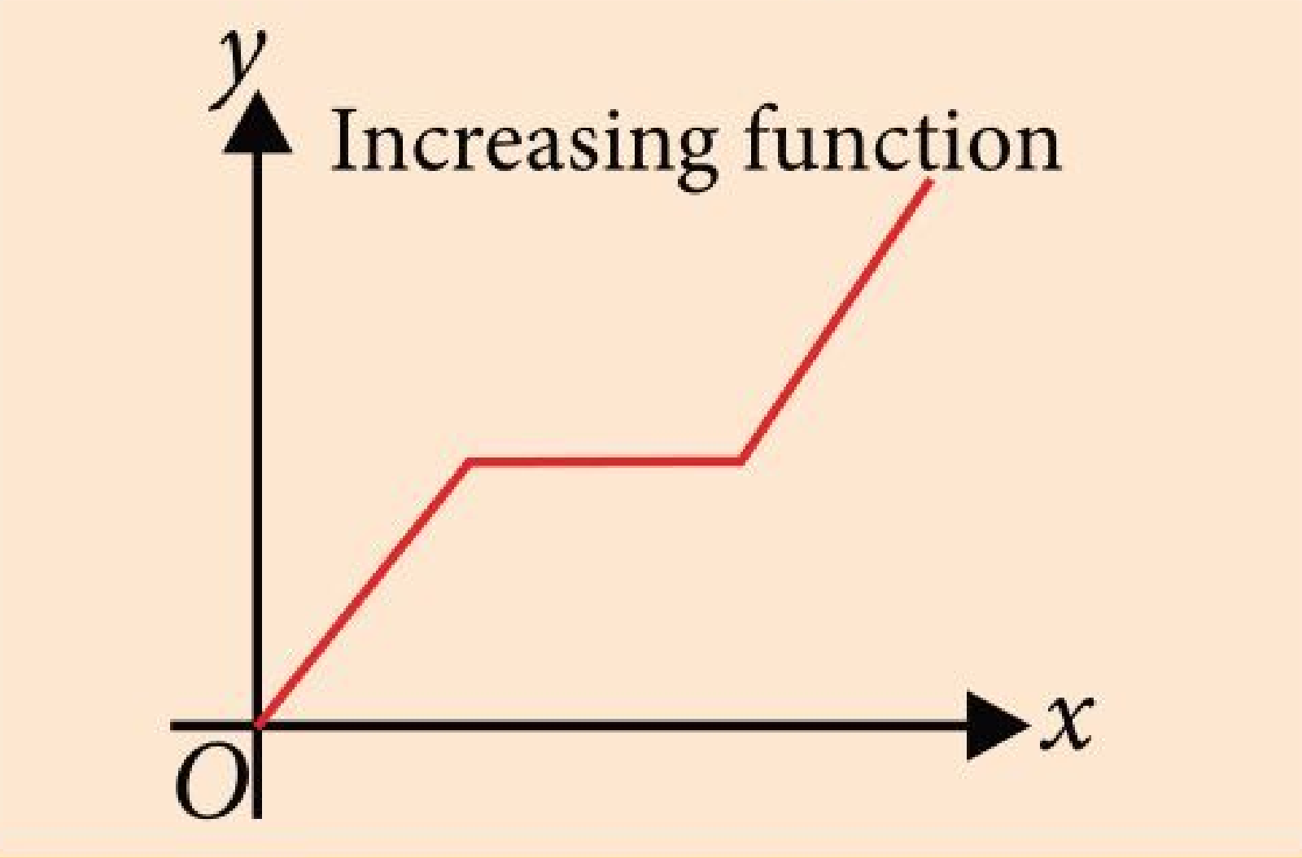
Increasing and Decreasing Functions



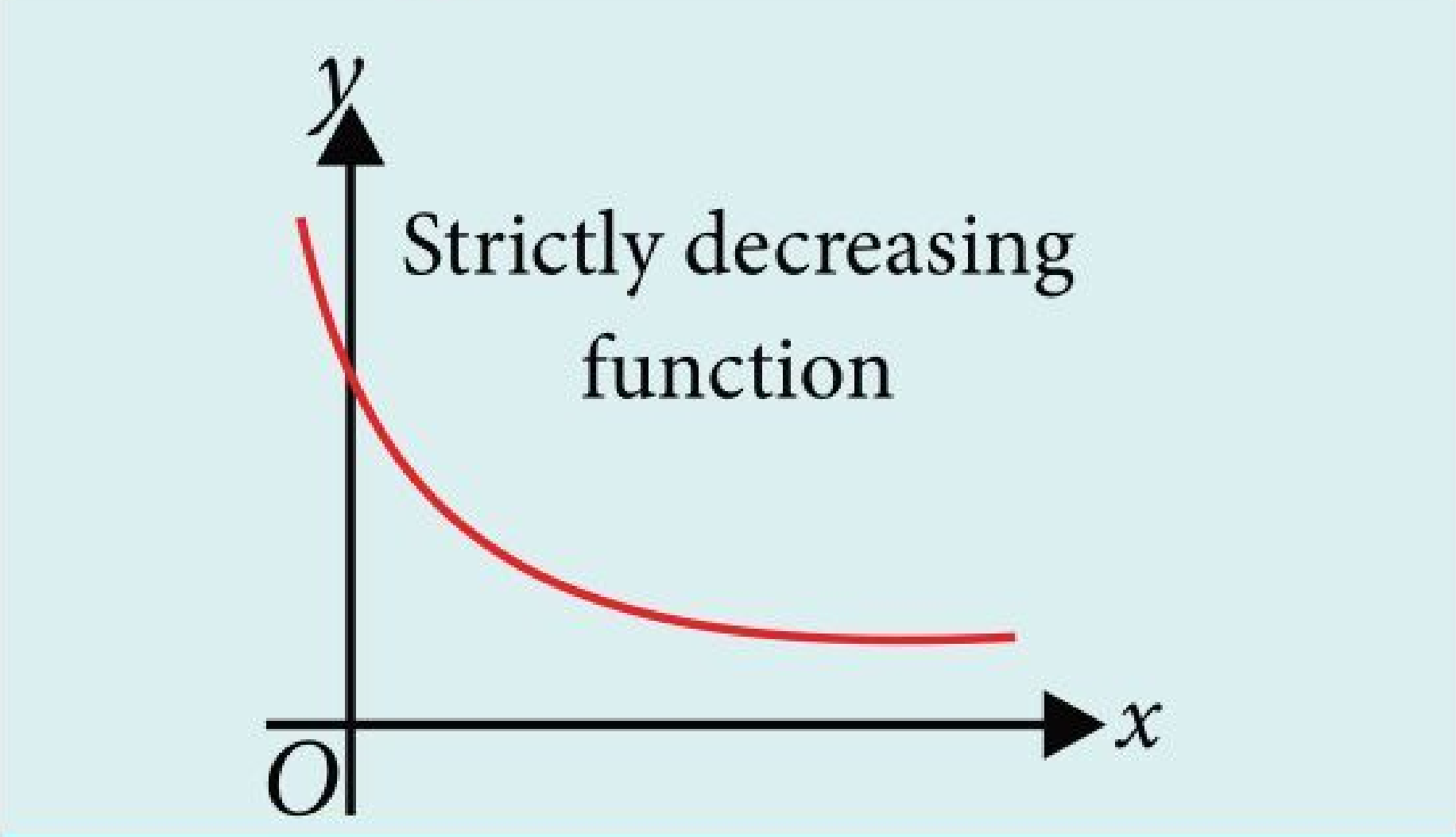
Strictly increasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in I$



Decreasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) \geq f(x_2)$ for all $x_1, x_2 \in I$



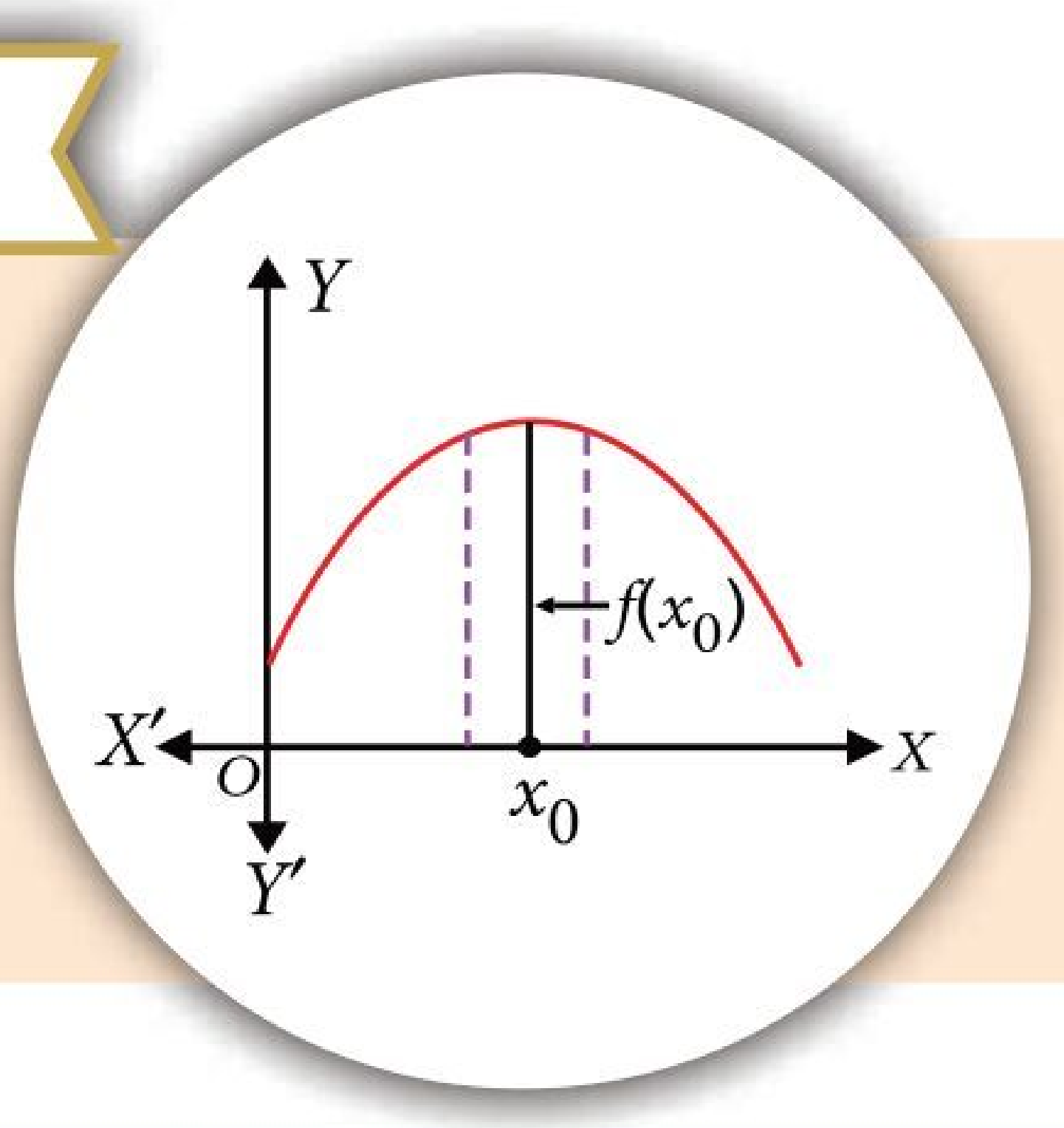
Increasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) \leq f(x_2)$ for all $x_1, x_2 \in I$



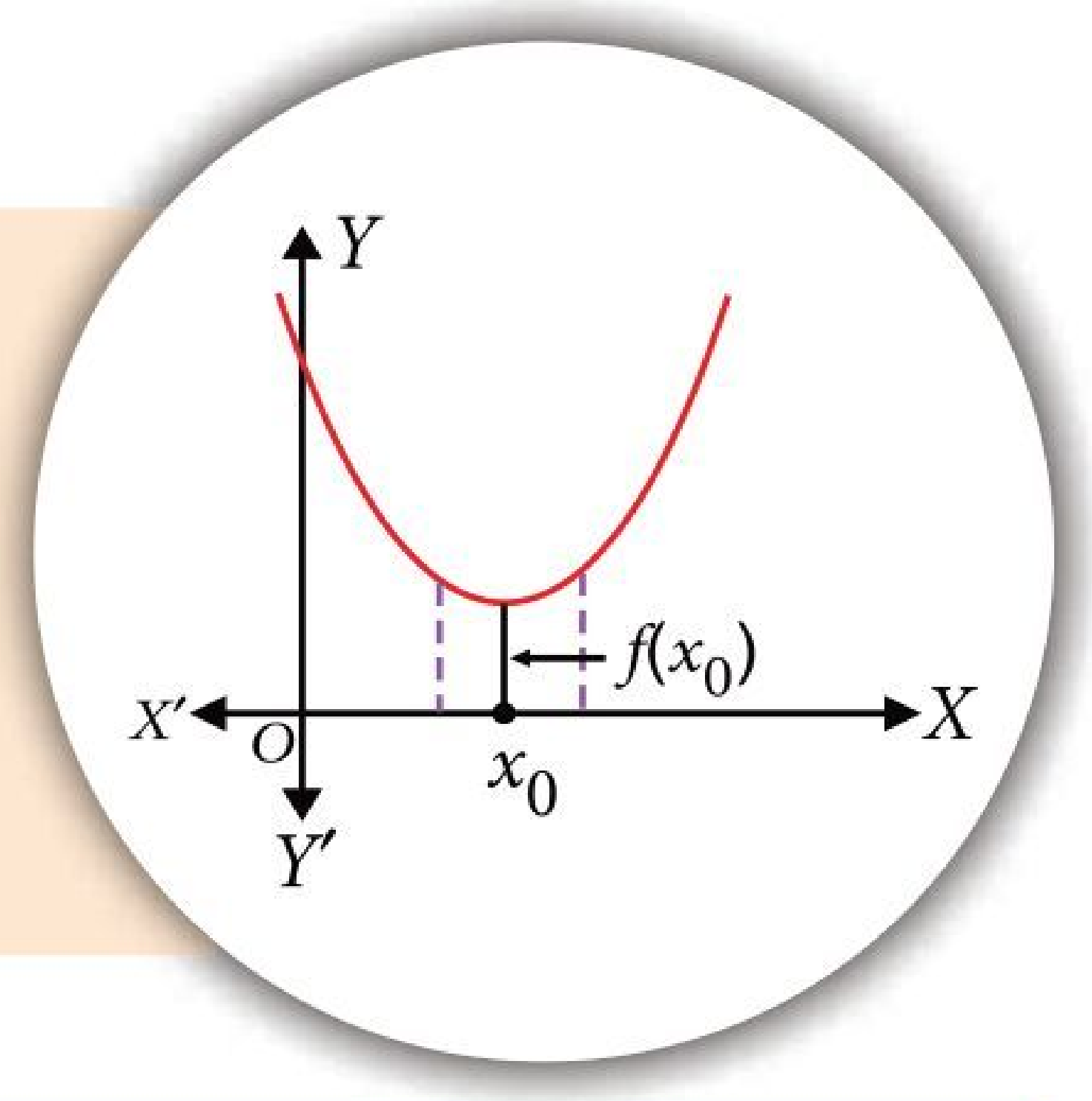
Strictly decreasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in I$

Maxima and Minima

Maximum value of a function : $f(x)$ is said to be maxima in I, if there exists a point $x_0 \in I$ such that $f(x_0) > f(x)\ \forall\ x \in I$.



Minimum Value of a Function : $f(x)$ is said to be minima in I, if there exists a point $x_0 \in I$ such that $f(x_0) < f(x)\ \forall\ x \in I$.



Differential Equation

Methods of Solving First Order and First Degree Differential Equations

Homogeneous differential equation

If $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$, where $f(x, y)$, $g(x, y)$ are homogeneous functions of the same degree in x and y, then put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ then apply variable separable methods.

Variable separable

If $\frac{dy}{dx} = f(x)g(y)$, then $\int \frac{dy}{g(y)} = \int f(x)\,dx + c$

Reducible to Variable Separable

If $dy/dx = f(ax + by + c)$, then put $ax + by + c = z$ and $b\frac{dy}{dx} = \frac{dz}{dx} - a$ and then apply variable separable method.

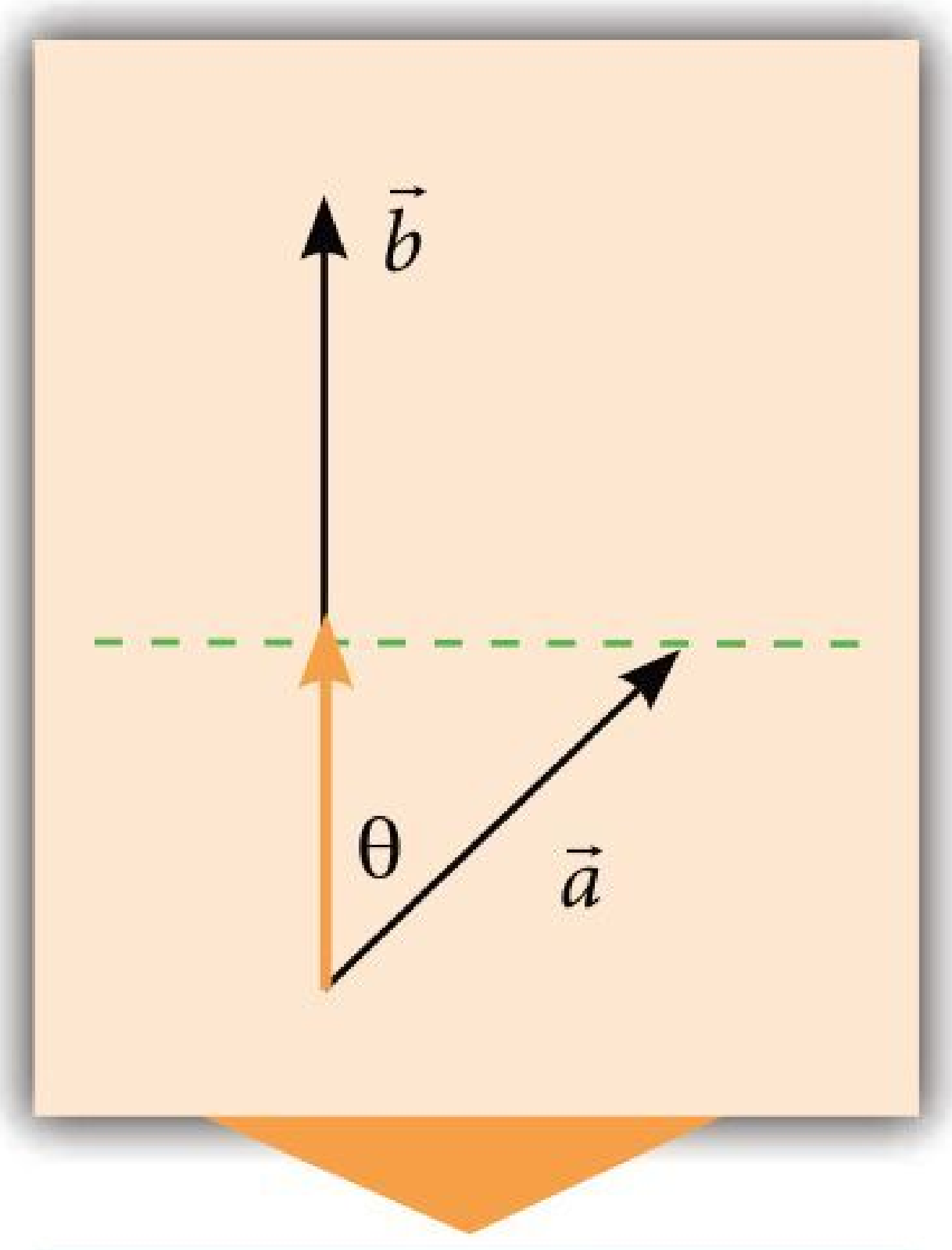
Linear differential equation

If $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x, then $ye^{\int P dx} = \int Qe^{\int P dx} dx + c$, where $e^{\int P dx}$ is the integrating factor (I.F.).

OR

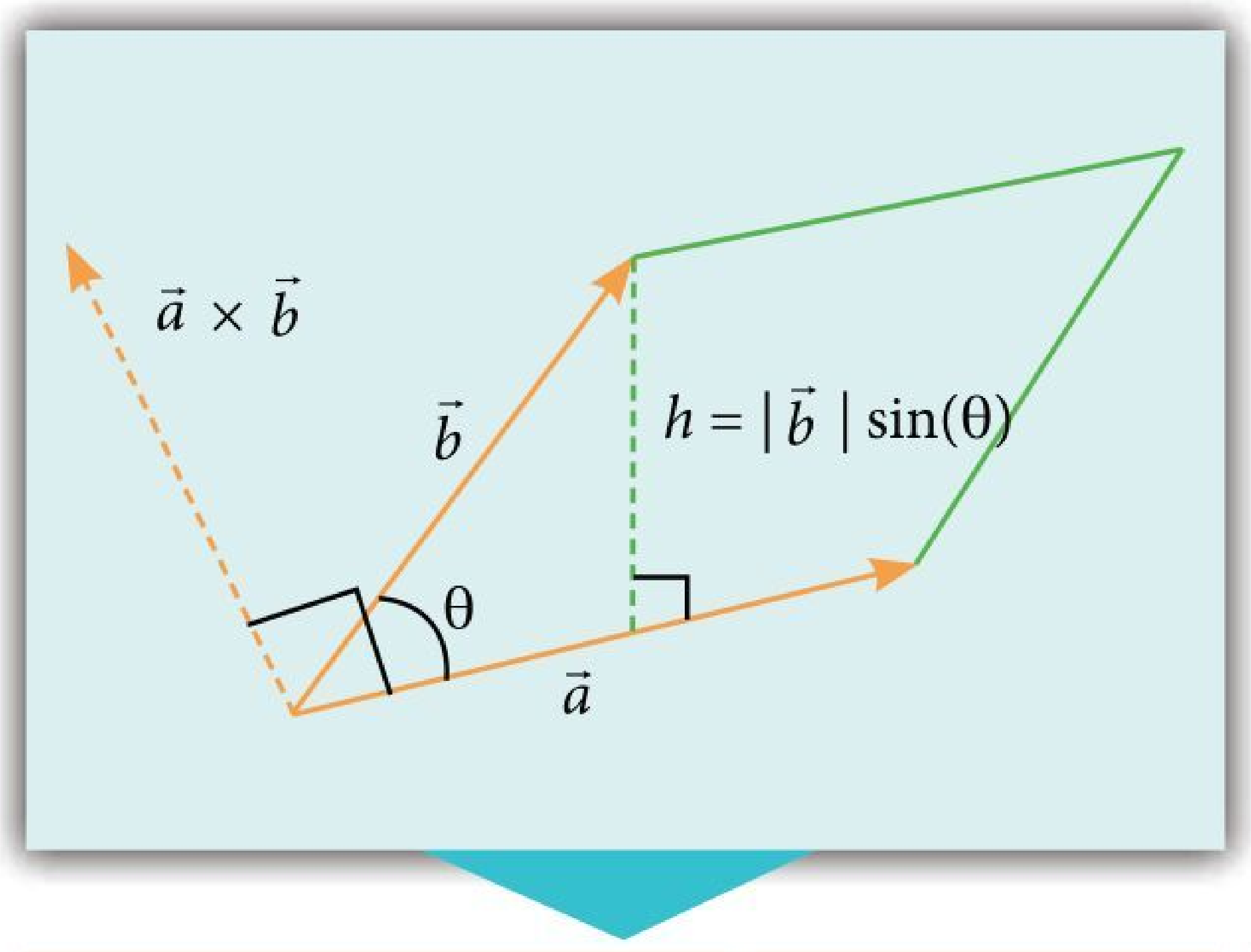
If $\frac{dx}{dy} + Px = Q$, where P, Q are functions of y, then $xe^{\int P dy} = \int Qe^{\int P dy} dy + c$, where $e^{\int P dy}$ is the integrating factor (I.F.).

Vector Algebra



Dot product

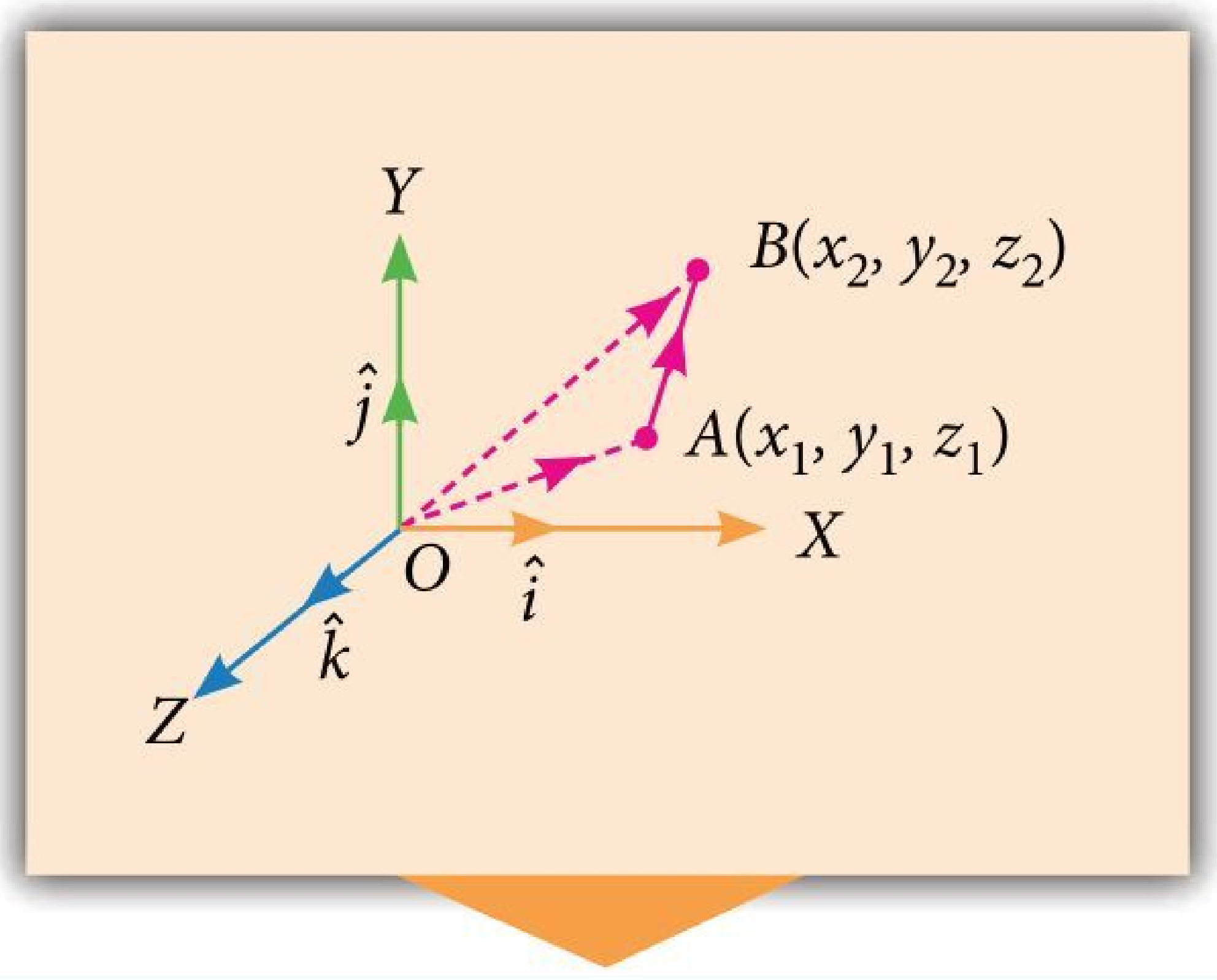
$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$


$\vec{a}\times\vec{b}$
$\vec{b}$
$h = |\vec{b}|\sin(\theta)$
θ
$\vec{a}$

Cross product

$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n},$$

where $\hat{n}$ is a unit vector


Y
$\hat{j}$
$B(x_2, y_2, z_2)$
$A(x_1, y_1, z_1)$
O
$\hat{i}$
X
$\hat{k}$
Z

Vector joining two points

$$|\overrightarrow{AB}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

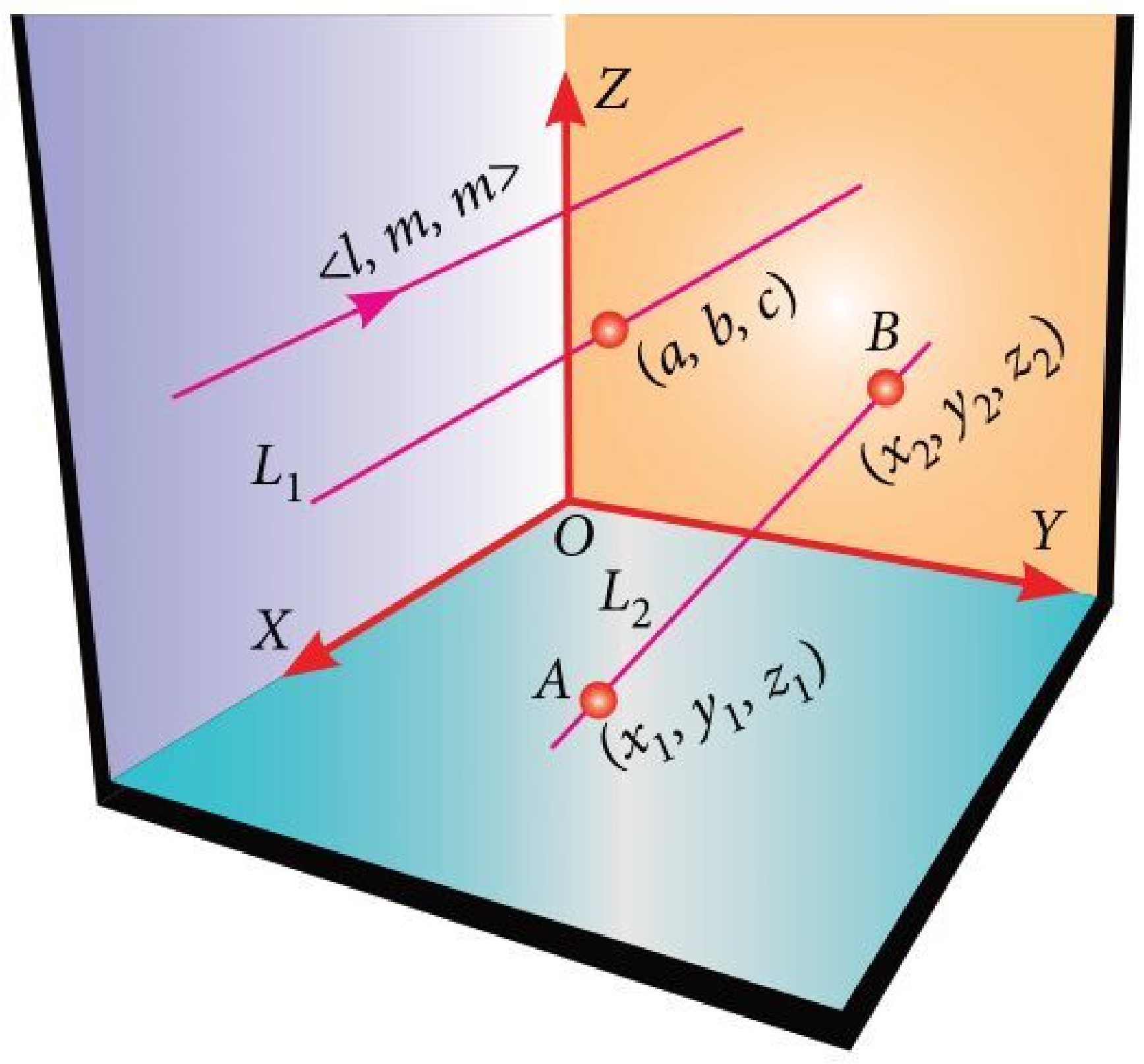
Three Dimensional Geometry

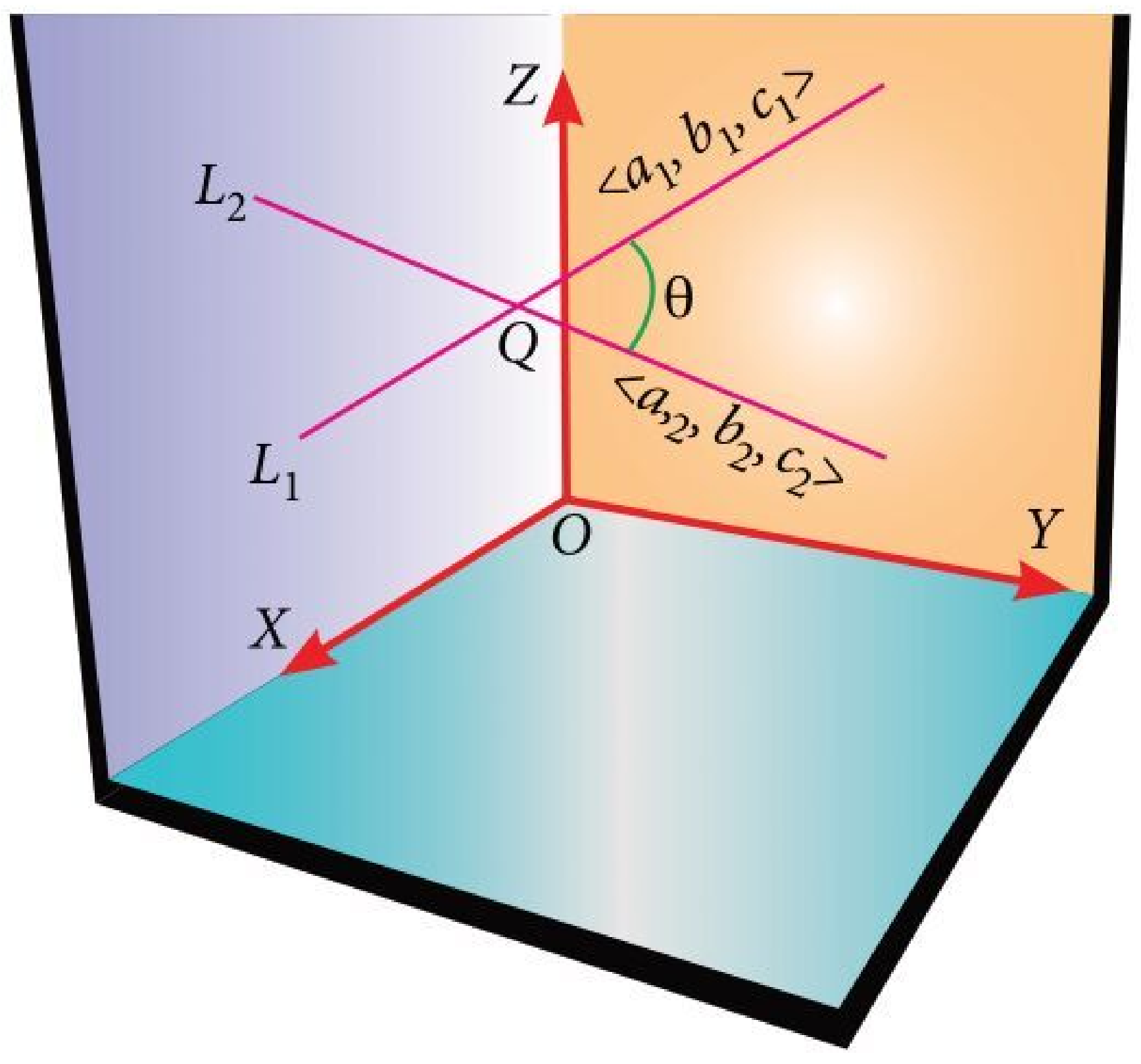
Line passing through point (a, b, c) parallel to line having direction cosines l, m, n is

$$L_1 : \frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$$

Equation of a line passing through two points A & B is

$$L_2 : \frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$


Z
<l, m, m>
(a, b, c)
B
(x_2, y_2, z_2)
L_1
O
L_2
Y
X
A
(x_1, y_1, z_1)


Z
L_2
$<a_1, b_1, c_1>$
θ
Q
$<a_2, b_2, c_2>$
L_1
O
Y
X

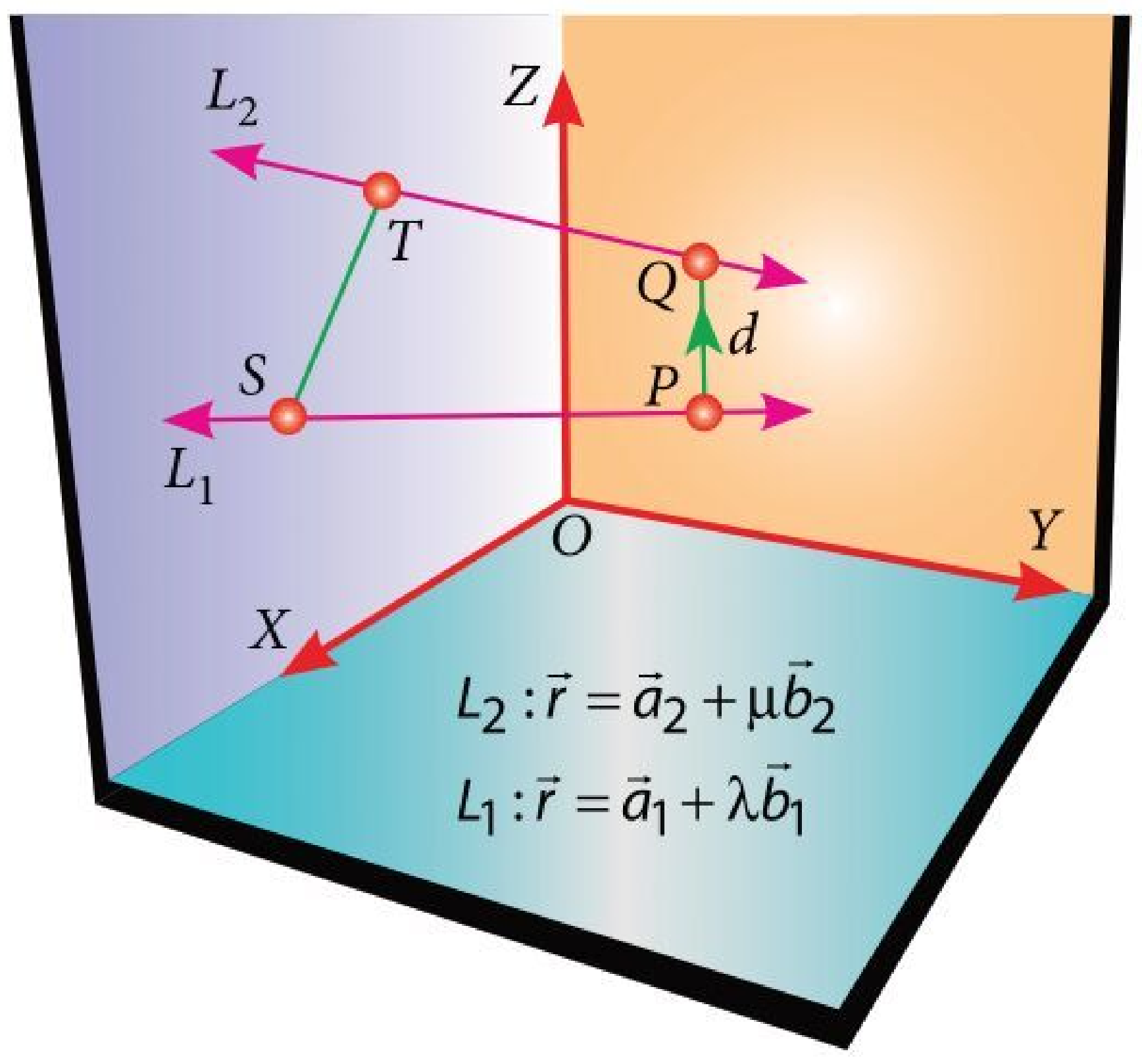
Angle between two lines is given by $\cos\theta = \dfrac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\,\sqrt{a_2^2+b_2^2+c_2^2}}$

If L_1 and L_2 are Perpendicular $(\theta = 90°)$, then $a_1a_2 + b_1b_2 + c_1c_2 = 0$

If L_1 and L_2 are Parallel $(\theta = 0°)$, then $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

Distance between two skew lines is given by

$$d = \left|\frac{(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|$$


L_2
Z
T
Q
d
S
P
L_1
O
Y
X
$L_2 : \vec{r} = \vec{a}_2 + \mu\vec{b}_2$
$L_1 : \vec{r} = \vec{a}_1 + \lambda\vec{b}_1$

Linear Programming

Formulation of LPP

The three steps in the mathematical formulation of an L.P.P. are as follows:

Step 1

Identify the objective function as a linear combination of variables (x and y) and construct all constraints *i.e.* linear equations and inequations involving these variables. Thus, an L.P.P. can be stated mathematically as

Maximise (or minimise) $Z = ax + by$

Subject to the constraints:

$a_ix + b_iy \le$ (or $\ge$ or $=$ or $>$ or $<$) c_i, where $i = 1,\ldots, n$,

$x \ge 0, y \ge 0$ (non-negative constraints).

Step 2

Find the solutions (feasible region) of these equations and inequations by some mathematical method.

Step 3

Find an optimal solution *i.e.*, select particular values of the variables x and y that give the desired value (maximum/minimum) of the objective function.

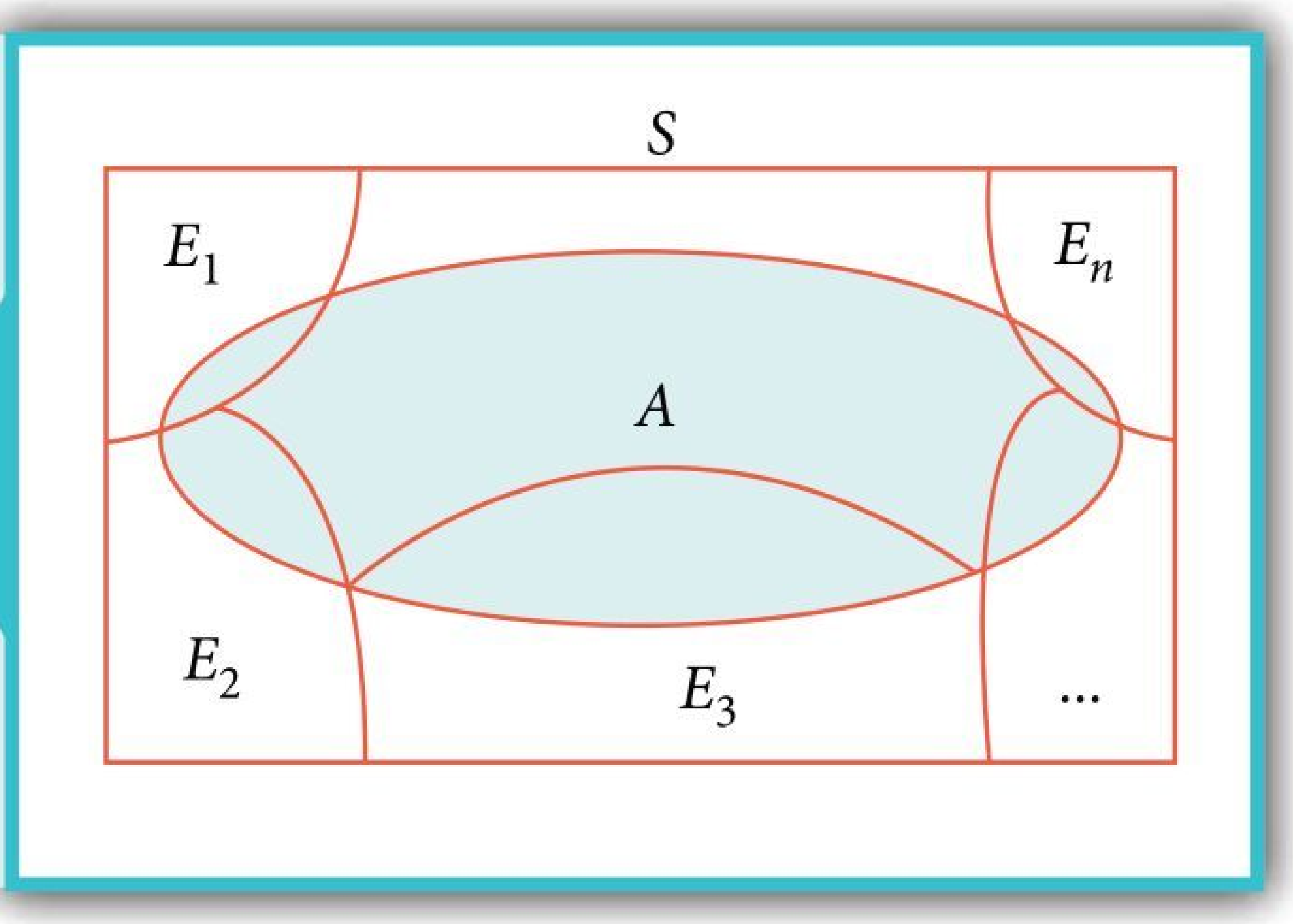
Probability

Theorem of Total Probability

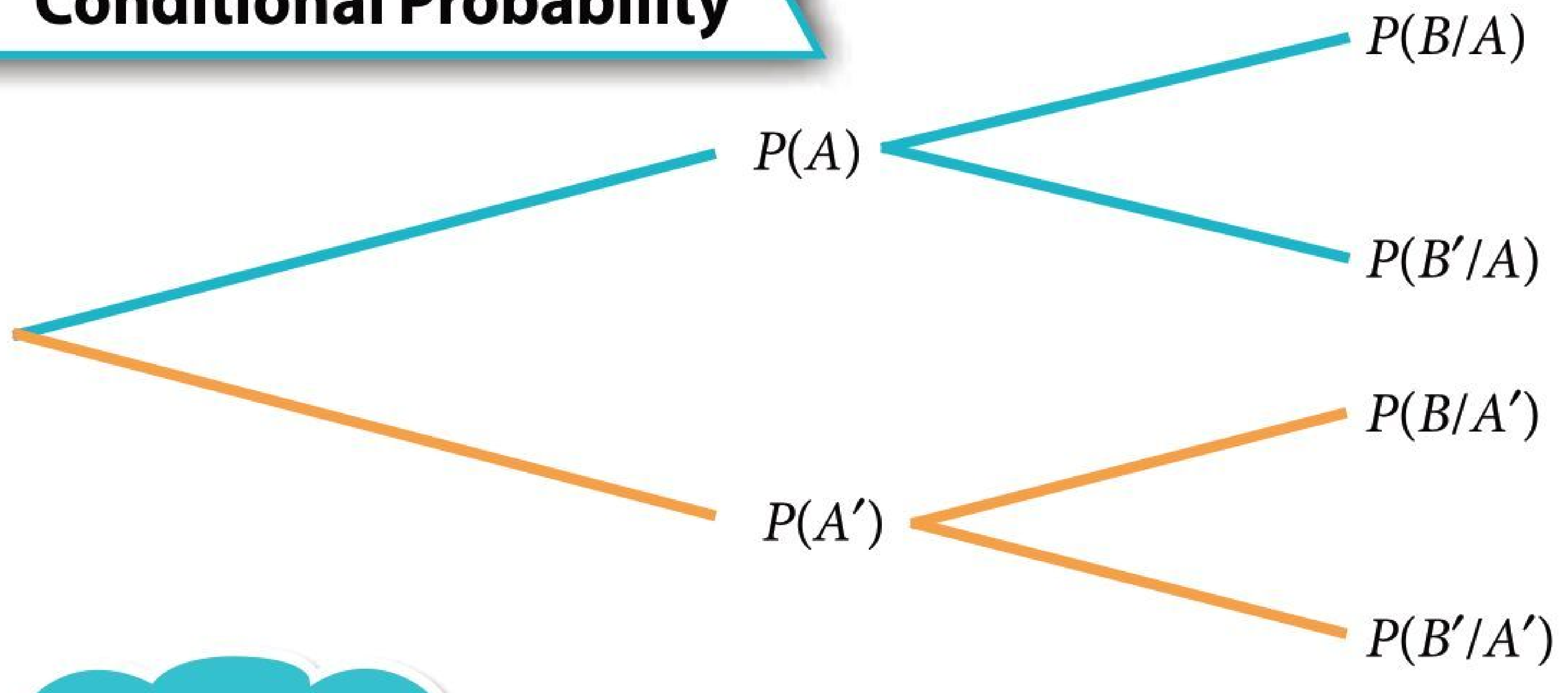
Let $\{E_1, E_2, \ldots\ldots, E_n\}$ be a partition of the sample space S, and suppose that each of the events $E_1, E_2, \ldots\ldots, E_n$ has nonzero probability of occurrence. Let A be any event associated with S, then

$$P(A) = P(E_1)\, P(A \mid E_1) + P(E_2)\, P(A \mid E_2) + \ldots + P(E_n)\, P(A \mid E_n)$$

$$= \sum_{j=1}^{n} P(E_j) P(A|E_j)$$



Conditional Probability



$P(A \cap B) = P(B/A)\, P(A)$

$P(A \cap B') = P(B'/A)\, P(A)$

$P(A' \cap B) = P(B/A')\, P(A')$

$P(A' \cap B') = P(B'/A')\, P(A')$

Bayes' Theorem

$P(E_i|A) = \dfrac{P(E_i)\, P(A|E_i)}{\Sigma P(E_i)\, P(A|E_i)}$, where $i = 1, 2, 3, \ldots\ldots, n$, and $E_1, E_2, E_3, \ldots\ldots, E_n$ are mutually exclusive.

QUANTITATIVE APTITUDE

For Various Competitive Exams

1. If $P=\dfrac{\sqrt{8}+\sqrt{7}}{\sqrt{8}-\sqrt{7}}$ and $Q=\dfrac{\sqrt{8}-\sqrt{7}}{\sqrt{8}+\sqrt{7}}$, then what is the value of P^2+PQ+Q^2?

(a) 30 (b) 899 (c) $\sqrt[4]{56}$ (d) 901

2. For what value of 'n' the number $2^{74}+2^{2058}+2^{2n}$ is a perfect square?

(a) 491 (b) 893 (c) 1029 (d) 2020

3. The seven digit number 2$A$5$A$756 is divisible by 3. What is the sum of the possible values of A?

(a) 8 (b) 12 (c) 16 (d) 18

4. In fresh grapes, 70% of weight is because of water and in dried grapes 40% weight is because of water. In a basket 60% of the weight is fresh grapes and rest are dried grapes. What is the percentage reduction in weight if all the fresh grapes turn into dried grapes?

(a) 30% (b) 50% (c) 40% (d) 20%

5. Mr. Kishore deposited a total amount of ₹ 65000 in there different schemes A, B and C with rates of interest 12% per annum, 16% per annum and 18% per annum, respectively and earned a total interest of ₹ 10,180 in one year. If the amount invested in scheme A was 72% of the amount invested in scheme C, then what was the amount invested in scheme B?

(a) ₹ 25000 (b) ₹ 22000 (c) ₹ 18000 (d) ₹ 17500

6. Arjun borrows ₹ 45000 from a bank at 10% compound interest. He repays it in three annual installments that are in arithmetic progression. He ends up paying ₹ 54000 totally. How much did he pay in year 1?

(a) ₹ 21000 (b) ₹ 19500 (c) ₹ 16500 (d) ₹ 18000

7. A shopkeeper marks up his goods by 20% and then gives a discount of 20%. Besides he cheats both his supplier and customer by 100 g *i.e.* he takes 1100 g from his supplier and sells only 900 g to his customer. What is his net profit percentage?

(a) 25% (b) 24.5% (c) 17.33% (d) 32.5%

8. Sunny marks up his goods by 40% and gives a discount of 10%. Apart from this, he uses a faulty balance which reads 800 g for 1000 g. What is his net profit / loss percentage?

(a) 0.8% (b) 57.2% (c) 37.6% (d) 25%

9. The present age of Romila is one fourth of that of her father. After 6 years the father's age will be twice the present age of Kapil. If Kapil celebrated fifth birth day 8 years ago, What is Romila's present age?

(a) 7 years (b) 5 years (c) 8 years (d) 8.5 years

10. There are three partners A, B and C start a business together. They invest ₹ 30000, ₹ 24000 and ₹ 42000 respectively in the beginning. After 4 months, B withdraw ₹ 6000 and C withdraw ₹ 10000. They get a profit of ₹ 11960 at the end of the year. B's share in the profit is

(a) ₹ 2700 (b) ₹ 2803 (c) ₹ 2900 (d) ₹ 2785

11. The average weight of three men A, B and C is 84 kg. D joins them and the average weight of the four becomes 80 kg. If E, whose weight is 3 kg more than that of D, replaces A, the average weight of B, C, D and E becomes 79 kg. The weight of A is

(a) 65 kg (b) 70 kg (c) 75 kg (d) 80 kg

12. Ten years ago, the ages of the members of a joint family of eight people added up to 231 years. Three years later, one member died at the age of 60 years and a child was born during the same years. After another three years, one more member died, again at 60, and a child was born during the same year. The current average age of this eight-member joint family is closet to

(a) 21 years (b) 25 years (c) 24 years (d) 23 years

13. A manufacturer has 200 litres of acid solution which has 15% acid content. How many litres of acid solution with 30% acid content may be added so that acid content in the resulting mixture will be more than 20% but less than 25%?

(a) More than 100 litres but less than 300 litres.
(b) More than 120 litres but less than 400 litres.
(c) More than 100 litres but less than 400 litres.
(d) More than 150 litres but less than 600 litres.

14. From a 3 : 5 solution of milk and water, 20% is taken out and replaced by milk. How many times should this process be done to make the ratio of milk to water as 17 : 8?

(a) Four times (b) Thrice
(c) Twice (d) Once

15. A tank of volume 432 cubic feet has one inlet pipe and two outlet pipes. The inlet pipe fills water into the tank at the rate of 4 cubic inches/min. and the two outlet pipes empty it out at the rate of 14 cubic inches/min. and 6 cubic inches/min. respectively. If all the three pipes are opened when the tank is full, in how much time will it be emptied?

(a) 777.6 h (b) 3 days (c) 15 days (d) 77.76 h

16. P_1, P_2 and P_3 are pipes attached to a cistern. P_1 and P_2 can fill it in 20 and 30 minutes, respectively, while P_3 can empty it in 15 minutes. If P_1, P_2 and P_3 be kept open alternatively for 1 minute each, how soon will the cistern be filled?

(a) 158 minutes (b) 152 minutes
(c) 167 minutes (d) 169 minutes

17. The sum of the 6th and 15th term of an arithmetic progression is equal to the sum of 7th, 10th and 12th term of the same progression. Which element of the series should necessarily be equal to zero?

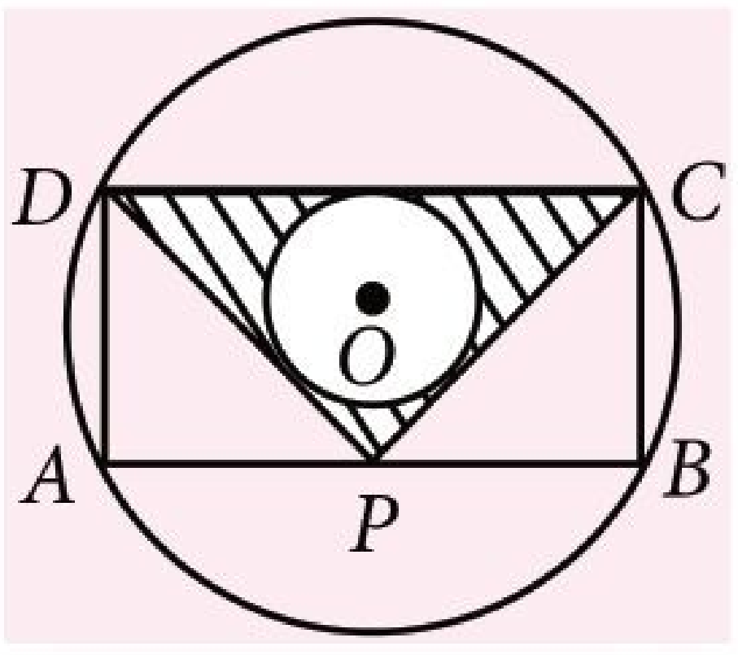
(a) 10th (b) 8th (c) 1st (d) 5th

18. Find the sum to '*n*' terms and also to infinite terms of the series $10+5+\frac{5}{2}+\frac{5}{4}+.....$

(a) $20\left(1-\frac{1}{2^n}\right), 20$ (b) $30\left(1-\frac{1}{2^n}\right), 30$

(c) $20\left(\frac{1}{2^n}-1\right), 20$ (d) $30\left(\frac{1}{2n}-1\right), 30$

19. Find the area of ΔPCD if P lies on the line AB where O is the centre of the outer circle with radius 6.5 cm. $ABCD$ is a parallelogram, one of whose side is 5 cm.



(a) 30 cm^2 (b) 45 cm^2 (c) 36 cm^2 (d) 50 cm^2

20. For the CBSE, a selection committee is to be chosen consisting of 5 Mathematician. Now there are 10 representatives from four zones. It has further been decide that if Aryabhatt is selected Ramanujan will not be selected and vice versa. In how many ways can this be done?

(a) 140 (b) 112 (c) 196 (d) 56

SOLUTIONS

1. (b) : $P^2 + PQ + Q^2 = P^2 + PQ + Q^2 + PQ - PQ$
$= P^2 + 2PQ + Q^2 - PQ = (P + Q)^2 - PQ$

Now, $PQ = \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}-\sqrt{7}}{\sqrt{8}+\sqrt{7}} = 1$

and $P + Q = \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}-\sqrt{7}} + \frac{\sqrt{8}-\sqrt{7}}{\sqrt{8}+\sqrt{7}}$

$$= \frac{\left(\sqrt{8}+\sqrt{7}\right)^2 + \left(\sqrt{8}-\sqrt{7}\right)^2}{\left(\sqrt{8}\right)^2 - \left(\sqrt{7}\right)^2} = \frac{2\left(\left(\sqrt{8}\right)^2 + \left(\sqrt{7}\right)^2\right)}{8-7}$$

$= 2\ (8 + 7) = 30$

So, Required Value $= (P + Q)^2 - PQ = 30^2 - 1^2 = 899$

2. (d) : We have to write the expression in the form of $a^2 + b^2 + 2ab$

So, first we will find that 2^{2n} will be of the form a^2 or $2ab$

Case 1: 2^{2n} is of $2ab$ form

So, $(2^{37})^2 + (2^{1029})^2 + 2^1 \times 2^{37} \times 2^{1029}$.

Now, $1 + 37 + 1029 = 38 + 1029 = 1067$, which is a odd value and can't be equated to 2^{2n} as the power is even.

So, this case is discarded

Case 2: 2^{2n} is of the a^2 form.

In this case 2^{2058} should be of the $2ab$ form as it contain 2^{74}.

$2^{74} + 2^{2058} + 2^{2n} = (2^{37})^2 + 2^1 \times 2^n \times 2^{37} + (2^n)^2$

So, $2058 = 1 + n + 37$

$\Rightarrow\ n = 2020$

3. (b) : In order for $2A5A756$ to be divisible by 3, the sum of the digits must be divisible by 3. The sum of the digits is $2A + 25$ and that needs to be a multiple of 3. The smallest possible value of $2A + 25$ is $2(0) + 25 = 25$ and the largest possible value is $2(9) + 25 = 43$. The multiples of 3 between 25 and 43 are 27, 30, 33, 36, 39, and 42. We can solve the equation $2A + 25 = 27$ or 30 or 33 or 36 or 39 or 42 to find A. Subtracting 25 gives us $2A = 2$, 5, 8, 11, 14, or 17. We then divide by 2, but ignore any decimals since A is a digit. The possible values of A then are: 1, 4 or 7 and they have a sum of 12.

4. (a) : When fresh grapes turns into dry grapes the weight reduces to 1/2 of the original weight (since pulp that consists 30% is now 60% of the weight in dried grapes).

So, the final weight as $x\%$ of the original weight be:

$$= \frac{1}{2}(60\%) + (40\%) = 70\%$$

Loss or reduction of weight = 100 – 70 = 30%

5. (b) : Suppose amount invested in scheme A = ₹ x

Amount invested in scheme B = ₹ y

Amount invested in scheme C = ₹ z

$$\therefore\ x + y + z = 65000 \qquad \ldots(1)$$

$$x = 72\%\text{ of } z = \frac{18}{25}z \qquad \ldots(2)$$

$$12x + 16y + 18z = 1018000$$

i.e., $6x + 8y + 9z = 509000 \qquad \ldots(3)$

Using (2) in (1) and (3), we get

$$\frac{43}{25}z + y = 65000 \qquad \ldots(4)$$

$$\Rightarrow\ \frac{108}{25}z + 8y + 9z = 509000$$

$$\Rightarrow\ \frac{333}{25}z + 8y = 509000 \qquad \ldots(5)$$

From (4), we have

$$\frac{344z}{25} + 8y = 520000 \qquad \ldots(6)$$

$$\therefore\ (6) - (5) \text{ gives } \frac{11z}{25} = 11000$$

$$\Rightarrow\ 11z = 275000$$

$$\therefore\ z = 25000,\ x = 18000,\ y = 22000$$

6. (b) : Let the repayments be ₹ $a - d$, ₹ a and ₹ $a + d$

$a - d + a + a + d = 54000$

$\Rightarrow\ 3a = 54000 \Rightarrow a = 18000$

The payment at the end of year 2 is ₹ 18000.

Borrowed amount = ₹ 45000

Amount remaining at the end of year 1

$= (45000 \times 1.1) - (18000 - d) = 31500 + d$

Amount remaining at the end of year 2

$= [(31500 + d) \times 1.1] - 18000$

$= 34650 + 1.1d - 18000 = 16650 + 1.1d$

Amount remaining at the end of year 3

$[(16650 + 1.1d) \times 1.1] = 18000 + d$

$\Rightarrow\ 18315 + 1.21d = 18000 + d$

$\Rightarrow\ 0.21d = -315\ \Rightarrow\ d = -1500$

The payments are ₹ 19500, ₹ 18000 and ₹ 16500

7. (c) : Assume his CP = ₹ 1000/1100 g

M.P. = ₹ 1200 and SP = ₹ 960/900 g

So, CP / 1000 g = ₹ 909.09

SP / 1000 g = ₹ 1066.66

So, profit = ₹ 157.57

$$\therefore\ \text{Profit percentage} = \frac{157.57}{909.09} \times 100 = 17.33\%$$

8. (a) : Let CP for 1000 g = ₹ 1000

So, M.P. = ₹ 1400, SP = ₹ 1260

Now, when he sells 1000g, he actually obtains the money only for 800 g.

So, when he sells articles worth ₹ 1000, money obtained after selling will be equal to 1260 × 0.8 = ₹ 1008

So, profit percentage = 0.8%

9. (b) : Let present age of Romila's father be 'x' year then age of Romila will be $x/4$ year

$(x + 6) = 2$(Present age of Kapil)

Present age of Kapil = 8 + 5 = 13 years

$x + 6 = 2(13)$

$x + 6 = 26$

$x = 26 - 6 = 20$ years

$$\text{Age of Romila } = \frac{20}{4} = 5 \text{ years}$$

10. (b) : Total amount invested by $A = 30000 \times 12$

= ₹ 360000

By $B = 24000 \times 4 + 18000 \times 8 = 96000 + 144000$

= ₹ 240000

By $C = 42000 \times 4 + 32000 \times 8 = 168000 + 256000$

= ₹ 424000

Ratio of profit becomes

360000 : 240000 : 424000 = 360 : 240 : 424

= 90 : 60 : 106 = 45 : 30 : 53

$\therefore$ Total profit $= 45x + 30x + 53x = 128x$

$$\therefore\ B\text{'s share} = \frac{30x}{128x} \times 11960 = ₹\ 2803 \text{ (approximatley)}$$

11. (c) : Sum of weights of A, B and $C = 84 \times 3$ kg = 252 kg

Weight of $D = (4 \times 80) - 252 = 320 - 252 = 68$ kg

Weight of $E = 68 + 3 = 71$ kg

$A + B + C + D = 320$

$B + C + D + E = 79 \times 4 = 316$

$A - E = 4$ kg

So, $A = 4 + E = 4 + 71 = 75$ kg

12. (c) : 10 years ago, total age of 8 members = 231.

After three years, sum of the ages

= 231 + (8 × 3) – 60 =195

Three more years later sum of ages

= 195 + (8 × 3) – 60 = 159

Recipe for Success

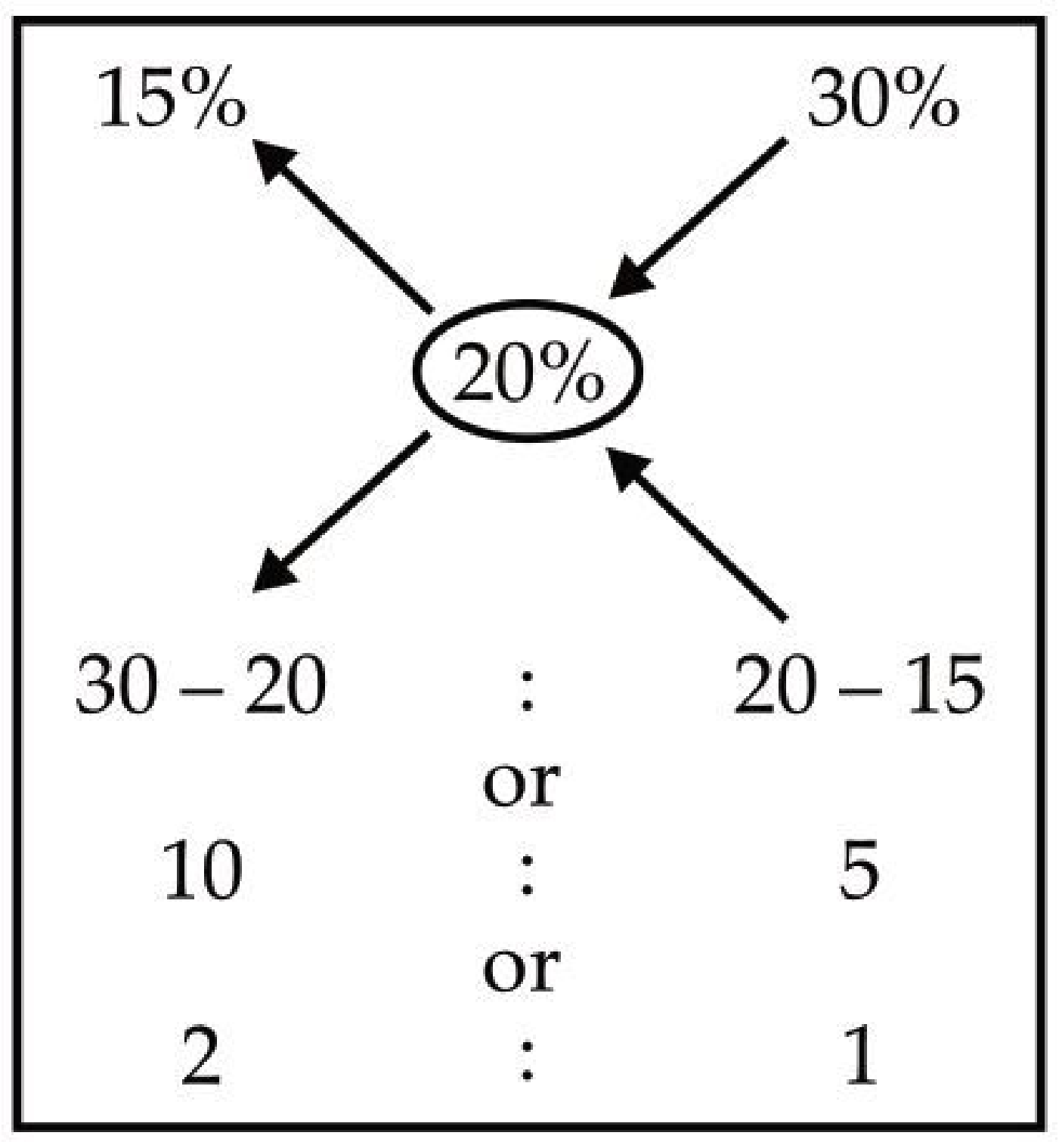
"Every morning you have two choices :
Continue to sleep with your dreams
or wake up and chase them"

– Charmelo Anthony

Total current age = 159 + (4 × 8) = 191

Current average = $\frac{191}{8}$ = 24 years (approx.)

13. (c) : For the resulting mixture to contain 20% acid content.

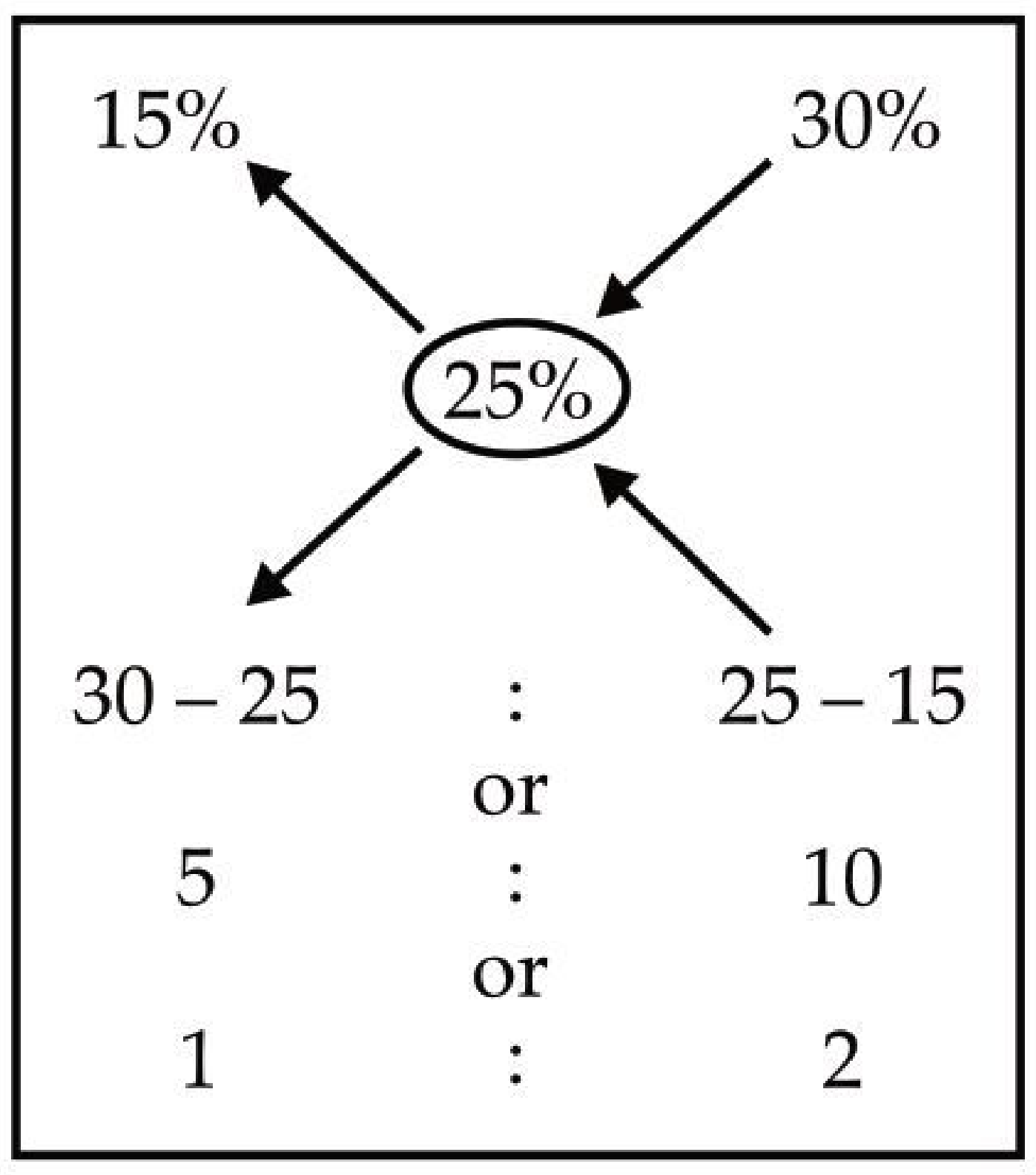


$\frac{2x}{3} = 200 \Rightarrow x = 300$

∴ The mixture with 30% acid content to be added

$= \frac{x}{3} = \frac{300}{3} = 100$ litres

For the resulting mixture to contain 25% acid content.



$\frac{x}{3} = 200 \Rightarrow x = 600$

∴ $\frac{2x}{3} = \frac{2 \times 600}{3} = 400$ litres

Hence, the acid solution with 30% acid content that may be added so that acid content in the resulting mixture will be more than 20% but less than 25% should be more than 100 litres but less than 400 litres.

14. (b) : Fraction of water initially = 5/8

Fraction of water after replacement = 8/25

As in this case we are replacing water with milk hence quantity of water keeps on decreasing constantly hence replacement can be only applied to water in this case.

Applying the formula

$$\frac{8}{25} = \frac{5}{8}\left(1 - \frac{20}{100}\right)^n \Rightarrow \frac{64}{125} = \left(\frac{4}{5}\right)^n$$

$$\Rightarrow \left(\frac{4}{5}\right)^3 = \left(\frac{4}{5}\right)^n \Rightarrow n = 3$$

15. (a) : Time = $\frac{432 \times 1728}{(14 + 6) - 4}$ = 46656 minutes

= 777.6 hours

[Note: 1 feet = 12 inchs = 12^3 = 1728]

16. (c) : Let total capacity be 60 units [LCM (20, 30, 15)]

P_1 in 1 min can fill = 3 units

P_2 in 1 min can fill = 2 units

P_3 in 1 min can fill = – 4 units

When opened alternatively they will fill 3 + 2 – 4 = 1 unit of tank in 3 minutes.

So, 55 units cistern will be filled in 165 minutes.

In 166th, P_1 will fill 3 units and in 167th, P_2 will fill 2 units to completely fill the tank.

17. (b) : Let the first term and common difference of the A.P. be a and d respectively.

Then, $(a + 5d) + (a + 14d) = (a + 6d) + (a + 9d) + (a + 11d)$

$\Rightarrow 2a + 19d = 3a + 26d \Rightarrow a + 7d = 0$

∴ 8th term is 0.

18. (a) : $a = 10$; $r = 1/2$

Sum of first n terms in a geometric progressions is given by

$$= \left[\frac{a(1-r^n)}{1-r}\right] = \left[\frac{10\left\{1-\left(\frac{1}{2}\right)^n\right\}}{\frac{1}{2}}\right] = 20\left[1-\left(\frac{1}{2}\right)^n\right]$$

Sum upto infinite terms is given by $\frac{a}{1-r} = \frac{10}{1-\frac{1}{2}} = 20$

19. (a) : If radius = 6.5 cm, then BD = 13 cm.

By Pythagoras theorem,

The other side of the rectangle has to be 12.

(Since 5, 12 and 13 from a Pythagoras triplet)

So, area $(\Delta PCD) = \left(\frac{1}{2}\right) \times 12 \times 5 = 30\text{ cm}^2$

It does not matter which side is 12 or 5. Area remains the same.

20. (c) : Aryabhatt is selected, then Ramanujan cannot. Hence, there are ${}^8C_4$ ways of selecting the team, *i.e.*, 70 ways.

Similarly, if Ramanujan is selected, there are 70 ways. And if both are not selected, there are ${}^8C_5$ = 56 ways.

Hence, total number of ways = 70 + 70 + 56 = 196

◆◆

Brain Teaser

LOGICAL REASONING
for Various Competitive Exams

Direction Q.(1 and 2): Find the wrong term in the given series.

1. 80, 42, 24, 13.5, 8.75, 6.375, 5.1875
(a) 8.75 (b) 13.5 (c) 24 (d) 6.375

2. AN36,BO25, CP16, DS9, ER4
(a) AN36 (b) BO25
(c) CP16 (d) DS9

Direction Q.(3 and 4): The two words or number on the right of (: :) are related in the some way as the words on the left of (: :) Identify what will come in the place of blank.

3. DEFIANT : OBEDIENT : : DEPRAVITY : ……....
(a) GOODNESS (b) SADNESS
(c) SERIOUSNESS (d) OBEDIENT

4. If '598' is to '490' then '637' is to ……………
(a) 763 (b) 315 (c) 463 (d) 222

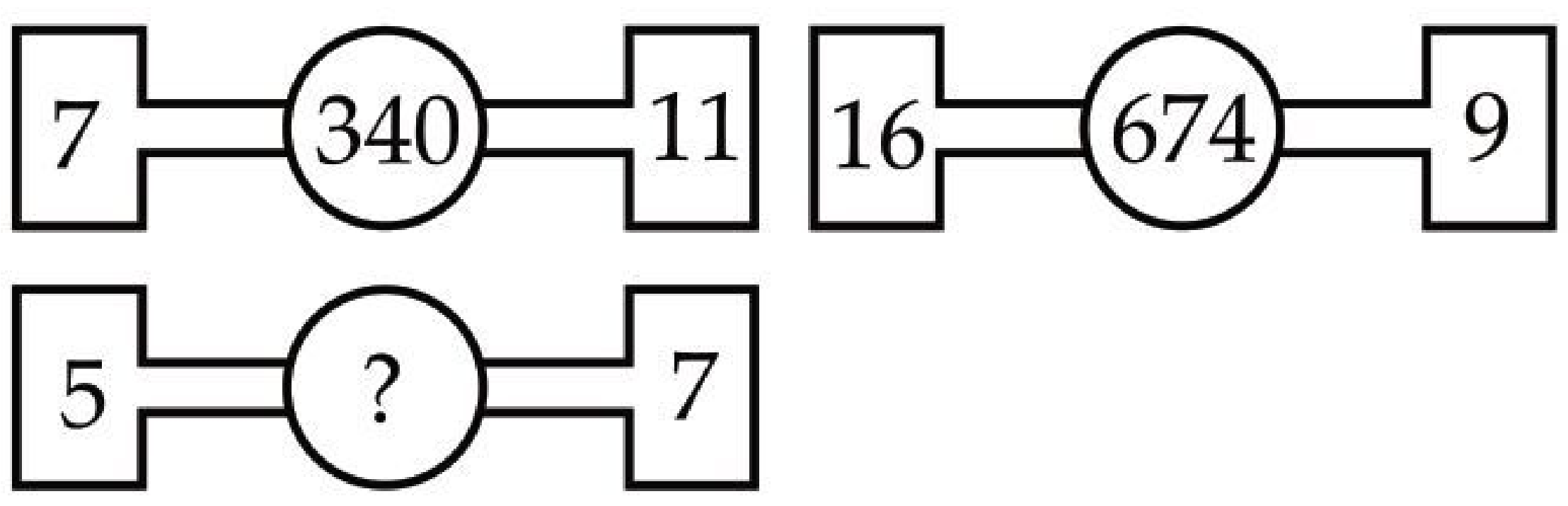
Direction Q. (5 and 6) : Three of the following four are alike in a certain way and so form a group. Which is the one that does not belong to that group?

5.
(a) fluent (b) dense (c) smooth (d) runny

6.
(a) Raisin (b) Rain (c) Shower (d) Snow

Direction Q.(7 and 8): In each of the following figures, find the number/letter which replaces the sign of '?'

7.



(a) 144 (b) 142 (c) 148 (d) 150

8.

B	?	G	257	P	50	C	5
122	D	485	K	122	V	17	?

(a) 12, J (b) 10, K (c) 15, I (d) 122, K

Direction Q.(9 and 10): In a certain code language, RAT is coded as 1521; LION is coded as 2500; TIGER is coded as 3481.

9. What is the code of MOUSE?
(a) 3249 (b) 4900 (c) 5329 (d) 5184

10. What is the code of JACKAL?
(a) 1296 (b) 1444 (c) 1521 (d) 1600

Direction Q.(11 and 12): Study the following information carefully and answer the questions given below:
If 'A @ B' means 'A is father of B'.
If 'A + B' means 'A is son of B'.
If 'A $ B' means 'A is daughter of B'.
If 'A % B' means 'A is mother of B'.
If 'A & B' means 'A is husband of B'.

11. Which among the following options is true if the expression 'P + Q % R $ S + T & W' is definitely true?
(a) Q is the only daughter of T.
(b) P is the grandson of S.
(c) R is the granddaughter of W.
(d) T is the father-in-law of P.

12. How is V related to P in the expression 'P & S % T @ U + V'?
(a) Grandmother (b) Daughter-in-law
(c) Grandson (d) Wife

13. Two squads of soldiers A and B, facing East and West respectively received the following commands– Left Turn, Left Turn, Left Turn, Right Turn, Left Turn. Which directions would the squads A and B respectively face at the end?
(a) East, West (b) West, East
(c) North, South (d) South, North

Direction Q.(14 and 15): In these questions, relationship between different elements is shown in the statements. These statements are followed by two conclusions.

Mark answer
(a) if only conclusion (I) is true.
(b) if only conclusion (II) is true.
(c) if either conclusion (I) or (II) is true.
(d) if both conclusions (I) and (II) are true.

14. **Statements:** $A > B = C$; $E = F$; $E \geq D > C$
Conclusions:
I. $B < E$ II. $F < B$

15. **Statements:** $A > B = C$; $E = F$; $E \geq D > C$
Conclusions:
I. $C < F$ II. $A > C$

16. Seema correctly remembers that she took leave after 21st October and before 27th October. Her colleague Rita took leave on 23rd October but Seema was present on that day. If 24th October was a public holiday and 26th October was Sunday, on which day in October did Seema take leave?
(a) 22nd October
(b) 25th October
(c) 22nd or 25th October
(d) Data inadequate

Direction Q.(17 to 19): Study the given information carefully and answer the questions given below:

There are six people – P, Q, R, S, T and U each having different weight. P is heavier than U but lighter than Q. T is heavier than R but lighter than P. Q is lighter than S. R is not the lightest. The second lightest person weighs 52 kilogram while the third heaviest is of 64 kilogram.

17. Who among the following may be of 58 kilogram in weight?
(a) P (b) Q (c) T (d) R

18. How many persons are heavier than R?
(a) One (b) Two (c) Three (d) Four

19. Which of the following may represent the weight of Q ?
(a) 62 kg (b) 66 kg (c) 60 kg (d) 58 kg

20. Seven events – A, B, C, D, E, F and G were organised on different days of the same week starting from Monday and ending on Sunday but not necessarily in the same order. Which event was organised on Sunday?

I. There were two events between A and B. Event A was organised after event B. Event G was organised immediately before event D. Event E was organised on Wednesday.

II. Event F was organised immediately before event C. There was only one event between F and A. Event G was organised on Monday.

(a) Data in statement (I) alone is sufficient to answer the question, while the data in statement (II) alone is not sufficient to answer the question.
(b) Data in statement (II) alone is sufficient to answer the question, while the data in statement (I) alone is not sufficient to answer the question.
(c) Data either in statement (I) alone or in statement (II) alone is sufficient to answer the question.
(d) Data in both the statements (I) and (II) together are necessary to answer the question.

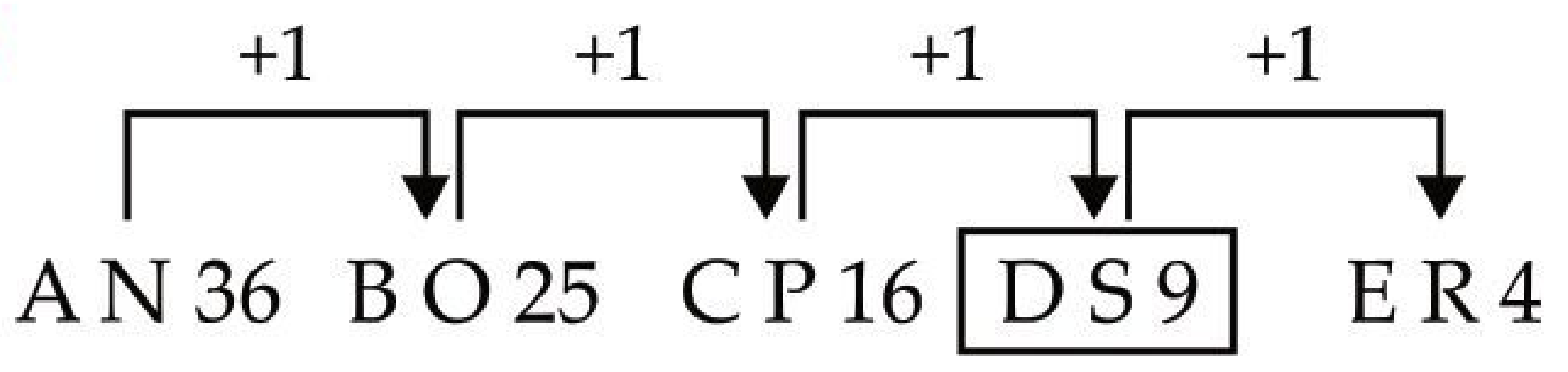
SOLUTIONS

1. (c) :

$$80 \xrightarrow{-38} 42 \xrightarrow{-19} \boxed{24} \xrightarrow{-9.5} 13.5 \xrightarrow{-4.75}$$
$$8.75 \xrightarrow{-2.375} 6.375 \xrightarrow{-1.1875} 5.1875$$

∴ Wrong term 24.

2. (d) :



3. (a) : Defiant and obedient are antonyms.

4. (d) : The relationship is : 598 is to (5 × 98).

5. (b) : Dense is an antonym to all other words.

6. (a) : All other are related to each other as all are weather events.

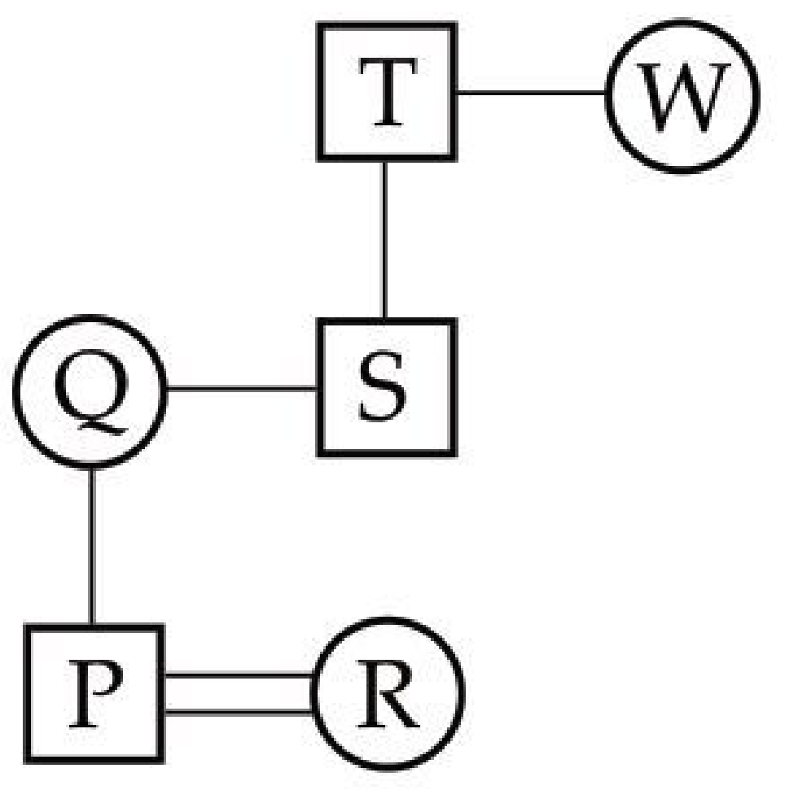
7. (c) : Diagram 1 : $(7 + 11)^2 + (7 - 11)^2 = 324 + 16 = 340$
Diagram 2 : $(16 + 9)^2 + (16 - 9)^2 = 674$
Diagram 3 : $(5 + 7)^2 + (5 - 7)^2 = 144 + 4 = 148$.

8. (b) : B + 2 = D; D + 3 = G, G + 4 = K; K + 5 = P
P + 6 = V; V + 7 = C, C + 8 = K.
Corresponding numbers are one more than the square of alphabets from the right. Like number for B is $2^2 + 1 = 5$.

9. (c) : $(M + O + U + S + E)^2$
$= (13 + 15 + 21 + 19 + 5)^2 = 5329$

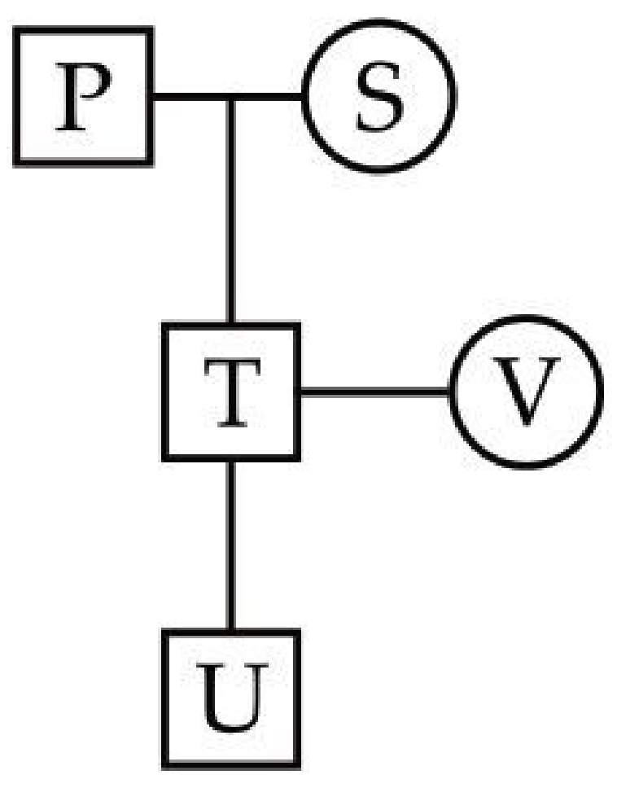
10. (b) : $(J + A + C + K + A + L)^2$
$= (10 + 1 + 3 + 11 + 1 + 12)^2 = 1444$

11. (c) : Statement: P + Q % R $ S + T & W can be drawn as



R is granddaughter of W.

12. (b) : Statement: P & S % T @ U + V can be drawn as



V is daughter-in-law of P.

13. (d) : A is facing South and B is facing North.

14. (a) : Statements: $A > B = C$; $E = F$; $E \geq D > C$
$A > B = C < D \leq E = F$
I. $B < E$: True
II. $F < B$: Not true
Hence, only conclusion (I) is true.

15. (d) : Statements: $A > B = C$; $E = F$; $E \geq D > C$
$A > B = C < D \leq E = F$
I. $C < F$: True
II. $A > C$: True
Hence, both conclusions (I) and (II) are true.

16. (c) : According to Seema, she took leave may be on 22nd, 23rd, 24th, 25th or 26th October. (on 23rd Seema was present as her colleague Rita took leave on 23rd October).
Hence, the common dates are 22nd or 25th October.

17. (c) :

$$S > Q > \underset{\substack{\downarrow \\ 64\text{ kg}}}{P} > T > \underset{\substack{\downarrow \\ 52\text{ kg}}}{R} > U$$

Therefore, T may be of 58 kg.

18. (d) : Four persons – S, Q, P and T are heavier than R.

19. (b) : Q is heavier than P and P's weight is 64 kg. Therefore, Q's weight may be 66 kg.

20. (d) : No statement alone is sufficient to answer. From both the statements,

Day	Event
Monday	G
Tuesday	D
Wednesday	E
Thursday	B
Friday	F
Saturday	C
Sunday	A

∴ Event A was organised on Sunday.

◆ ◆

PUBLIC NOTICE

JEE MAIN 2023

The Department of Higher Education, Ministry of Education, Government of India has entrusted the responsibility of conducting the Joint Entrance Examination (JEE Main) to the NTA from 2019 onwards.

The Joint Entrance Examination (JEE (Main) comprises two papers. **Paper 1 (B.E./B.Tech.)** is conducted for admission to Undergraduate Engineering Programs (B.E/B. Tech) at NITs, IIITs, other Centrally Funded Technical Institutions (CFTIs), Institutions/Universities funded/recognized by participating State Governments. It is also an eligibility test for JEE (Advanced), which is conducted for admission to IITs. **Paper 2** is conducted for admission to **B. Arch and B. Planning** courses in the Country.

For Academic Session 2023-24, it has been decided that the **JEE (Main) – 2023 will be conducted in two Sessions** i.e. Session 1 (January 2023) and Session 2 (April 2023). The details for the **Session 1 (January 2023)** are given below :

Online Submission of Application Form	**15 December 2022 to 12 January 2023 (up to 09:00 P.M.)**
Last date of successful transaction of fee through Credit/Debit Card/ Net-Banking/UPI	**12 January 2023 (up to 11:50 P.M.)**
Announcement of the City of Examination	**Second week of the January 2023**
Downloading of Admit Cards from the NTA website	**Third week of the January 2023**
Date of Examination	**24, 25, 27, 28, 29, 30 and 31 January 2023**
Centre, Date, and Shift	As indicated on Admit Card
Display of Recorded Responses and Answer Keys	To be announced later on the website
Website(s)	www.nta.ac.in , https://jeemain.nta.nic.in/
Declaration of Result on the NTA website	*To be announced later on the website*

JEE (Main) - 2023 will be conducted in **13 languages** i.e. **English, Hindi, Assamese, Bengali, Gujarati, Kannada, Malayalam, Marathi, Odia, Punjabi, Tamil, Telugu, and Urdu.**

In the first session of JEE (Main) - 2023, only Session 1 will be visible and candidates can opt for that. In the next session, Session 2 will be visible, and the candidates can opt for that Session. The application window for Session 2 will be re-opened as per the details available in the Information Bulletin and will also be notified separately through a Public Notice.

For Papers, Scheme, Timing, eligibility and other information, candidates are requested to please check the Information Bulletin of JEE (Main) – 2023 available on the website: https://jeemain.nta.nic.in/

beat the TIME TRAP

Duration : 60 minutes

SECTION-I

Single Option Correct Type

1. In a triangle *ABC*, if $A=\frac{\pi}{7}, B=\frac{2\pi}{7}$ and $C=\frac{4\pi}{7}$, then $a^2+b^2+c^2$ is (*R* is circumradius of ΔABC)

(a) $4R^2$ (b) $6R^2$ (c) $7R^2$ (d) $8R^2$

2. Let $f(x)$ be a function such that $f(x) = x - [x]$, where $[x]$ is the greatest integer less than or equal to *x*. Then the number of solution of the equation $f(x)+f\left(\frac{1}{x}\right)=1$ is

(a) 0 (b) 1 (c) 2 (d) infinite

3. If z_1, z_2, z_3 and z_4 be the consecutive vertices of a square, then $z_1^2+z_2^2+z_3^2+z_4^2$ equals

(a) $z_1z_2 + z_2z_3 + z_3z_4 + z_4z_1$

(b) $z_1z_2 + z_1z_3 + z_1z_4 + z_2z_3 + z_2z_4 + z_3z_4$

(c) 0

(d) none of these

4. If $f(x)=\frac{e^{2x}-(1+4x)^{1/2}}{\ln(1-x^2)}$ for $x \neq 0$, then *f* has

(a) an irremovable discontinuity at $x = 0$

(b) a removable discontinuity at $x = 0$ and $f(0) = \frac{1}{4}$

(c) a removable discontinuity at $x = 0$ and $f(0)=-\frac{1}{4}$

(d) none of these

5. $7^9 + 9^7$ is divisible by

(a) 128 (b) 24 (c) 64 (d) 72

6. The distance of the point (1, –2, 3) from the plane $x-y+z=5$ measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z-1}{-6}$ is

(a) 1 unit (b) 2 units

(c) 4 units (d) None of these

7. How many numbers between 5000 and 10000 can be formed using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 each digit appearing not more than once in each number?

(a) $5 \times {}^8P_3$ (b) $5 \times {}^8C_8$

(c) $5! \times {}^8P_3$ (d) $5 \times {}^8C_3$

8. If $\Delta = \begin{vmatrix} \sin x & \sin(x+h) & \sin(x+2h) \\ \sin(x+2h) & \sin x & \sin(x+h) \\ \sin(x+h) & \sin(x+2h) & \sin x \end{vmatrix}$, then value of $\lim\limits_{h\to 0}\left(\frac{\Delta}{h^2}\right)$ is

(a) $6 \sin x \cos^2 x$ (b) $6 \cos x \sin^2 x$

(c) $9 \sin x \cos^2 x$ (d) $9 \cos x \sin^2 x$

9. The mean and variance of 5 observations of an experiment are 4 and 5.2 respectively. From these observations three are 1, 2 and 6 and $\lambda = |x_1 - x_2| + 8$, where x_1 and x_2 are remaining observations. Then number of solution of equation $10 - x^2 - 2x = \lambda$ is

(a) 1 (b) 2 (c) 3 (d) 4

10. If $(\sin y)^{\sin\frac{\pi x}{2}}+\frac{\sqrt{3}}{2}\sec^{-1}2x + 2^x\tan\ln(x+2) = 0$, then $y'(-1) =$

(a) $\frac{1}{\pi^2-3}$ (b) $\frac{1}{\pi\sqrt{\pi^2-3}}$

(c) $\frac{3}{2\pi\sqrt{\pi^2-3}}$ (d) $\frac{-3}{\pi\sqrt{\pi^2-3}}$

SECTION-II

Numerical Answer Type

11. The value of $\lim\limits_{x\to\pi/4}\frac{8-\sqrt{2}(\cos x+\sin x)^5}{1-\sin 2x}$ is _____.

12. If *A* is the area bounded by $y = x^{2/3} - x^{4/3}$ and *x*-axis, then the value of $35A/4$ is _____.

13. The length of the common chord of two circles of radii 15 and 20, whose centres are 25 units apart, is _____.

14. If the straight lines joining the origin to the points of intersection of $2x + 3y = k$ and $3x^2 - xy + 3y^2 + 2x - 3y - 4 = 0$ are at right angle, then $6k - \frac{52}{k}$ is ______.

15. Water is dropped at the rate of 2 m^3/s into a cone of semi vertical angle 45°. If the rate at which periphery of water surface changes when the height of the water in the cone is 2 m is d, then the value of 5 d is ______.

SOLUTIONS

1. (c) : $a^2 + b^2 + c^2 = 4R^2(\sin^2 A + \sin^2 B + \sin^2 C)$

$= 2R^2(1 - \cos 2A + 1 - \cos 2B + 1 - \cos 2C)$

$$= 2R^2\left[3 - \left(\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{8\pi}{7}\right)\right]$$

$$= 2R^2\left[3 - \left(\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}\right)\right]$$

$$= 2R^2\left[3 - \left(2\sin\frac{\pi}{7}\cdot\cos\frac{2\pi}{7} + 2\sin\frac{\pi}{7}\cdot\cos\frac{4\pi}{7} + 2\sin\frac{\pi}{7}\cdot\cos\frac{6\pi}{7}\right)\frac{1}{2\sin\frac{\pi}{7}}\right]$$

$$= 2R^2\left[3 - \frac{1}{2\sin\frac{\pi}{7}}\left(\sin\frac{3\pi}{7} - \sin\frac{\pi}{7} + \sin\frac{5\pi}{7} - \sin\frac{3\pi}{7} + \sin\frac{7\pi}{7} - \sin\frac{5\pi}{7}\right)\right]$$

$$= 2R^2\left[3 + \frac{1}{2}\right] = 7R^2$$

2. (d) : Given, $f(x) = x - [x], x \in R - \{0\}$

Now, $f(x) + f\left(\frac{1}{x}\right) = 1 \Rightarrow x - [x] + \frac{1}{x} - \left[\frac{1}{x}\right] = 1$

$$\Rightarrow \left(x + \frac{1}{x}\right) - \left([x] + \left[\frac{1}{x}\right]\right) = 1$$

$$\Rightarrow \left(x + \frac{1}{x}\right) = [x] + \left[\frac{1}{x}\right] + 1 \quad \text{...(i)}$$

Clearly, R.H.S. is an integer ∴ L. H. S. is also an integer.

Let $x + \frac{1}{x} = k$ (an integer) $\Rightarrow x^2 - kx + 1 = 0$

$$\therefore \quad x = \frac{k \pm \sqrt{k^2 - 4}}{2}$$

For real values of x, $k^2 - 4 \geq 0 \Rightarrow k \geq 2$ or $k \leq -2$

Also, $k = 2$ and -2 does not satisfy equation (i)

∴ The equation (i) will have solutions if $k > 2$ or $k < -2$, $k \in Z$.

Hence, equation (i) has infinite number of solutions.

3. (a) : We know that, $\frac{z_2 - z_1}{z_4 - z_1} = \left|\frac{z_2 - z_1}{z_4 - z_1}\right| e^{\frac{i\pi}{2}} = i$

$[\because |z_2 - z_1| = |z_4 - z_1|]$

$\Rightarrow (z_4 - z_1)^2 + (z_2 - z_1)^2 = 0 \quad$ ...(i)

Similarly, $\frac{z_4 - z_3}{z_2 - z_3} = i$

$\Rightarrow (z_4 - z_3)^2 + (z_2 - z_3)^2 = 0 \quad$ ...(ii)

On adding (i) and (ii), we get

$$2[z_1^2 + z_2^2 + z_3^2 + z_4^2 - z_1z_2 - z_4z_1 - z_4z_3 - z_2z_3] = 0$$

$$\Rightarrow z_1^2 + z_2^2 + z_3^2 + z_4^2 = z_1z_2 + z_2z_3 + z_3z_4 + z_4z_1$$

4. (d) : $\lim_{x\to 0}\frac{e^{2x} - (1+4x)^{1/2}}{\log(1-x^2)} = \lim_{x\to 0}\frac{e^{2x} - (1+4x)^{1/2}}{\frac{\log(1-x^2)}{-x^2}(-x^2)}$

$$= \lim_{x\to 0}\frac{(1+4x)^{1/2} - e^{2x}}{x^2}$$

$$= \lim_{x\to 0}\frac{\left(1 + \frac{1}{2}4x + \frac{1}{2}\cdot\left(\frac{1}{2} - 1\right)\frac{1}{2!}16x^2 + \ldots\right) - \left(1 + \frac{2x}{1!} + \frac{4x^2}{2!} + \ldots\right)}{x^2}$$

$= -4$

5. (c)

6. (a) : The equation of the line passing through $P(1, -2, 3)$ and parallel to the given line is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6}$$

Suppose it meets the plane $x - y + z = 5$ at the point Q given by $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$

i.e., $(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$

This lies on $x - y + z = 5$. Therefore,

$2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5 \Rightarrow -7\lambda = -1 \Rightarrow \lambda = \frac{1}{7}$

So, the co-ordinates of Q are (9/7, –11/7, 15/7).

Hence, required distance $= PQ = \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = 1$ unit.

7. (a) : We are to form 4-digit numbers with either 5 or 6 or 7 or 8 or 9 at the thousand's place.

$$\text{4-digit number :}\quad \underset{1}{\overset{\times}{\downarrow}} \quad \underset{2}{\overset{\times}{\downarrow}} \quad \underset{3}{\overset{\times}{\downarrow}} \quad \underset{4}{\overset{\times}{\downarrow}}$$

Number of ways of filling 1st place = 5

After filling 1st place, we have 8 digits to be used.

Number of ways of filling 2nd, 3rd, 4th places = ${}^8P_3$

∴ Total number of numbers formed = $5 \times {}^8P_3$

8. (c) : We apply the following transformations to Δ in sequence: $C_3 \to C_3 - C_2$, $C_2 \to C_2 - C_1$, $C_2 \to \frac{C_2}{h}$ and $C_3 \to \frac{C_3}{h}$. Now using $\lim\limits_{h \to 0}$ on the resulting determinant, we have

$$\lim_{h\to 0}\frac{\Delta}{h^2} = \begin{vmatrix} \sin x & \cos x & \cos x \\ \sin x & -2\cos x & \cos x \\ \sin x & \cos x & -2\cos x \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, $R_2 \to R_2 + 2R_3$, we get

$$\lim_{h\to 0}\frac{\Delta}{h^2} = \begin{vmatrix} 3\sin x & 0 & 0 \\ 3\sin x & 0 & -3\cos x \\ \sin x & \cos x & -2\cos x \end{vmatrix}$$

$= 9 \sin x \cos^2 x$ [Expanding along R_1]

9. (a) : Mean $(\bar{x}) = 4$, variance $= 5.2$

Let x_1, x_2 be the remaining values.

Mean, $\bar{x} = \frac{1+2+6+x_1+x_2}{5} \Rightarrow x_1 + x_2 = 11$ ...(i)

Variance, $\sigma^2 = 5.2 = \frac{1^2+2^2+6^2+x_1^2+x_2^2}{5} - (\bar{x})^2$

$\Rightarrow x_1^2 + x_2^2 = 65$ ...(ii)

$\Rightarrow |x_1 - x_2| = 3$

Now, $\lambda = |x_1 - x_2| + 8 \Rightarrow \lambda = 11$

$\Rightarrow 10 - x^2 - 2x = 11 \Rightarrow (x+1)^2 = 0$ i.e., only one solution.

10. (c) : $x = -1 \Rightarrow \sin y = -\frac{\sqrt{3}}{\pi}$, $\cos y = \frac{\sqrt{\pi^2 - 3}}{\pi}$

Differentiating the given relation,

$$(\sin y)^{\sin\frac{\pi x}{2}}\left[\frac{\pi}{2}\cos\frac{\pi x}{2}\ln\sin y + \sin\frac{\pi x}{2}\cdot\cot y\cdot y'\right] + \frac{\sqrt{3}}{2}$$

$$\cdot\frac{2}{|2x|\sqrt{4x^2-1}} + 2^x\left[\ln 2\tan\ln(x+2) + \frac{\sec^2\ln(x+2)}{x+2}\right] = 0$$

Putting $x = -1$ and using $\sin y(-1) = \frac{-\sqrt{3}}{\pi}$,

$\cos y(-1) = \frac{\sqrt{\pi^2-3}}{\pi}$, we get,

$\therefore\ y'(-1) = \frac{\sin^2 y(-1)}{2\cos y(-1)} = \frac{3}{2\pi\sqrt{\pi^2-3}}$.

11. (10) : $\lim\limits_{x\to\pi/4}\frac{8-\sqrt{2}(\cos x+\sin x)^5}{1-\sin 2x}$

$= \sqrt{2}\lim\limits_{x\to\pi/4}\frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}$

$= \sqrt{2}\lim\limits_{x\to\pi/4}\frac{2^{5/2}-((\sin x+\cos x)^2)^{5/2}}{1-\sin 2x}$

$= \sqrt{2}\lim\limits_{x\to\pi/4}\frac{2^{5/2}-(1+\sin 2x)^{5/2}}{2-(1+\sin 2x)}$

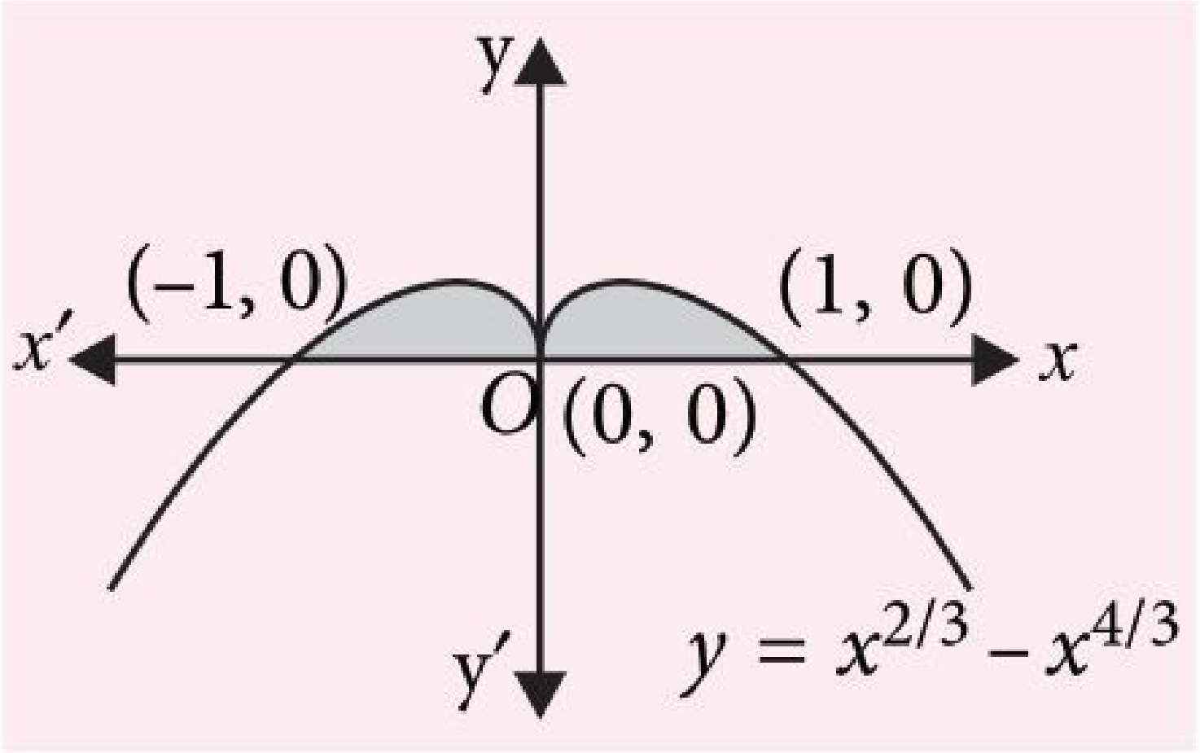
$= \sqrt{2}\lim\limits_{u\to 2}\frac{2^{5/2}-u^{5/2}}{2-u}$ $(\because\ u = 1 + \sin 2x)$

$= \sqrt{2}\,\frac{5}{2}.2^{3/2} = 10$

12. (3) : We have, $y = f(x) = x^{2/3} - x^{4/3}$

It meets the x-axis at $x = 0, \pm 1$.

Also, function is even.



$\therefore$ Required area, A

$= 2\left[\int_0^1 (x^{2/3} - x^{4/3})\,dx\right]$

$= 2\left[\frac{3x^{5/3}}{5} - \frac{3x^{7/3}}{7}\right]_0^1 = \frac{12}{35}$

13. (24) : Let C_1 and C_2 be the centres of the two circles of radii r and R respectively.

Then, $C_1C_2 = 25$, $r = 15$ and $R = 20$ [Given]

We observe that $C_1C_2^2 = r^2 + R^2$

Thus, the two circles, intersect orthogonally.

$\therefore$ Length of the common chord

$= \frac{2rR}{\sqrt{r^2+R^2}} = \frac{2\times 15\times 20}{25} = 24$

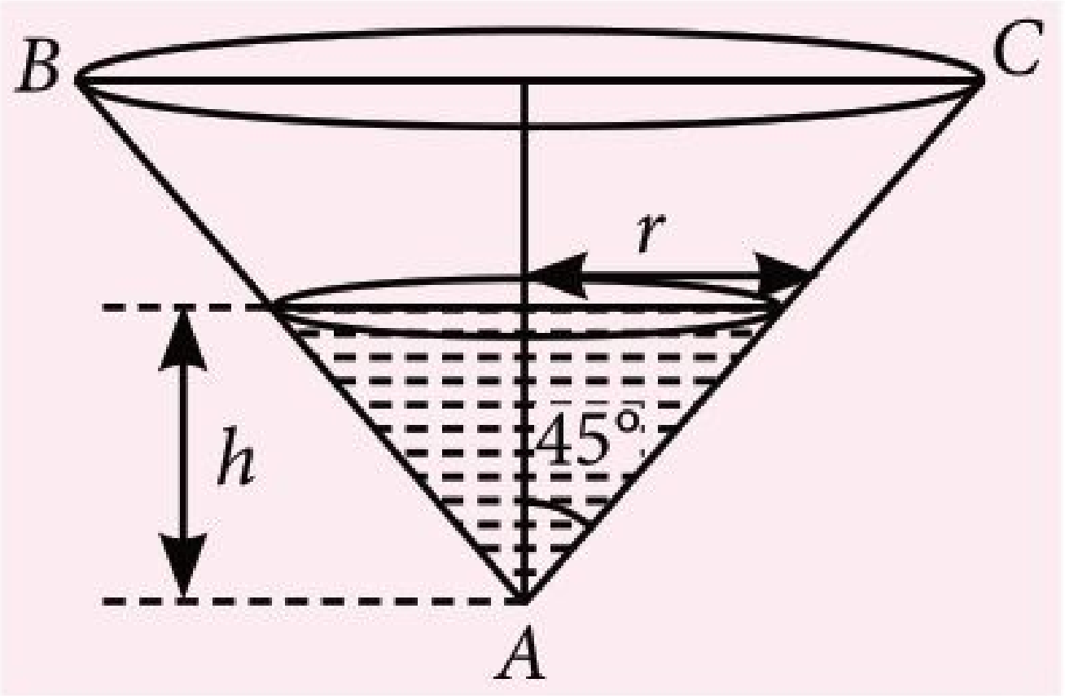
14. (5) : The pair of lines is

$$3x^2 - xy + 3y^2 + (2x - 3y)\frac{(2x+3y)}{k} - 4\frac{(2x+3y)^2}{k^2} = 0$$

Now, coefficient of x^2 + coefficient of $y^2 = 0$

$\Rightarrow 3+3+\frac{4-9}{k} - \frac{4(4+9)}{k^2} = 0 \Rightarrow 6k - \frac{52}{k} = 5$

15. (5) : We have,



$\frac{dV}{dt} = 2$ or $\frac{d}{dt}\left(\frac{1}{3}\pi r^3\right) = 2$

$[\because r = h$ as $\theta = 45°]$

$\Rightarrow \pi r^2\frac{dr}{dt} = 2 \Rightarrow \frac{dr}{dt} = \frac{2}{\pi r^2}$ ...(i)

Now, perimeter $= 2\pi r = p$ (say)

$\therefore\ \frac{d}{dt}(2\pi r) = 2\pi\frac{dr}{dt} = 2\pi\left(\frac{2}{\pi r^2}\right) = \frac{4}{r^2}$ ...(ii) [Using (i)]

When $h = 2$ m, $r = 2$ m

Hence, $\frac{dp}{dt} = \frac{4}{4} = 1$ m/s.

◆ ◆

CBSE warm-up!

CLASS-XI

Practice questions for CBSE Exams as per the latest pattern and reduced syllabus by CBSE for the academic session 2022-23.

Practice Paper 2022-23

Time Allowed : 3 hours
Maximum Marks : 80

General Instructions :

- *(a) This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
- *(b) Section A has 18 MCQ's and 2 Assertion-Reason based questions of 1 mark each.*
- *(c) Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
- *(d) Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
- *(e) Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
- *(f) Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.*

SECTION - A (MULTIPLE CHOICE QUESTIONS)

Each question carries 1 mark.

1. If $A + B = \frac{5\pi}{4}$, then value of $(1 + \tan A)(1 + \tan B)$ equals

(a) 1 (b) 2 (c) −2 (d) −1

2. The mean deviation of the data 2, 9, 9, 3, 6, 9, 4 from the mean is

(a) 2.23 (b) 2.57 (c) 3.23 (d) 3.57

3. In a single throw of two dice, determine the probability of not getting the same number on the two dice.

(a) $\frac{5}{6}$ (b) $\frac{4}{6}$ (c) $\frac{3}{6}$ (d) $\frac{1}{6}$

4. If $y = x \tan x$, then dy/dx is equal to

(a) $\frac{\tan x}{x - x^2 - y^2}$ (b) $\frac{y}{x - x^2 - y^2}$

(c) $\frac{\tan x}{y - x}$ (d) $\frac{\cos x \sin x + x}{\cos^2 x}$

5. Find the point on x-axis which is equidistant from the point $A(3, 2, 2)$ and $B(5, 5, 4)$.

(a) $(16, 0, 0)$ (b) $\left(\frac{5}{4}, 0, 0\right)$

(c) $(9, 0, 0)$ (d) $\left(\frac{49}{4}, 0, 0\right)$

6. Let $A = \{(x, y) : y = e^{2x}, \forall x \in R\}$ and $B = \{(x, y) : y = e^{-2x}, \forall\ x \in R\}$ then $A \cap B$ is

(a) a set containing more than two elements
(b) Singleton set
(c) Empty set (d) None of these

7. If $|z^2 - 1| = |z|^2 + 1$, then z lies on

(a) imaginary axis (b) real axis
(c) origin (d) None of these

8. The set $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$ is equal to

(a) $B \cap C'$ (b) $A \cap C$
(c) $B' \cap C'$ (d) None of these

9. If $|3 - 4x| \geq 9$, then $x \in$

(a) $(-\infty, -3) \cup (3, \infty)$ (b) $\left(-\infty, \frac{-3}{2}\right] \cup (3, \infty)$

(c) $\left(-\infty, \frac{-3}{2}\right) \cup (0, \infty)$ (d) $\left(-\infty, \frac{-3}{2}\right] \cup [3, \infty)$

10. If $A + B = 45°$, then $(\cot A - 1)(\cot B - 1)$ is equal to

(a) 1 (b) $\frac{1}{2}$ (c) – 1 (d) 2

11. Find the value of n such that $\frac{{}^{n}P_4}{{}^{n-1}P_4} = \frac{5}{3}, n > 4$.

(a) 11 (b) 10 (c) 13 (d) 12

12. Which term of the G.P. $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$ is $\frac{1}{128}$?

(a) 9th (b) 8th (c) 7th (d) 5th

13. Find the 4th term from the end in the expansion of $\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^9$.

(a) $\frac{674}{x}$ (b) $\frac{670}{x^3}$ (c) $\frac{672}{x^3}$ (d) $\frac{-670}{x^3}$

14. Solution of $2x - 1 = |x + 7|$ is

(a) – 2 (b) 8

(c) – 2, 8 (d) None of these

15. If A and B are non-empty sets, then $(A \cap B) \cup (A - B)$ is equal to

(a) B (b) A (c) A' (d) B'

16. Convert 6 radians into degree measure.

(a) 343°38′11″ (b) 348°33′11″

(c) 433°38′11″ (d) 343°37′12″

17. The argument of $\frac{1 - i\sqrt{3}}{1 + i\sqrt{3}}$ is

(a) 210° (b) 180° (c) 240° (d) 60°

18. Given 4 flags of different colours, how many different signals can be generated, if a signal requires the use of 2 flags one below the other?

(a) 12 (b) 13 (c) 14 (d) 15

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A) :** A line through the focus and perpendicular to the directrix is called the x-axis of the parabola.

Reason (R) : The point of intersection of parabola with the axis is called the vertex of the parabola.

20. **Assertion (A) :** The range of $f(x) = \cos[x]$, for $\frac{-\pi}{2} < x < \frac{\pi}{2}$ is $\{\cos 2, \cos 1, 1\}$.

Reason (R) : The function $f : R \rightarrow R$ defined by $f(x) = [x], x \in R$ assumes the value of the greatest integer, less than or equal to x, is called greatest integer function.

SECTION - B

This section comprises of very short answer type questions (VSA) of 2 marks each.

21. Find the domain of the function

$$f(x) = \log_2\left(-\log_{1/2}\left(1 + \frac{1}{\sqrt[4]{x}}\right) - 1\right).$$

22. For $|x| < 1$, let $y = 1 + x + x^2 + \ldots \infty$, then prove that $\frac{dy}{dx} - y = xy^2$.

23. Using binomial theorem, prove that $3^{3n} - 26n - 1$ is divisible by 676.

OR

Simplify: $(x + 2y)^8 + (x - 2y)^8$.

24. If $A = \left\{\frac{-1}{2}, 0, \frac{1}{4}, \frac{3}{4}, 2\right\}, B = \left\{0, \frac{1}{4}, \frac{3}{4}, 2, \frac{5}{2}\right\}$, and $C = \left\{\frac{-1}{2}, \frac{1}{4}, 1, 2, \frac{5}{2}\right\}$, then verify the associative property of intersection of sets.

25. Find the equation of a line which divides the join of (1, 0) and (3, 0) in the ratio 2 : 1 and perpendicular to it.

OR

Without using distance formula check whether the points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) are the vertices of a triangle or not?

SECTION - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. A, B and C are the subsets of a universal set U. If $A = \{2, 4, 6, 8, 12, 20\}, B = \{3, 6, 9, 12, 15\}, C = \{5, 10, 15, 20\}$ and U is the set of all whole numbers, draw a Venn diagram showing the relation of U, A, B and C.

27. Find the coordinates of the orthocentre of a triangle whose vertices are $(-1, 3)$, $(2, -1)$ and $(0, 0)$.

OR

If the equation of the base of an equilateral triangle is $x + y = 2$ and the vertex is $(2, -1)$, then find the length of the side of the triangle.
[Hint: Find length of perpendicular (p) from $(2, -1)$ to the line and use $p = l \sin 60°$, where l is the length of side of the triangle.]

28. Using binomial theorem, show that $7^9 + 9^7$ is divisible by 64.

OR

Without expanding, find the value of
$(2x - 1)^5 + 5(2x - 1)^4 (1 - x) + 10(2x - 1)^3 (1 - x)^2 + 10(2x - 1)^2 (1 - x)^3 + 5(2x - 1)(1 - x)^4 + (1 - x)^5$.

29. If $|z| = 1$, prove that $\frac{z-1}{z+1} (z \neq -1)$ is a purely imaginary number. What will you conclude if $z = 1$?

30. Solve the inequation : $\frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3}$

OR

Solve the following linear inequations and draw the graph of the solution.

(i) $3x + 17 \leq 2(1 - x)$

(ii) $\frac{2x-3}{4} + 1 < x + \frac{4}{3}; x \in R$

31. If n arithmetic means are inserted between 20 and 80 such that the ratio of first mean to the last mean is 1 : 3, then find the value of n.

SECTION - D

This section comprises of long answer type questions (LA) of 5 marks each.

32. From a group of 2 boys and 3 girls, two children are selected at random. Describe the following events.

(i) A = Both selected children are girls.

(ii) B = The selected group consists of one boy and one girl.

(iii) C = At least one boy is selected.

Which pair (s) of events is (are) mutually exclusive?

33. Evaluate : $\lim_{x \to 0} \frac{\tan x + 4 \tan 2x - 3 \tan 3x}{x^2 \tan x}$

OR

Let $f(x) = \frac{\sin x + \cos x}{\sin x - \cos x}$, then find $f'(x)$.

34. If $\sin(\theta + \alpha) = a$ and $\sin(\theta + \beta) = b$, then prove that $\cos 2(\alpha - \beta) - 4ab \cos(\alpha - \beta) = 1 - 2a^2 - 2b^2$.
[Hint: Express $\cos(\alpha - \beta) = \cos((\theta + \alpha) - (\theta + \beta))$].

35. Find the variance and standard deviation of the following frequency distribution :

Variable (x_i)	Frequency (f_i)
2	4
4	4
6	5
8	15
10	8
12	5
14	4
16	5

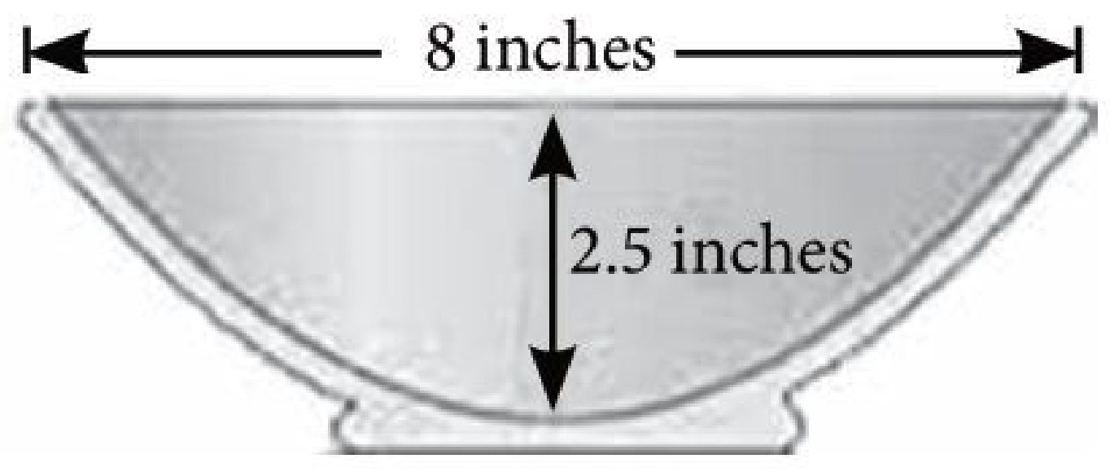
OR

Mean and standard deviation of 100 observations were found to be 40 and 10, respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively, find the correct standard deviation.

SECTION - E

This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii) and (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36. **Case-study 1:** Read the following passage and answer the questions given below.
Ram's mother gave a bowl of soup to Ram. The cross section of bowl was in parabolic shape. The bowl has a diameter of 8 inches and is 2.5 inches deep



(i) Find the equation of the parabola.

(ii) What is the length of latus rectum ?

(iii) What is the area of triangle formed by the lines joining the vertex of the parabola to the ends of its latus rectum?

OR

Find the equation of the lines joining the vertex of the parabola to the points on it, which have ordinate 10.

37. **Case-study 2 :** Read the following passage and answer the questions given below.

Ram and Ankit are cousins. They study in the same school but in different classes. Ram is older than Ankit. Assume the ages of Ram and Ankit are x and y respectively.

(i) If n geometric means be inserted between x and y, then what will be the n^{th} geometric mean ?

(ii) What is the product of n geometric means between x and $1/x$?

(iii) If arithmetic mean of their ages exceed the geometric mean of their ages by 2 and Ram is 12 years older than Ankit, then what will be the ages of Ram and Ankit?

OR

Find two geometric means between the number 1 and 64.

38. **Case-study 3:** Read the following passage and answer the questions given below.

Anil and Sunil were playing with Word Scrabble game. Anil made the word "OBJECTIVE" and Sunil made the word "SUBJECTIVE.

Let X and Y respectively denote the set of letters of the words formed by Anil and Sunil.

(i) Find $X \cap Y$ and $n(X \cup Y)$.

(ii) Show that $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

SOLUTIONS

1. **(b) :** $\tan(A + B) = \tan\dfrac{5\pi}{4} = \tan\left(\pi + \dfrac{\pi}{4}\right) = \tan\dfrac{\pi}{4}$

$\therefore$ $\tan A + \tan B = 1 - \tan A \tan B$

$\Rightarrow$ $\tan A + \tan B + \tan A \tan B = 1$

$\Rightarrow$ $(1 + \tan A) + \tan B\,(1 + \tan A) = 1 + 1$

(by adding 1 on both sides)

$\Rightarrow$ $(1 + \tan A)(1 + \tan B) = 2$

2. **(b) :** Mean $\bar{x} = \dfrac{2+9+9+3+6+9+4}{7} = \dfrac{42}{7} = 6$

$$\text{M.D.}\,(\bar{x}) = \frac{\Sigma|x_i - \bar{x}|}{n} = \frac{4+3+3+3+0+3+2}{7} = 2.57$$

3. **(a) :** $n(S) = (6 \times 6) = 36$

Let E = event of getting the same number on two dice.
Then, $E = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$.

$$\therefore\ P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$\Rightarrow$ P(not getting the same number on the two dice)

$= \left(1 - \dfrac{1}{6}\right) = \dfrac{5}{6}$.

4. **(d) :** We have, $y = x \tan x$

$$\therefore\ \frac{dy}{dx} = x(\tan x)' + \tan x \cdot (x)' = x\sec^2 x + \tan x \cdot 1$$

$$= \frac{x}{\cos^2 x} + \frac{\sin x}{\cos x} = \frac{x + \sin x \cos x}{\cos^2 x}$$

5. **(d) :** The point on the x-axis is of the form $P(x, 0, 0)$. Since the points A and B are equidistant from P. Therefore $PA^2 = PB^2$

$\Rightarrow (x - 3)^2 + (0 - 2)^2 + (0 - 2)^2 = (x - 5)^2 + (0 - 5)^2 + (0 - 4)^2$

$\Rightarrow$ $4x = 25 + 25 + 16 - 17 \Rightarrow x = \dfrac{49}{4}$

Thus, the point P on the x-axis is $\left(\dfrac{49}{4}, 0, 0\right)$ which is equidistant from A and B.

6. **(b) :** Given $A = \{(x, y) : y = e^{2x}, \forall\, x \in R\}$
and $B = \{(x, y) : y = e^{-2x}, \forall x \in R\}$

$$\Rightarrow\ e^{2x} = \frac{1}{e^{2x}} \Rightarrow e^{4x} = 1 \Rightarrow e^{4x} = e^0 \Rightarrow x = 0 \text{ and } y = 1$$

As e^{2x} and e^{-2x} both intersect each other at (0, 1), so $A \cap B$ is a singleton set.

7. **(a) :** Let $z = x + iy$

Then, $|z^2 - 1| = |z|^2 + 1$

$\Rightarrow$ $|x^2 - y^2 - 1 + i2xy| = |x + iy|^2 + 1$

$\Rightarrow$ $(x^2 - y^2 - 1)^2 + 4x^2y^2 = (x^2 + y^2 + 1)^2$

$\Rightarrow$ $4x^2 = 0 \Rightarrow x = 0$

Hence z lies on y-axis or imaginary axis.

8. **(a) :** $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$

$= (A \cup B \cup C) \cap (A' \cup B \cup C) \cap C'$

$= [(A \cap A') \cup (B \cup C)] \cap C'$

$= [(\phi \cup B \cup C)] \cap C' = (B \cup C) \cap C'$

$= (B \cap C') \cup (C \cap C') = (B \cap C') \cup \phi = B \cap C'$

9. **(d) :** We have $|3 - 4x| \geq 9$

$\Rightarrow$ $3 - 4x \leq -9$ or $3 - 4x \geq 9 \Rightarrow -4x \leq -12$ or $-4x \geq 6$

$$\Rightarrow\ x \geq 3 \text{ or } x \leq \frac{-3}{2} \Rightarrow x \in \left(-\infty, \frac{-3}{2}\right] \cup [3, \infty)$$

10. **(d) :** We have, $A + B = \dfrac{\pi}{4}$

$$\Rightarrow\ \cot(A + B) = \cot\frac{\pi}{4} \Rightarrow \frac{\cot A\ \cot B - 1}{\cot A + \cot B} = 1$$

$\Rightarrow$ cot$A$ cot$B$ – 1 = cot$A$ + cot$B$
$\Rightarrow$ cot$A$ cot$B$ – cot$A$ – cot$B$ + 1 = 2
$\Rightarrow$ cot$A$ (cot$B$ – 1) – 1 (cot$B$ – 1) = 2
$\Rightarrow$ (cot$A$ – 1) (cot$B$ – 1) = 2

11. (b) : Given that $\dfrac{{}^nP_4}{{}^{n-1}P_4} = \dfrac{5}{3}$

$\Rightarrow$ $3n(n-1)(n-2)(n-3) = 5(n-1)(n-2)(n-3)(n-4)$
$\Rightarrow$ $3n = 5(n-4)$ [as $(n-1)(n-2)(n-3) \neq 0, n > 4$]
$\Rightarrow$ $n = 10$

12. (a) : Here, first term $(a) = 2$ and common ratio $(r) = 1/2$

Let the n^{th} term be $\dfrac{1}{128}$.

Then, $a_n = \dfrac{1}{128} \Rightarrow ar^{n-1} = \dfrac{1}{128} \Rightarrow 2\left(\dfrac{1}{2}\right)^{n-1} = \dfrac{1}{128}$

$$\Rightarrow \left(\frac{1}{2}\right)^{n-1} = \frac{1}{2^8} \Rightarrow \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^8$$

$\Rightarrow$ $n - 1 = 8 \Rightarrow n = 9$

Thus, 9th term of the given G.P. is $\dfrac{1}{128}$.

13. (c) : 4th term from the end is 9 – 4 + 2, *i.e.*, 7th term from the beginning.

$$\therefore\ T_7 = {}^9C_6\left(\frac{x^3}{2}\right)^3\left(\frac{-2}{x^2}\right)^6 = {}^9C_3\,\frac{x^9}{8}\cdot\frac{64}{x^{12}}$$

$$= \frac{9\times8\times7}{3\times2\times1}\times\frac{8}{x^3} = \frac{672}{x^3}$$

14. (b) : $2x - 1 = |x + 7| = \begin{cases} x+7 & ,\ \text{if } x \geq -7 \\ -(x+7) & ,\ \text{if } x < -7 \end{cases}$

$\therefore$ If $x \geq -7$, $2x - 1 = x + 7 \Rightarrow x = 8$
If $x < -7$, $2x - 1 = -(x + 7) \Rightarrow 3x = -6$
$\Rightarrow$ $x = -2$, Not possible.

15. (b) : $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')$
[$\because (A - B) = (A \cap B')$]
$= A \cap (B \cup B')$ (By distributive law)
$= A \cap U = A$ ($\because B \cup B' = U$)

16. (a) : We know that π radian = 180°.

Hence, 6 radians $= \dfrac{180}{\pi} \times 6$ degree $= \dfrac{1080\times7}{22}$ degree

$= 343\dfrac{7}{11}$ degree $= 343° + \dfrac{7\times60}{11}$ minute [as $1° = 60'$]

$= 343° + 38' + \dfrac{2}{11} \times 60$ seconds [as $1' = 60''$]

$= 343° + 38' + 10.9'' = 343°38'\,11''$ approximately.
Hence, 6 radians = $343°38'\,11''$ approximately.

17. (c) : Let $z = \dfrac{1-i\sqrt{3}}{1+i\sqrt{3}} = \dfrac{1-i\sqrt{3}}{1+i\sqrt{3}} \times \dfrac{1-i\sqrt{3}}{1-i\sqrt{3}}$

$$= \frac{1-3-2\sqrt{3}\,i}{1+3} = \frac{-2-2\sqrt{3}\,i}{4} = \frac{-1}{2} - \frac{\sqrt{3}}{2}i$$

$\because$ $x < 0, y < 0$
$\therefore$ z lies in 3rd Quadrant.

$\therefore$ $\tan\theta = \dfrac{-\sqrt{3}/2}{-1/2} = \sqrt{3} = \tan 60°$

$= \tan(180° + 60°) = \tan 240° \Rightarrow \theta = 240°$

18. (a) : There will be as many signals as there are ways of filling 2 vacant places ☐☐ in succession by the 4 flags of different colours. Hence, by the multiplication principle, the required number of signals = 4 × 3 = 12.

19. (d) : A line through the focus and perpendicular to the directrix is called the axis of the parabola. The point of intersection of parabola with the axis is called the vertex of the parabola.

20. (a)

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7	3	4	8	9	2	1	5	6				4	1	3	6	9	5	2	7	8
1	8	5	7	6	3	9	4	2				8	6	9	1	7	2	5	3	4
2	6	9	5	4	1	3	7	8				5	7	2	3	4	8	1	9	6
9	4	1	2	5	6	7	8	3				2	8	6	9	5	3	4	1	7
8	5	6	3	7	9	4	2	1				3	9	5	7	1	4	6	8	2
3	2	7	1	8	4	5	6	9				7	4	1	8	2	6	9	5	3
6	7	3	9	2	5	8	1	4	2	9	3	6	5	7	2	3	1	8	4	9
4	1	8	6	3	7	2	9	5	4	6	7	1	3	8	4	6	9	7	2	5
5	9	2	4	1	8	6	3	7	5	8	1	9	2	4	5	8	7	3	6	1
						4	6	2	8	5	9	3	7	1						
						3	7	8	1	2	6	5	4	9						
						9	5	1	3	7	4	2	8	6						
4	1	2	5	9	6	7	8	3	9	1	5	4	6	2	8	5	1	3	9	7
9	3	5	2	8	7	1	4	6	7	3	2	8	9	5	2	7	3	6	4	1
8	6	7	1	4	3	5	2	9	6	4	8	7	1	3	9	6	4	5	2	8
5	8	3	7	1	4	9	6	2				5	4	9	1	2	8	7	3	6
7	4	6	9	2	8	3	1	5				1	3	7	5	9	6	2	8	4
2	9	1	3	6	5	8	7	4				6	2	8	3	4	7	1	5	9
3	2	8	4	5	1	6	9	7				2	7	6	4	8	5	9	1	3
6	7	9	8	3	2	4	5	1				9	8	1	7	3	2	4	6	5
1	5	4	6	7	9	2	3	8				3	5	4	6	1	9	8	7	2

21. We have $f(x)=\log_2\left(-\log_{1/2}\left(1+\frac{1}{\sqrt[4]{x}}\right)-1\right)$

$f(x)$ is defined if $-\log_{1/2}\left(1+\frac{1}{\sqrt[4]{x}}\right)-1>0$

or if $\log_{1/2}\left(1+\frac{1}{\sqrt[4]{x}}\right)<-1$ or if $\left(1+\frac{1}{\sqrt[4]{x}}\right)>(1/2)^{-1}$

or if $1+\frac{1}{\sqrt[4]{x}}>2$ or if $\frac{1}{\sqrt[4]{x}}>1$

or if $x^{1/4} < 1$ or if $0 < x < 1$ $\quad\therefore\quad D(f) = (0, 1)$

22. We have, $y = 1 + x + x^2 + \ldots \infty$

It's a G.P. with common ratio x and first term 1.

$$\therefore \quad y=\frac{1}{1-x} \qquad [\because\ |x| < 1] \quad \ldots(\text{i})$$

$$\Rightarrow \frac{dy}{dx}=\frac{0-1(1-x)'}{(1-x)^2}=\frac{1}{(1-x)^2}$$

$$\therefore \frac{dy}{dx}-y=\frac{1}{(1-x)^2}-\frac{1}{1-x} \qquad [\text{From (i)}]$$

$$=\frac{1-(1-x)}{(1-x)^2}=\frac{x}{(1-x)^2}=\frac{x}{(1-x)(1-x)}=xy^2$$

23. We have $3^{3n} = 27^n = (1 + 26)^n$

$= {}^nC_0(1)^n + {}^nC_1\,(1)^{n-1}\,(26) + {}^nC_2\,(1)^{n-2}\,(26)^2$
$\quad + {}^nC_3\,(1)^{n-3}\,(26)^3 + \ldots + {}^nC_n\,(26)^n$

$= 1 + 26n + {}^nC_2\,(26)^2 + {}^nC_3\,(26)^3 + \ldots + {}^nC_n\,(26)^n$

$\therefore \quad 3^{3n} - 26n - 1 = (26)^2\,[{}^nC_2 + {}^nC_3\,(26) + \ldots$
$\quad + {}^nC_n\,(26)^{n-2}]$

$= 676\,[{}^nC_2 + {}^nC_3\,(26) + \ldots + {}^nC_n\,(26)^{n-2}]$

which shows that $3^{3n} - 26n - 1$ is divisible by 676.

OR

$(x + 2y)^8 + (x - 2y)^8 = 2[{}^8C_0\,x^8 + {}^8C_2\,x^6\,(2y)^2 + {}^8C_4\,x^4\,(2y)^4$
$\quad + {}^8C_6\,x^2\,(2y)^6 + {}^8C_8\,(2y)^8]$

$[\because (x + a)^n + (x - a)^n = 2({}^nC_0\,x^n + {}^nC_2\,x^{n-2}\,a^2$
$\quad + {}^nC_4\,x^{n-4}\,a^4 + \ldots)]$

$\Rightarrow \quad (x + 2y)^8 + (x - 2y)^8 = 2[x^8 + 28x^6 \times 4y^2 + 70x^4 \times 16y^4 + 28x^2 \times 64y^6 + 256y^8]$

$= 2[x^8 + 112x^6y^2 + 1120x^4y^4 + 1792x^2y^6 + 256y^8]$

24. Here, we need to verify the following

$A \cap (B \cap C) = (A \cap B) \cap C$

Clearly, $B\cap C=\left\{\frac{1}{4},2,\frac{5}{2}\right\}$ and $A\cap B=\left\{0,\frac{1}{4},\frac{3}{4},2\right\}$

$$\therefore\ A\cap(B\cap C)=\left\{-\frac{1}{2},0,\frac{1}{4},\frac{3}{4},2\right\}\cap\left\{\frac{1}{4},2,\frac{5}{2}\right\}=\left\{\frac{1}{4},2\right\} \quad \ldots(\text{i})$$

$$\text{and } (A\cap B)\cap C=\left\{0,\frac{1}{4},\frac{3}{4},2\right\}\cap\left\{-\frac{1}{2},\frac{1}{4},1,2,\frac{5}{2}\right\}=\left\{\frac{1}{4},2\right\} \quad \ldots(\text{ii})$$

From (i) and (ii), we get $A \cap (B \cap C) = (A \cap B) \cap C$

Hence, the associative property of intersection of sets is verified.

25. Let C be the point which divides the join of $A(1, 0)$ and $B(3, 0)$ in the ratio 2 : 1. Then, the coordinates of C are $\left(\frac{2\times3+1\times1}{2+1},\frac{2\times0+1\times0}{2+1}\right)=\left(\frac{7}{3},0\right)$

Since AB is along X-axis, therefore a line perpendicular to AB is parallel to Y-axis. As it passes through $C(7/3, 0)$, therefore its equation is $x=\frac{7}{3} \Rightarrow 3x = 7$

Hence, the equation of the required line is $3x = 7$.

OR

Let the given points be $A\left(0,\frac{8}{3}\right)$, $B(1, 3)$ and $C(82, 30)$

Now, area of ΔABC

$$=\frac{1}{2}\left|0\cdot(3-30)+1\cdot\left(30-\frac{8}{3}\right)+82\cdot\left(\frac{8}{3}-3\right)\right|$$

$$=\frac{1}{2}\left|\frac{82}{3}-\frac{82}{3}\right|=0$$

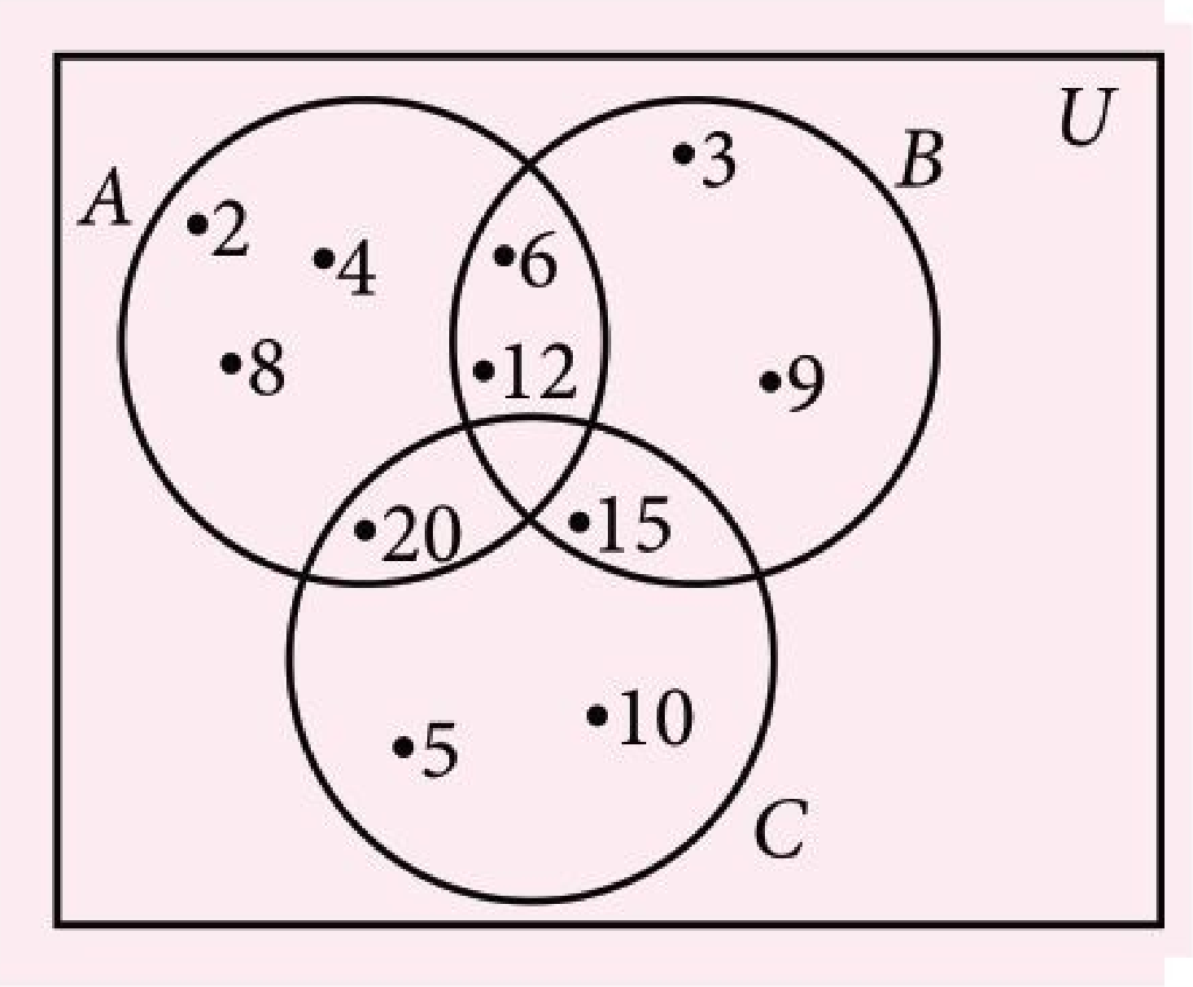
So, no triangle can be formed from the given points.

Hence, the given points are collinear.

26. $A \cap B = \{6, 12\}$, $B \cap C = \{15\}$

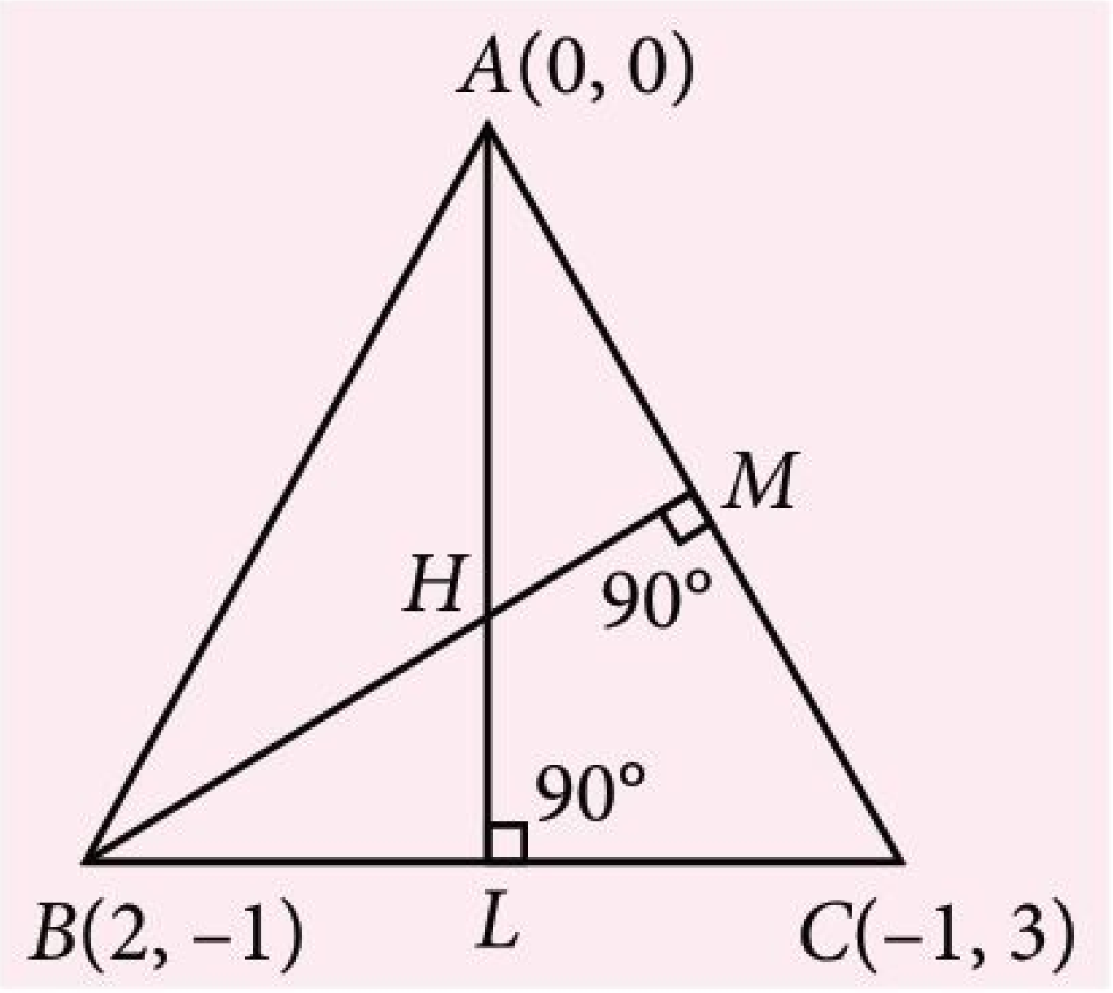
$C \cap A = \{20\}$, $A \cap B \cap C = \phi$

$\therefore$ Venn diagram showing the relation between U, A, B and C is



27. Let $A = (0, 0)$, $B = (2, -1)$ and $C = (-1, 3)$

Since $AL \perp BC$, therefore equation of line AL is



$$y-0=-\frac{2+1}{-1-3}(x-0)$$

$$\left[\because \ \text{Slope of } BC=\frac{-1-3}{2+1}\right]$$

$\Rightarrow \quad 4y = 3x$

or $\quad 3x - 4y = 0 \ldots(\text{i})$

Since $BM \perp AC$, therefore, equation of line BM is

$$y+1=-\frac{0+1}{0-3}(x-2) \text{ or } 3(y + 1) = x - 2 \text{ or } x - 3y = 5 \quad \ldots(\text{ii})$$

Solving equations (i) and (ii), we get $x = -4$, $y = -3$

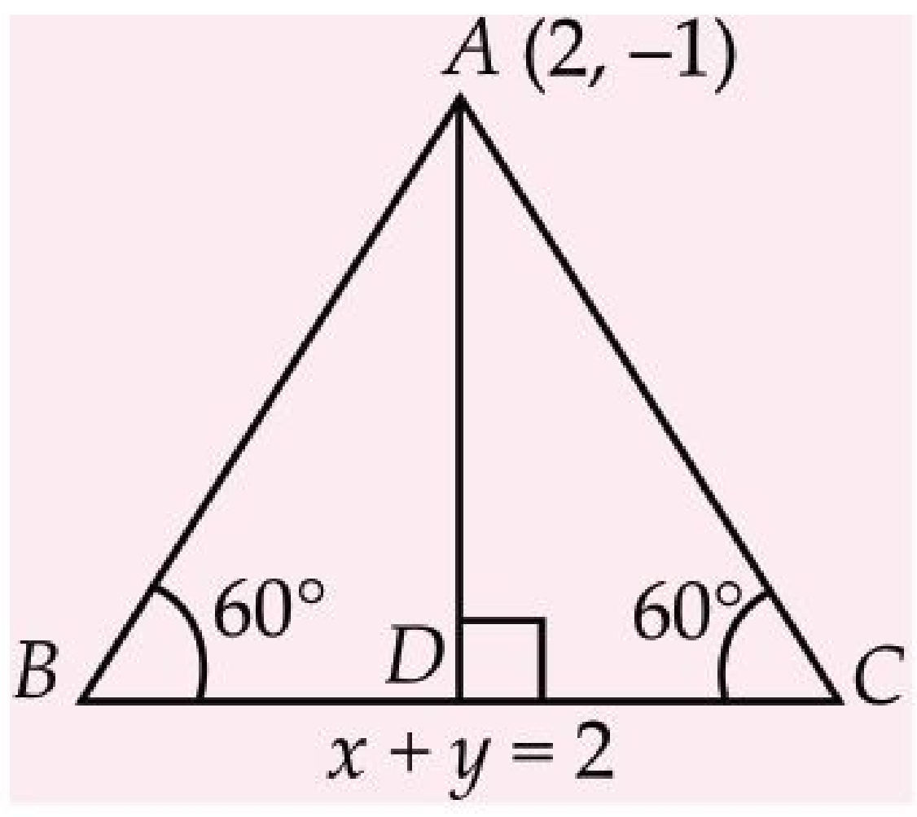
Hence orthocentre of the triangle is $(-4, -3)$.

OR

Since ΔABC is an equilateral triangle.

$\therefore \quad \angle A = \angle B = \angle C = 60°$

$(AD \perp BC)$



In ΔABD, $\sin 60° = \dfrac{AD}{AB}$

$\Rightarrow \; AD = AB\dfrac{\sqrt{3}}{2}$ ...(i)

The length of perpendicular from (2, –1) to the line $x + y = 2$ is given by $AD = \left|\dfrac{2+(-1)-2}{\sqrt{1^2+1^2}}\right| = \dfrac{1}{\sqrt{2}}$

From (i), $\dfrac{1}{\sqrt{2}} = AB\dfrac{\sqrt{3}}{2} \Rightarrow AB = \sqrt{\dfrac{2}{3}}$

28. We have, $7^9 + 9^7 = 9^7 + 7^9 = (1 + 8)^7 - (1 - 8)^9$

$= ({}^7C_0 + {}^7C_1 8 + {}^7C_2 8^2 + \ldots\ldots + {}^7C_7 8^7)$

$\quad - ({}^9C_0 - {}^9C_1 8 + {}^9C_2 8^2 - \ldots\ldots - {}^9C_9 8^9)$

$= [1 + (7 \times 8) + (21 \times 8^2) + \ldots\ldots + 8^7] -$

$\quad [1 - (9 \times 8) + (36 \times 8^2) - \ldots\ldots - 8^9)]$

$= (7 \times 8 + 9 \times 8) + (21 \times 8^2 - 36 \times 8^2) + \ldots\ldots$

$= (2 \times 64) + (21 - 36) \times 64 + \ldots\ldots.$

$= 64[2 + (21 - 36) + \ldots\ldots]$

which is divisible by 64.

OR

Clearly, $(2x - 1)^5 + 5(2x - 1)^4 (1 - x) + 10(2x - 1)^3 (1 - x)^2 + 10(2x - 1)^2 (1 - x)^3 + 5(2x - 1)(1 - x)^4 + (1 - x)^5$

$= {}^5C_0(2x - 1)^5 + {}^5C_1(2x - 1)^{5-1}(1 - x) + {}^5C_2(2x - 1)^{5-2}(1 - x)^2 + {}^5C_3(2x - 1)^{5-3}(1 - x)^3 + {}^5C_4(2x - 1)^{5-4}(1 - x)^4 + {}^5C_5(2x - 1)^{5-5}(1 - x)^5$

$= [(2x - 1) + (1 - x)]^5 = x^5.$

29. Let $z = x + iy$

Given, $|z| = 1 \Rightarrow |z|^2 = 1 \Rightarrow x^2 + y^2 = 1$ ...(i)

Now, $\dfrac{z-1}{z+1} = \dfrac{x+iy-1}{x+iy+1} = \dfrac{(x-1)+iy}{(x+1)+iy}$

$= \dfrac{(x-1)+iy}{(x+1)+iy} \times \dfrac{(x+1)-iy}{(x+1)-iy}$

$= \dfrac{x^2-1-iy(x-1)+iy(x+1)-i^2y^2}{(x+1)^2-i^2y^2}$

$= \dfrac{x^2-1+y^2+iy(x+1-x+1)}{(x+1)^2+y^2}$

$= \dfrac{x^2+y^2-1}{(x+1)^2+y^2} + i\dfrac{2y}{(x+1)^2+y^2}$

$= \dfrac{1-1}{(x+1)^2+y^2} + i\dfrac{2y}{(x+1)^2+y}$ [From (i)]

$= i\dfrac{2y}{(x+1)^2+y^2}$ [$\because x^2 + y^2 = 1$]

i.e., a purely imaginary number.

If $z = 1$, then $\dfrac{z-1}{z+1} = 0$, which is both real and purely imaginary.

30. We have, $\dfrac{2x-3}{4} + 9 \ge 3 + \dfrac{4x}{3}$

$\Rightarrow \dfrac{2x-3}{4} - \dfrac{4x}{3} \ge 3 - 9$

[Transposing $\dfrac{4x}{3}$ to L.H.S. and 9 to R.H.S.]

$\Rightarrow \dfrac{3(2x-3)-16x}{12} \ge -6$

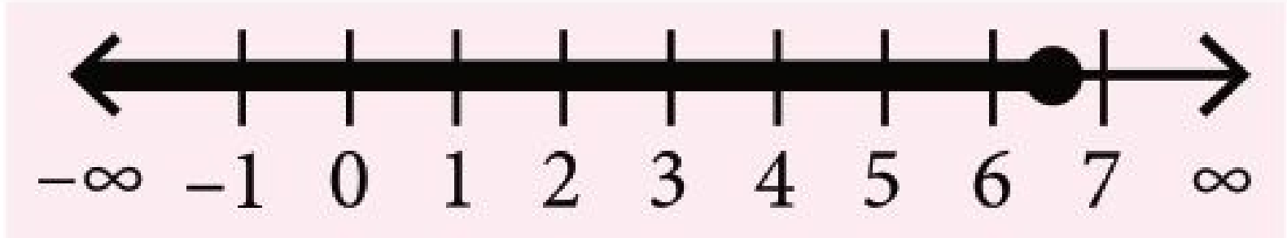
$\Rightarrow \dfrac{6x-9-16x}{12} \ge -6 \Rightarrow \dfrac{-9-10x}{12} \ge -6$

$\Rightarrow \; -9 - 10x \ge -72$ [Multiplying both sides by 12]

$\Rightarrow \; -10x \ge -72 + 9 \Rightarrow -10x \ge -63$

$\Rightarrow \dfrac{-10x}{-10} \le \dfrac{-63}{-10} \Rightarrow x \le \dfrac{63}{10} \Rightarrow x \in (-\infty, 63/10]$

Hence, the solution set of the given inequation is $(-\infty, 63/10]$. This can be graphed on real line as shown below:



OR

(i) We have, $3x + 17 \le 2(1 - x)$

$\Rightarrow \; 3x + 17 \le 2 - 2x \Rightarrow 3x + 2x \le 2 - 17$

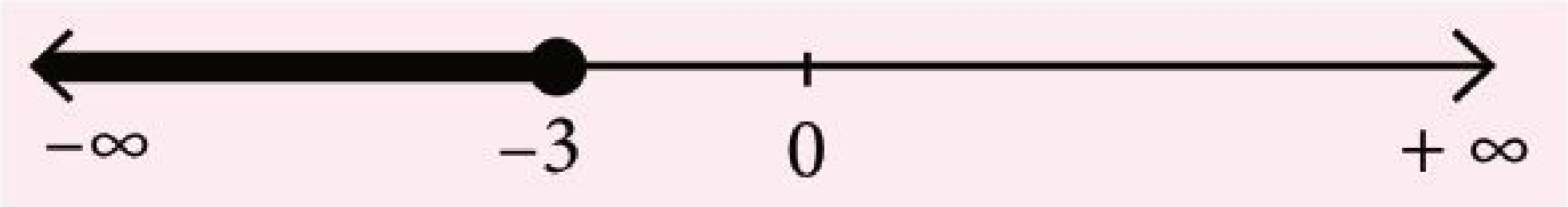
[Transposing $-2x$ to L.H.S. and 17 to R.H.S.]

$\Rightarrow \; 5x \le -15$

$\Rightarrow \dfrac{5x}{5} \le \dfrac{-15}{5}$ [Dividing both sides by 5]

$\Rightarrow \; x \le -3 \Rightarrow x \in (-\infty, -3]$

Hence, the solution set of the given inequation is $(-\infty, -3]$, which can be graphed on real line as shown below :



(ii) We have, $\dfrac{2x-3}{4} + 1 < x + \dfrac{4}{3}; x \in R$

$\Rightarrow \dfrac{2x-3+4}{4} < \dfrac{3x+4}{3} \Rightarrow \dfrac{2x+1}{4} < \dfrac{3x+4}{3}$

Multiplying both sides by 12 *i.e.*, L.C.M. of 4 and 3, we have,

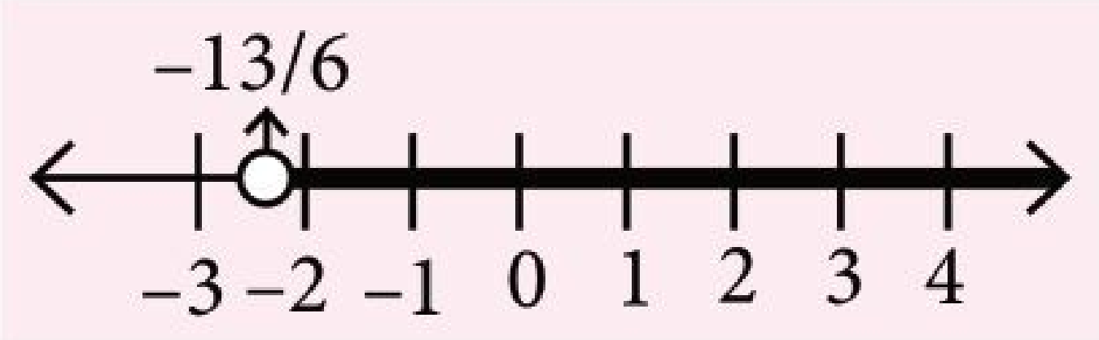
$$\frac{12}{4}(2x+1) < \frac{12}{3}(3x+4) \Rightarrow 3(2x+1) < 4(3x+4)$$

$\Rightarrow$ $6x + 3 < 12x + 16 \Rightarrow$ $6x - 12x < 16 - 3$

$\Rightarrow$ $-6x < 13 \Rightarrow x > -\frac{13}{6}$ [Dividing both sides by – 6]

$\therefore$ Solution set $= \{x : x > -\frac{13}{6}, x \in R\}$ *i.e.*, $\left(-\frac{13}{6}, \infty\right)$

This set can be represented on number line as shown below:



31. Let A_1 A_2, ..., A_n be the n A.M.'s

$\therefore$ 20, A_1, A_2, ..., A_n, 80 are in A.P.

$\therefore$ $80 = 20 + (n + 2 - 1)d \Rightarrow 60 = (n + 1)d$

$\Rightarrow d = \frac{60}{n+1}$

Now, $A_1 = a + d = 20 + \frac{60}{n+1} = 20\left(\frac{n+4}{n+1}\right)$

and $A_n = a + nd = 20 + \frac{60n}{n+1} = 20\left(\frac{4n+1}{n+1}\right)$

According to the question,

$$\frac{A_1}{A_n} = \frac{1}{3} \Rightarrow \frac{20(n+4/n+1)}{20(4n+1/n+1)} = \frac{1}{3} \Rightarrow \frac{n+4}{4n+1} = \frac{1}{3}$$

$\Rightarrow$ $4n + 1 = 3n + 12 \Rightarrow n = 11$

32. Let B_1, B_2 be two boys and G_1, G_2, G_3 be three girls. Then, the sample space associated with the random experiment is

$S = \{B_1B_2, B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, G_1G_2, G_1G_3, G_2G_3\}$

(i) A = Both selected children are girls
$= \{G_1G_2, G_1G_3, G_2G_3\}$

(ii) B = The selected group consists of one boy and one girl $= \{B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3\}$

(iii) C = At least one boy is selected
$= \{B_1B_2, B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3\}$

Clearly, $A \cap B = \phi$ and $A \cap C = \phi$. So, A and B, A and C are two pairs of mutually exclusive events.

Note that $B \cap C \neq \phi$ therefore they are not mutually exclusive.

33. We have, $\lim\limits_{x\to 0} \frac{\tan x + 4\tan 2x - 3\tan 3x}{x^2 \tan x}$

$$= \lim_{x\to 0} \frac{\tan x + 4\left(\frac{2\tan x}{1-\tan^2 x}\right) - 3\left(\frac{3\tan x - \tan^3 x}{1-3\tan^2 x}\right)}{x^2 \tan x}$$

$$= \lim_{x\to 0} \frac{1 + \frac{8}{1-\tan^2 x} - 3\left(\frac{3-\tan^2 x}{1-3\tan^2 x}\right)}{x^2}$$

$$= \lim_{x\to 0} \frac{(1-\tan^2 x)(1-3\tan^2 x) + 8(1-3\tan^2 x) - 3(3-\tan^2 x)(1-\tan^2 x)}{x^2(1-\tan^2 x)(1-3\tan^2 x)}$$

$$= \lim_{x\to 0} \frac{1 - 4\tan^2 x + 3\tan^4 x + 8 - 24\tan^2 x - 9 + 12\tan^2 x - 3\tan^4 x}{x^2(1-\tan^2 x)(1-3\tan^2 x)}$$

$$= \lim_{x\to 0} \frac{-16\tan^2 x}{x^2(1-\tan^2 x)(1-3\tan^2 x)}$$

$$= -16 \lim_{x\to 0} \left(\frac{\tan x}{x}\right)^2 \times \frac{1}{1-\tan^2 x} \times \frac{1}{1-3\tan^2 x} = -16$$

OR

Here $f(x) = \frac{\sin x + \cos x}{\sin x - \cos x}$ ...(i)

Differentiating (i) w.r.t. 'x', we get

$$f'(x) = \frac{d}{dx}\left[\frac{\sin x + \cos x}{\sin x - \cos x}\right]$$

$$= \frac{(\sin x - \cos x)\frac{d}{dx}(\sin x + \cos x) - (\sin x + \cos x)\frac{d}{dx}(\sin x - \cos x)}{(\sin x - \cos x)^2}$$

(Using quotient rule)

$$= \frac{(\sin x - \cos x)(\cos x - \sin x) - (\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2}$$

$$= \frac{-(\sin x - \cos x)^2 - (\sin x + \cos x)^2}{(\sin x - \cos x)^2}$$

$$= \frac{-\sin^2 x - \cos^2 x + 2\sin x\cos x - \sin^2 x - \cos^2 x - 2\sin x\cos x}{(\sin x - \cos x)^2}$$

$$= \frac{-2(\sin^2 x + \cos^2 x)}{(\sin x - \cos x)^2} = \frac{-2}{(\sin x - \cos x)^2}$$

34. Given, $\sin(\theta + \alpha) = a$ ...(i)

and $\sin(\theta + \beta) = b$ ...(ii)

$\therefore$ $\cos(\theta+\alpha) = \sqrt{1-a^2}$ and $\cos(\theta+\beta) = \sqrt{1-b^2}$

Since, $\cos(\alpha - \beta) = \cos[(\theta + \alpha) - (\theta + \beta)]$

$= \cos(\theta + \alpha)\cos(\theta + \beta) + \sin(\theta + \alpha)\sin(\theta + \beta)$

[$\because$ $\cos(A - B) = \cos A \cos B + \sin A \sin B$]

$$= \sqrt{1-a^2}\sqrt{1-b^2} + a\cdot b = ab + \sqrt{(1-a^2)(1-b^2)}$$

$$\Rightarrow \cos(\alpha-\beta) = ab + \sqrt{1-a^2-b^2+a^2b^2}$$

Now, L.H.S. = $\cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta)$

= $2\cos^2(\alpha - \beta) - 1 - 4ab\cos(\alpha - \beta)$

$$= 2\left[ab+\sqrt{1-a^2-b^2+a^2b^2}\right]^2 - 1 - 4ab\left[ab+\sqrt{1-a^2-b^2+a^2b^2}\right]$$

$$= 2\left[(ab)^2+1-a^2-b^2+a^2b^2+2ab\sqrt{1-a^2-b^2+a^2b^2}\right] - 1 - 4a^2b^2 - 4ab\sqrt{1-a^2-b^2+a^2b^2}$$

$$= 2a^2b^2+2-2a^2-2b^2+2a^2b^2+4ab\sqrt{1-a^2-b^2+a^2b^2} - 1 - 4a^2b^2 - 4ab\sqrt{1-a^2-b^2+a^2b^2}$$

= $1 - 2a^2 - 2b^2$ = R.H.S.

35.

Variable (x_i)	Frequency (f_i)	f_ix_i	$x_i - \bar{x}$ $= x_i - 9$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
2	4	8	–7	49	196
4	4	16	–5	25	100
6	5	30	–3	9	45
8	15	120	–1	1	15
10	8	80	1	1	8
12	5	60	3	9	45
14	4	56	5	25	100
16	5	80	7	49	245
	$N = \Sigma f_i$ $= 50$	Σf_ix_i $= 450$			$\Sigma f_i(x_i - \bar{x})^2$ $= 754$

Here, $N = 50$, $\Sigma f_ix_i = 450$

$$\therefore\ \bar{x} = \frac{1}{N}(\Sigma f_ix_i) = \frac{450}{50} = 9$$

We have, $\Sigma f_i(x_i - \bar{x})^2 = 754$

$$\therefore\ \text{Variance} = \frac{1}{N}[\Sigma f_i(x_i-\bar{x})^2] = \frac{754}{50} = 15.08$$

$$\text{S.D.} = \sqrt{\text{Variance}} = \sqrt{15.08} = 3.88$$

OR

We have given, $n = 100$,

Incorrect mean $= \bar{x} = 40$, Incorrect S.D. $= \sigma = 10$

$$\therefore\ \frac{\Sigma x_i}{n} = 40 \Rightarrow \frac{\Sigma x_i}{100} = 40$$

$\Rightarrow$ Incorrect $\Sigma x_i = 4000$

Hence, correct $\Sigma x_i = 4000 - 30 - 70 + 3 + 27$
$= 4030 - 100 = 3930$

$$\therefore\ \text{Correct mean} = \frac{3930}{100} = 39.3$$

Also, $\sigma^2 = \frac{\Sigma x_i^2}{n} - (40)^2 \Rightarrow 100 = \frac{\Sigma x_i^2}{100} - 1600$

$\Rightarrow$ Incorrect $\Sigma x_i^2 = 170000$

Mathematics bulletin

Using math to better treat cancer

Researchers at the University of Waterloo have identified a new method for scheduling radiation therapy that could be as much as 22 percent more effective at killing cancer cells than current standard radiation treatment regimens.

While many mathematical studies have examined how to optimize the scheduling of radiation treatment for maximum effectiveness against cancer, most of these studies assume "intratumoral homogeneity", which means that all of the cancer cells are the same. In recent years, scientists have realized that tumours are made up of many different kinds of cells. Most importantly, they include cancer stem cells, which are more resistant to radiation than other kinds of cells.

"The problem with any calculation involving cancer is that it's super hard to get exact values because things vary from cancer type to cancer type, patient to patient, even within the tumour," said **Cameron Meaney**, a Ph.D. candidate in Applied Mathematics at Waterloo and the lead researcher on the study. This new algorithm can generalize the differing radiation resistances of stem cells and non-stem cells, allowing doctors to predict how a tumour will respond to treatment before gathering exact data on an individual's cancer.

"The results of the algorithm are important because they shed light on the idea that heterogeneity in tumours matters for planning treatment," Meaney said.

The next step the researchers hope to see is an application of their algorithm to clinical studies.

Math is helping researchers understand how cancers grow and how cancer cells function. This knowledge equips scientists with tools to predict treatment response and in turn, improve those treatments overall. Math allows them to dive deeper into cancer cells. Investigators are applying complex math and geometric network algorithms to look for patterns in data and figure out which specific genes or gene networks might be involved in a drug response.

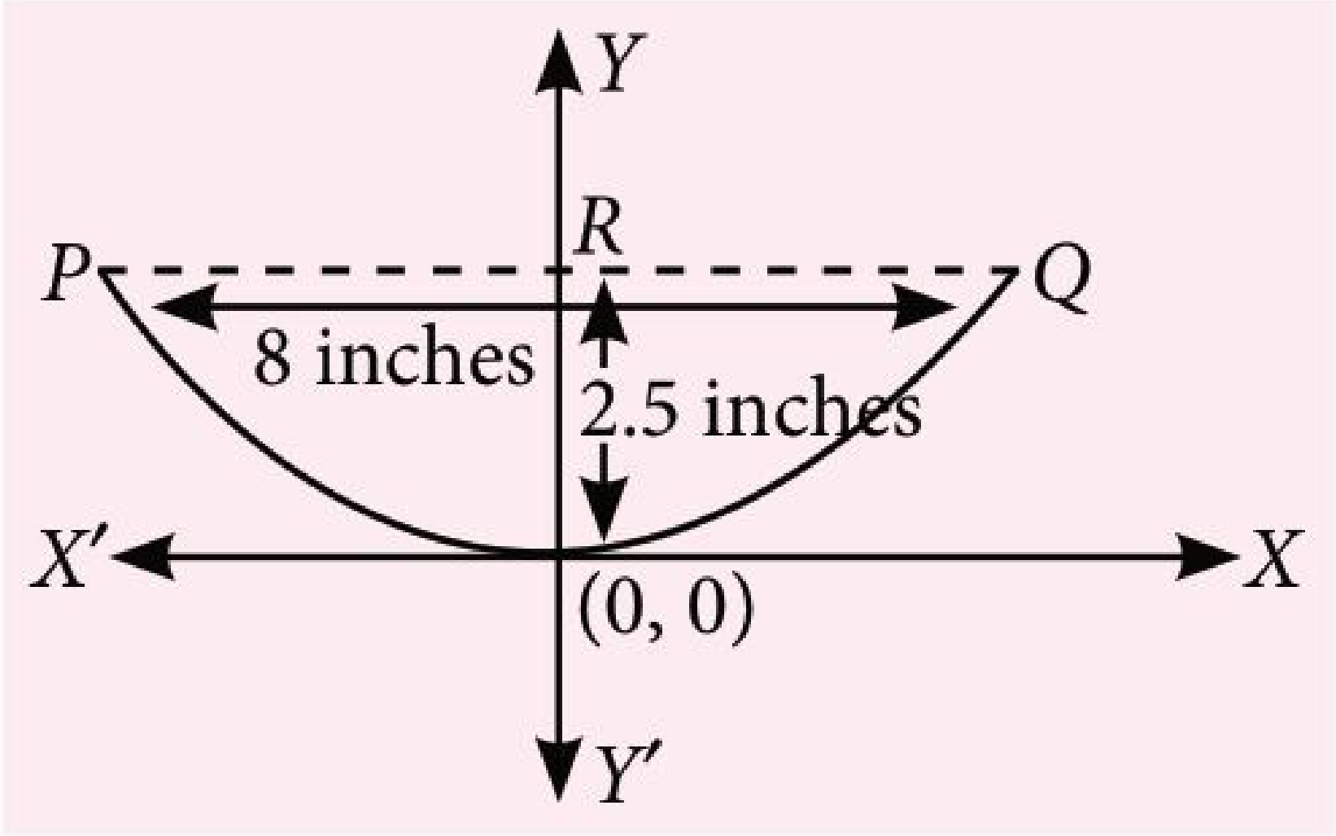
Now, correct Σx_i^2

$= 170000 - (30)^2 - (70)^2 + 3^2 + (27)^2 = 164938$

$$\therefore \quad \text{Correct S.D.} = \sqrt{\frac{164938}{100} - (39.3)^2}$$

$$= \sqrt{1649.38 - 1544.49} = \sqrt{104.89} = 10.24$$

36.



(i) : Since $PQ = 8$ inches $QR = 4$ inches
So, the coordinates of point $Q \equiv (4, 2.5)$
Let the equation of parabola is $x^2 = 4ay$.
Since, the point (4, 2.5) lies on parabola
So, $4^2 = 4a\,(2.5)$
$16 = 10a$

$$a = \frac{16}{10} = \frac{8}{5} = 1.6$$

$\therefore$ The equation of parabola is $x^2 = 6.4\,y$.

(ii) The length of latus rectum is $4a$.
$\therefore$ $4a = 4 \times 1.6 = 6.4$

(iii) Area of triangle $= \frac{1}{2} \times$ length of the latus rectum $\times$ distance between focus to vertex

$$= \frac{1}{2} \times 6.4 \times 1.6 \qquad (\because \text{ focus} \equiv (a, 0))$$

$= 5.12$ sq. units

OR

Let P and Q be points on the parabola $x^2 = 6.4\,y$ and OP, OQ be the lines joining the vertex O to the points P and Q whose ordinate are 10. Thus.
$x^2 = 6.4\,(10) = 64 \Rightarrow x = \pm 8$
Therefore, the coordinate of the points P and Q are (8, 10) and (–8, 10) respectively. Hence the lives are

$$y = \pm \frac{10}{8} x \Rightarrow 4y = \pm 5x$$

37. (i) : If n geometric mean $g_1, g_2, \ldots, g_n$ are to be inserted between two positive real numbers x and y, then $x, g_1, g_2 \ldots g_n, y$ are in G.P.
Then, $g_1 = xr, g_2 = xr^2 \ldots\ldots g_n = xr^n$

$$\text{So, } y = xr^{n+1} \Rightarrow \quad r = \left(\frac{y}{x}\right)^{\frac{1}{(n+1)}}$$

$$\text{Now, } n^{\text{th}} \text{ geometric mean } g_n = x\left(\frac{y}{x}\right)^{\frac{n}{n+1}}$$

(ii) : We have $x, g_1, g_2, g_3 \ldots g_n, \frac{1}{x}$

$$g_1 \cdot g_2 \cdot g_3 \cdot \ldots g_n = \left(x \times \frac{1}{x}\right)^{n/2} = 1$$

(iii) : Given that Ram is 12 years older than Ankit and arithmetic mean of their ages exceed the geometric mean of their ages by 2

$\therefore \quad x - y = 12$ ...(1)
and A.M. – G.M = 2 ...(2)

$$\Rightarrow \frac{x+y}{2} - \sqrt{xy} = 2$$

$$\Rightarrow x + y - 2\sqrt{xy} = 4 \Rightarrow (\sqrt{x} - \sqrt{y})^2 = 4$$

$$\Rightarrow \sqrt{x} - \sqrt{y} = 2 \qquad \text{...(3)}$$

Now, $x - y = 12$, $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y}) = 12$

$$\Rightarrow (\sqrt{x} + \sqrt{y}) \times (2) = 12 \qquad \text{[From (3)]}$$

$$\Rightarrow \sqrt{x} + \sqrt{y} = 6 \qquad \text{...(4)}$$

By (3) and (4), $x = 16$ and $y = 4$.

OR

Let y and x be two geometric mean such that 1, y, x, 64 , then
$y^2 = x$ ...(1)
and $x^2 = 64y$ ...(2)
Put $x = y^2$ in(2), we get $y^4 = 64y$
or $y^3 = 64 \Rightarrow y = 4$
Now, by (1), $x = (4)^2 \Rightarrow x = 16$
$\therefore$ Geometric means are 4, 16.

38. (i) Here, X = {O, B, J, E, C, T, I, V}
and Y = {S, U, B, J, E, C, T, I, V}
$\therefore$ $X \cap Y$ = {B, J, E, C, T, I, V}
$X \cup Y$ = {O, B, J, E, C ,T, I, V, S, U}

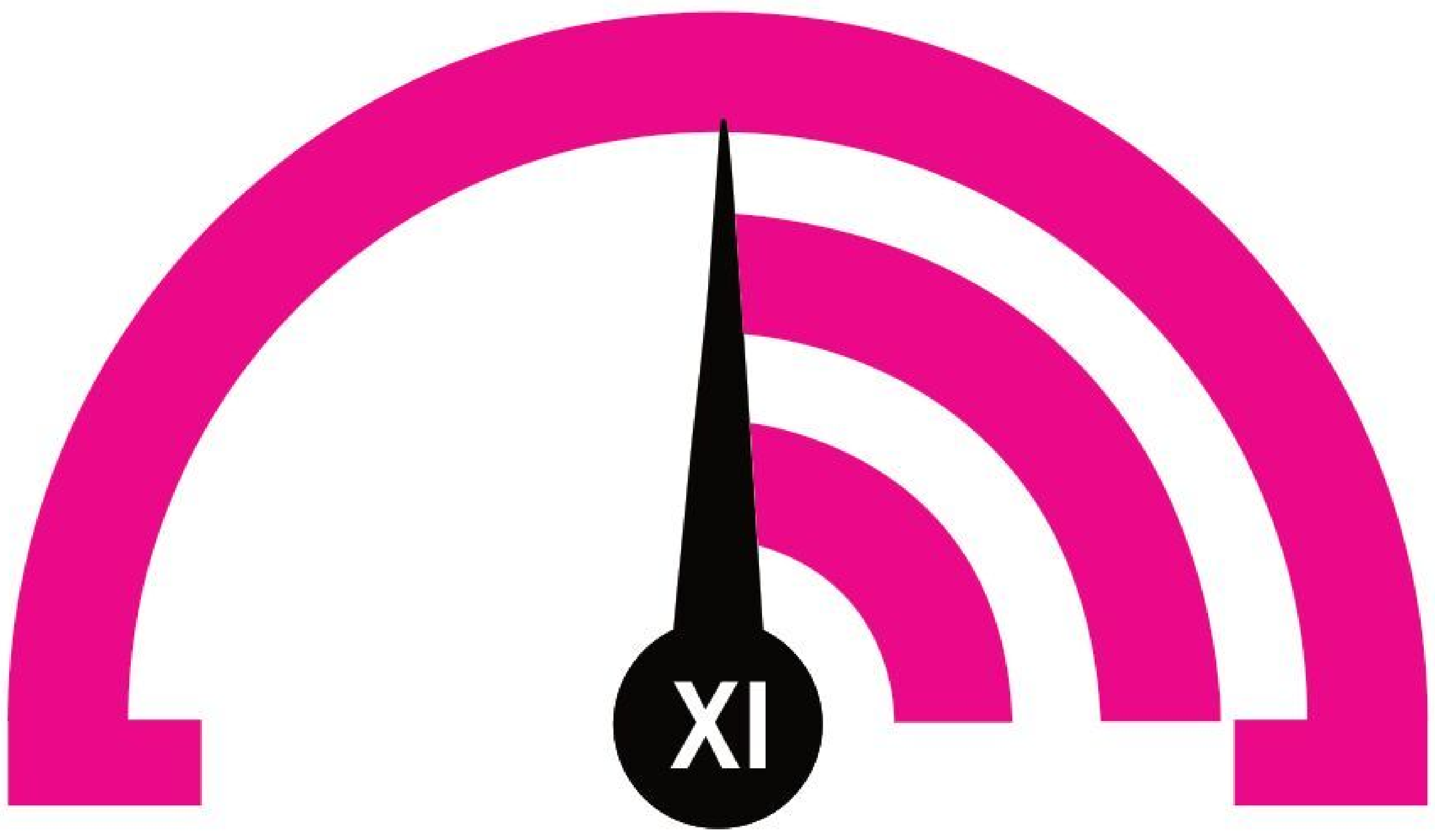
(ii) Since, $n(X \cup Y) = 10$, $n(X \cap Y) = 7$,
$n(X) = 8$, $n(Y) = 9$
R.H.S
$n(X) + n(Y) - n(X \cap Y) = 8 + 9 - 7 = 10 = n(X \cup Y)$
Hence proved.

◆◆

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1.	(d)	2.	(c)	3.	(c)	4.	(b)	5.	(a)
6.	(c)	7.	(b)	8.	(d)	9.	(a,b,c,d)	10.	(a,b,d)
11.	(a,c)	12.	(a,c)	13.	(d)	14.	(c)	15.	(b)
16.	(d)	17.	(9)	18.	(3)	19.	(2)	20.	(6)

MONTHLY TEST DRIVE



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Series 6 : Sequences and Series

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. Given that n A.M.'s are inserted between two sets of numbers a, $2b$ and $2a$, b where $a, b \in R$. Suppose further that m^{th} mean between these sets of numbers is same, then the ratio $a : b$ is equal to

(a) $(n - m + 1) : m$ (b) $(n - m + 1) : n$

(c) $n : (n - m + 1)$ (d) $m : (n - m + 1)$

2. If the sum of first 3 terms of an A.P. is 27 and the sum of their squares is 293. Then the absolute value of S_{15} is

(a) 315 (b) 585

(c) Both (a) and (b) (d) None of these

3. In an A.P., t_n denotes n^{th} term and S_n denotes the sum of first n terms. If $t_7 = \frac{1}{9}$ and $t_9 = \frac{1}{7}$, then value of S_{63} is

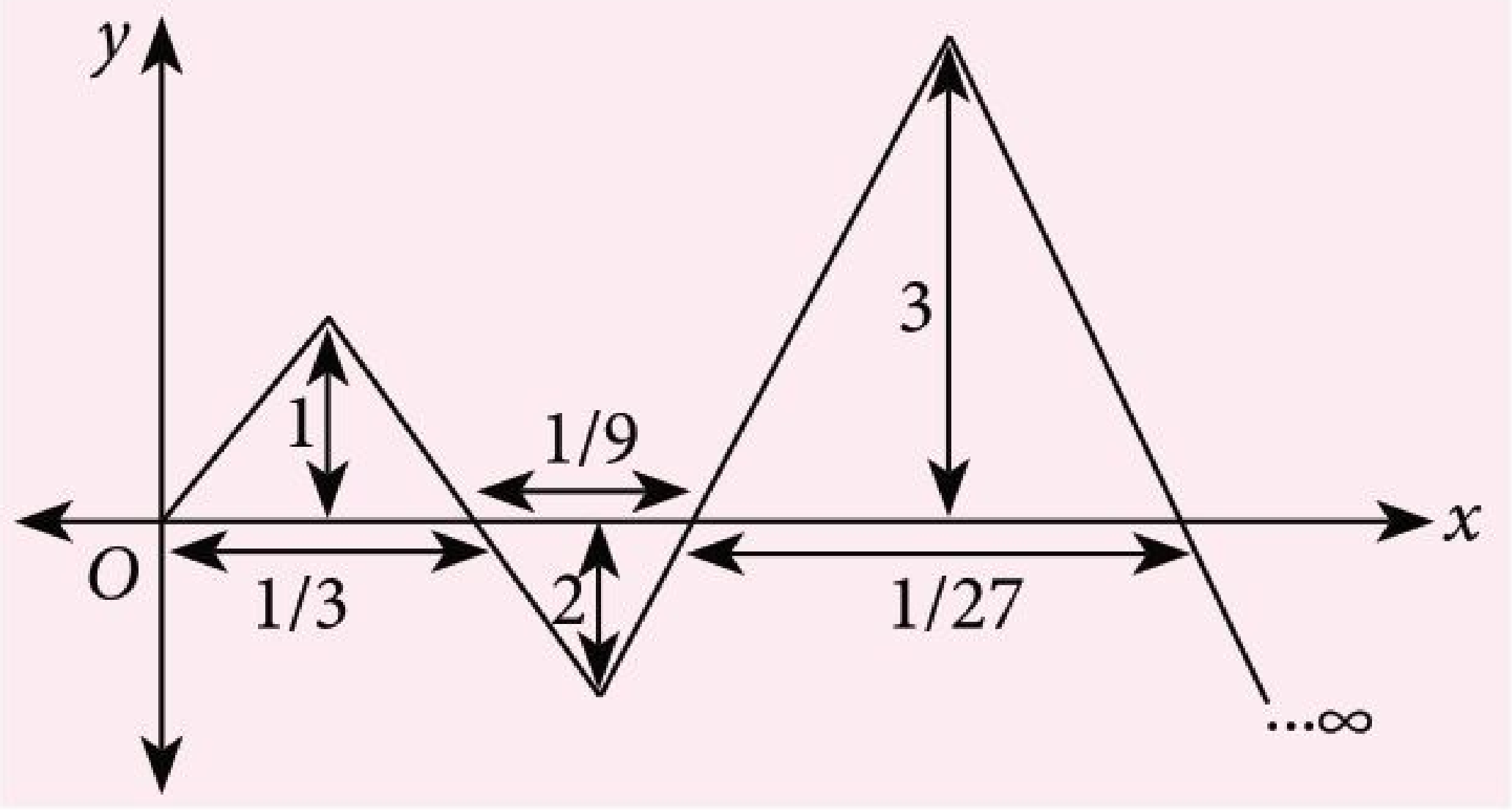
(a) 30 (b) 31 (c) 32 (d) 34

4. The first, second and middle terms of an A.P. are a, b, c respectively. Their sum is

(a) $\dfrac{2(c-a)}{b-a}$ (b) $\dfrac{2c(c-a)}{b-a} + c$

(c) $\dfrac{2c(b-a)}{c-a}$ (d) $\dfrac{2b(c-a)}{b-a}$

5. Infinite number of triangles are formed as shown in figure. If total area of these triangles is A, then $8A$ is equal to



(a) 3 (b) 4 (c) 1 (d) 2

6. If x, y, z are the three geometric means between 6, 54, then $z =$

(a) $9\sqrt{3}$ (b) 18 (c) $18\sqrt{3}$ (d) 27

One or More Than One Option(s) Correct Type

7. The sum to infinity of the series $1 + \frac{3}{4} + \frac{6}{4^2} + \frac{10}{4^3} + \frac{15}{4^4} + \ldots$ is

(a) $\frac{16}{9}$ (b) $\frac{64}{27}$ (c) $\frac{4}{3}$ (d) $\frac{32}{27}$

8. If x, y, z are three positive numbers, then the minimum value of $\frac{y+z}{x} + \frac{z+x}{y} + \frac{x+y}{z}$ is

(a) 1 (b) 2 (c) 3 (d) 6

9. If $a > 0$, $b > 0$, $c > 0$ and $2a + b + 3c = 1$, then

(a) $a^4b^2c^2$ is greatest when $a = \frac{1}{4}$

(b) $a^4b^2c^2$ is greatest when $b = \frac{1}{4}$

UNSCRAMBLED WORDS

DECEMBER 2022

1-c- VECTOR
2-e- TAUTOLOGY
3-a- INTO
4-h- ANGLE
5-i- INDUCTION
6-d- OUTCOME
7-f- INEQUALITY
8-j- RANGE
9-b- QUANTIFIERS
10-g- DOMAIN

(c) $a^4b^2c^2$ is greatest when $c = \frac{1}{12}$

(d) greatest value of $a^4b^2c^2$ is $\frac{1}{9\cdot 4^8}$

10. The difference between the sum of the cubes of first n natural numbers and the sum of first n natural numbers is always divisible by

(a) 2 (b) 3 (c) 4 (d) 6

11. The sum of n terms of the series

$$\frac{1}{1\cdot 2\cdot 3\cdot 4}+\frac{1}{2\cdot 3\cdot 4\cdot 5}+\frac{1}{3\cdot 4\cdot 5\cdot 6}+\ldots \text{ is}$$

(a) $\frac{n(n^2+6n+11)}{18(n+1)(n+2)(n+3)}$

(b) $\frac{n^3+6}{18(n+1)(n+2)(n+3)}$

(c) $\frac{1}{18}-\frac{1}{3(n+1)(n+2)(n+3)}$

(d) $\frac{1}{6}-\frac{1}{2(n+1)(n+2)(n+3)}$

12. If sum of n terms of an A.P. is given by $S_n = a + bn + cn^2$ where a, b, c are independent of n, then

(a) $a = 0$

(b) common difference of A.P. must be $2b$

(c) common difference of A.P. must be $2c$

(d) All of these

13. The sum of the series $1 + 3x + 6x^2 + 10x^3 + \ldots \infty$ will be $(|x|<1)$

(a) $\frac{1}{(1-x)^2}$ (b) $\frac{1}{1-x}$

(c) $\frac{1}{(1+x)^2}$ (d) $\frac{1}{(1-x)^3}$

Comprehension Type

Paragraph for Q. No. 14 and 15

Let $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots$ upto $\infty = A$, then

14. $\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\frac{1}{8^2}+\ldots$ upto ∞ is equal to

(a) $\frac{1}{2}A$ (b) A

(c) $\frac{1}{4}A$ (d) None of these

15. $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$ upto ∞ is equal to

(a) $\frac{1}{2}A$ (b) $\frac{3}{4}A$

(c) $\frac{1}{4}A$ (d) None of these

Matrix Match Type

16. Match the following columns :

	Column-I	Column-II	
(P)	$100\left(\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{99\cdot 100}\right)=$	(1)	7
(Q)	If x is the A.M. between two real numbers a and b, $y = a^{2/3} \cdot b^{1/3}$ and $z = a^{1/3}b^{2/3}$, then $\frac{y^3+z^3}{xyz} =$	(2)	9
(R)	If 198 A.M.'s are inserted between 1/4 and 3/4, then the sum of these A.M.'s is	(3)	99
(S)	If n is a positive integer such that n, $\frac{n(n-1)}{2}$ and $\frac{n(n-1)(n-2)}{6}$ are in A.P., then the value of n is	(4)	100
		(5)	2

	P	Q	R	S
(a)	1	2	5	3
(b)	3	5	2	4
(c)	3	5	3	1
(d)	3	5	3	5

Numerical Answer Type

17. For any odd integer $n \geq 1$, if the value of the sum $n^3 - (n-1)^3 + (n-2)^3 + \ldots + (-1)^{n-1}(1)^3$ equals 425, then find the value of n.

18. If $(a+b+c+d)^4 \leq 256\,abcd$ $(a, b, c, d > 0)$, then find the value of $\frac{a+b+c}{d}$.

19. If the ratio $\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\right)$ to $\left(1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\right)$ is $k : 1$, then find k.

20. For $a, b > 0$, let $5a - b$, $2a + b$, $a + 2b$ be in A.P. and $(b+1)^2$, $ab + 1$, $(a-1)^2$ are in G.P., then compute $(a^{-1} + b^{-1})$.

⁂

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B.Sc. IN DATA SCIENCE

B.Sc Data Science or Bachelor of Science in Data Science is a 3-year Under-Graduate degree course that is further split into six semesters which comes under the domains of Computer Science and Artificial Intelligence. B.Sc Data Science degree offers an extensive blend of Computer Science, Mathematics and Statistics. B.Sc Data Science degree grants a true understanding of data programming, Statistical Analysis and the usage and interpretation of data.

ELIGIBILITY

Candidates must have completed a recognized school board's intermediate (12^{th}) with a Science stream. Candidates must have completed their 12^{th} grade with one of the core disciplines being Physics, Chemistry and Mathematics.

JOB PROSPECTS

B.Sc Data Science Employment Areas are Financial sector, IT firms, Insurance sector, Large Businesses/ Telecommunication Business ,Manufacturing firms and Retail.

TOP COLLEGES LIST

Given below are some of the top institutes in India that offer the course.

- **IIT Madras, Chennai.**
- Loyola Academy Degree and PG College, Secunderabad.
- Sri Ramachandra Institute of Higher Education and Research, Chennai.
- NMIMS Mumbai, Mumbai.
- PSGCT, Coimbatore.
- Sathyabama Institute of Science and Technology, Chennai.

COLLEGE INFO : IIT MADRAS

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भारतीय प्रौद्योगिकी संस्थान मद्रास
Indian Institute of Technology Madras

IIT Madras is a public technical university located in Chennai, Tamil Nadu, India. It is recognized as an Institute of National Importance and has been consistently rated as one of India's most prestigious universities founded in 1959 with technical and financial assistance from the former government of West Germany it was the third IIT established by the Government of India. IIT Madras is ranked the top engineering institute in India by the Ministry of **Education's National Institutional Ranking Framework** since its inception in 2016.

SELECTION CRITERIA

Admission to IIT Madras online degree programs is based on a qualifier exam. Those who have qualified for JEE Advanced 2022 exam can apply for the BSc in Data Science program; provided they complete the first 4 weeks of the foundational course and pass in-person quizzes and exams.

IIT Madra introduced Data Science degree program in 2022 under its online degree program, candidate are expected to complete its all three levels i.e., Foundation Course, Diploma level and Degree level after which only they will be awarded with the degree.

APPLICATION DEADLINES

The registrations are open for January 2023 batch. Candidates who are interested can apply for the IIT Madras BS degree in Data Science and Applications till January 15, 2023. The application fee is INR 3,000, for general/ OBC candidates, INR 1500 (SC/ST/PWD) and INR 750/SC/ST, who are also PWD).

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CLASS-XII

Practice questions for CBSE Exams as per the latest pattern and reduced syllabus by CBSE for the academic session 2022-23.

Practice Paper 2022-23

Time Allowed : 3 hours
Maximum Marks : 80

General Instructions :

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.*

SECTION - A (MULTIPLE CHOICE QUESTIONS)

Each question carries 1 mark.

1. If $A=\begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is a singular matrix, then the value of k is

(a) 1 (b) –1 (c) ±4 (d) 0

2. If $y=\log(\cos e^x)$, then $\frac{dy}{dx}$ is

(a) $\cos e^{x-1}$ (b) $e^{-x}\cos e^x$
(c) $e^x \sin e^x$ (d) $-e^x \tan e^x$

3. The angle between the lines $\frac{x-5}{1}=\frac{y-7}{2}=\frac{z-11}{3}$ and $\frac{x}{2}=\frac{y+3}{2}=\frac{z-4}{-2}$ is

(a) 90° (b) 0°
(c) 180° (d) None of these

4. If the matrix $A=\begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix}$ is singular, then find x.

(a) 1 (b) –2 (c) –1 (d) 0

5. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, then find the probability of getting exactly one red ball.

(a) $\frac{13}{56}$ (b) $\frac{15}{56}$ (c) $\frac{17}{56}$ (d) $\frac{11}{56}$

6. If $f: R \to R$ is defined by

$$f(x)=\begin{cases} \dfrac{2\sin x-\sin 2x}{2x\cos x}, & \text{if } x\neq 0 \\ a, & \text{if } x=0 \end{cases},$$

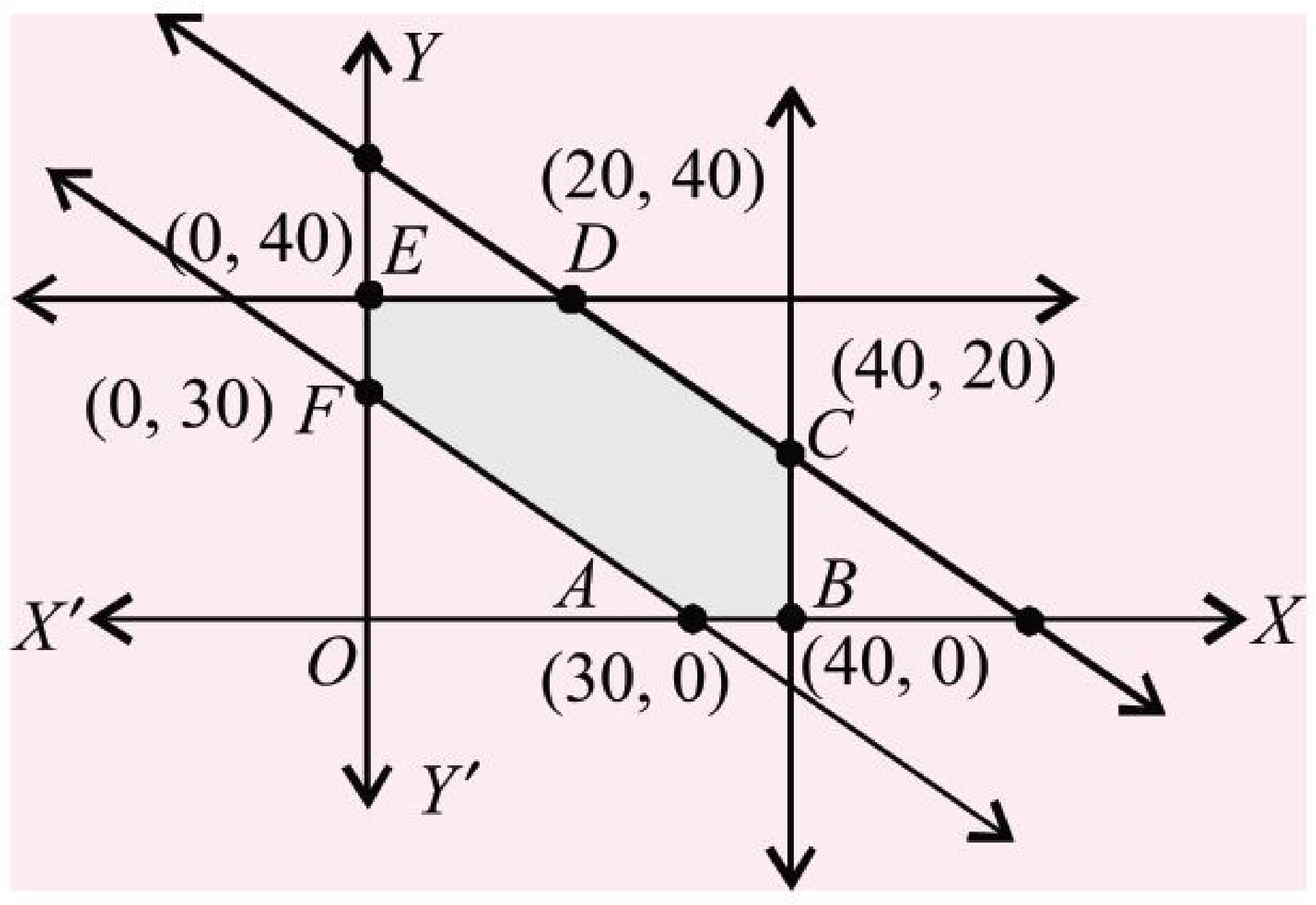
then the value of a so that f is continuous at $x = 0$ is

(a) 2 (b) 1 (c) –1 (d) 0

7. The position vectors of points A and B are $\hat{i}+3\hat{j}-7\hat{k}$ and $5\hat{i}-2\hat{j}+4\hat{k}$ respectively, then find the direction cosine of $\overrightarrow{AB}$ along Y-axis.

(a) $\frac{-3}{\sqrt{162}}$ (b) $\frac{-5}{\sqrt{162}}$ (c) $\frac{7}{\sqrt{162}}$ (d) $\frac{1}{\sqrt{162}}$

8. Feasible region for a LPP is shown in the following figure. Minimum of $Z = 3x + 4y + 270$ occurs at the point



(a) (20, 40) (b) (30, 0) (c) (0, 40) (d) (0, 30)

9. If $A(\text{adj } A) = 5I$, where I is identity matrix of order 3, then $|\text{adj } A| =$

(a) 125 (b) 25 (c) 10 (d) 5

10. If $\vec{a}$ and $\vec{b}$ are unit vectors enclosing an angle θ and $|\vec{a}+\vec{b}|<1$, then

(a) $\theta=\frac{\pi}{2}$ (b) $\theta<\frac{\pi}{3}$

(c) $\pi\geq\theta>\frac{2\pi}{3}$ (d) $\frac{\pi}{3}<\theta<\frac{2\pi}{3}$

11. The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0 \text{ is}$$

(a) 3 (b) 2

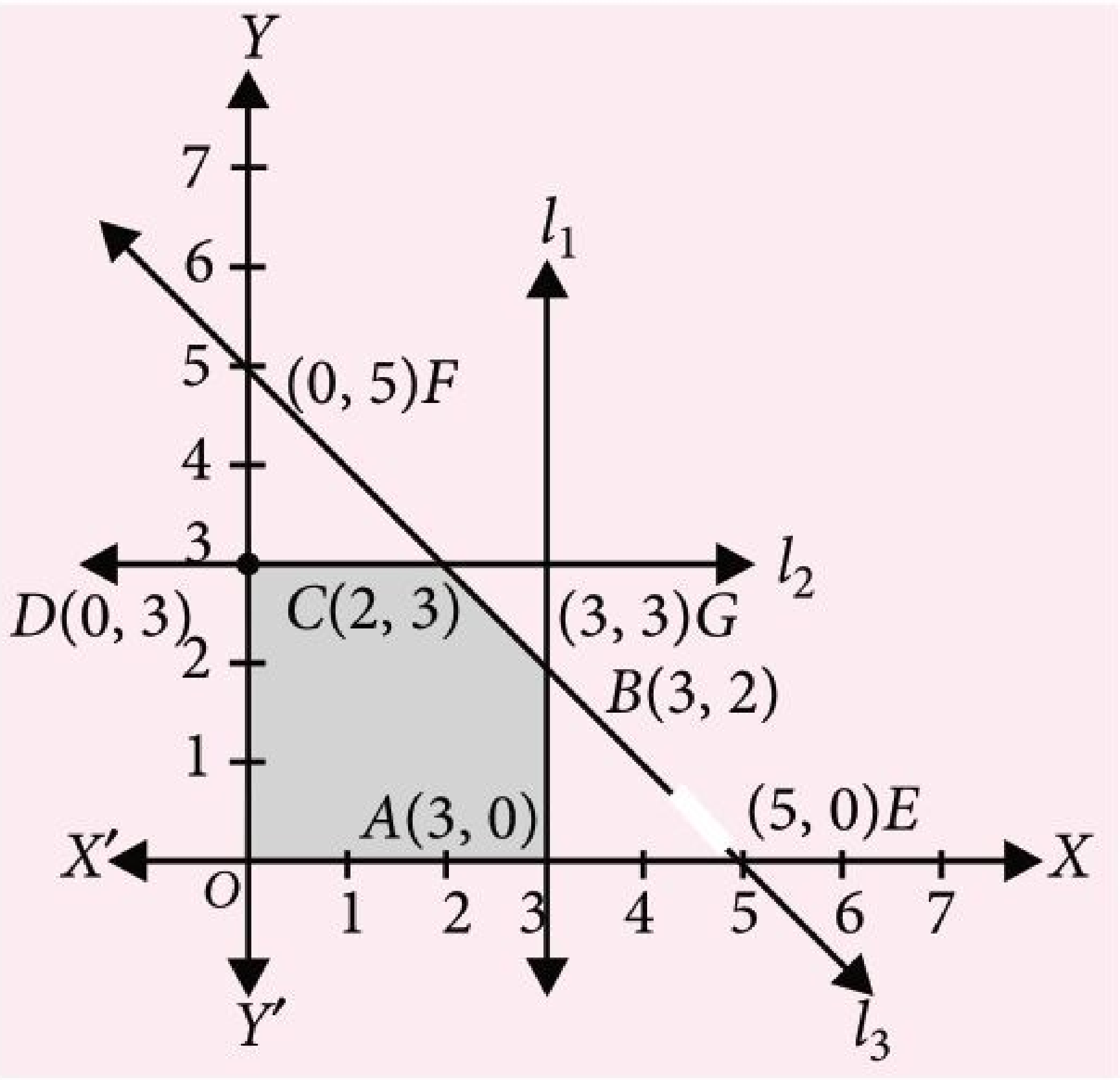
(c) 1 (d) Not defined

12. If $A=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$, then A^n is

(a) $\begin{bmatrix}1 & 2^n-2\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & n^2\\ 0 & 1\end{bmatrix}$

(c) $\begin{bmatrix}1 & 2n\\ 0 & 1\end{bmatrix}$ (d) $\begin{bmatrix}1 & n^2\\ 1 & 1\end{bmatrix}$

13. The corner points of the feasible region determined by the system of linear inequalities are



(a) (0, 0), (–3, 0), (3, 2), (2, 3)
(b) (3, 0), (3, 2), (2, 3), (0, –3)
(c) (0, 0), (3, 0), (3, 2), (2, 3), (0, 3)
(d) None of these

14. Find the value of $(\vec{a}\cdot\hat{i})^2+(\vec{a}\cdot\hat{j})^2+(\vec{a}\cdot\hat{k})^2$.

(a) $|\vec{a}|$ (b) $|\vec{a}|^2$ (c) 0 (d) 1

15. Find the integrating factor of the differential equation $\frac{dy}{dx}+\frac{y}{\sqrt{1-x^2}}=x$.

(a) $e^{\sin^{-1}x}$ (b) $e^{\cos^{-1}x}$

(c) $-e^{\sin^{-1}x}$ (d) $-e^{\cos^{-1}x}$

16. If $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=3\hat{i}+2\hat{j}-\hat{k}$, then find $(\vec{a}+3\vec{b})\cdot(2\vec{a}-\vec{b})$.

(a) –10 (b) –15 (c) 15 (d) 10

17. The length of the perpendicular drawn from the point (4, –7, 3) on the y-axis is

(a) 3 units (b) 4 units
(c) 5 units (d) 7 units

18. The number of all the possible matrices of order 3 × 3 with each entry 2 or 0 is

(a) 9 (b) 27 (c) 81 (d) 512

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion (A) :** The relation R in a set A = {1, 2, 3, 4} defined by $R = \{(x, y) : 3x - y = 0\}$ have the Domain = {1, 2, 3, 4} and Range = { 3, 6, 9, 12}.
Reason (R) : Domain & Range of the relation (R) is respectively the set of all second & first entries of the distinct ordered pair of the relation.

20. **Assertion (A) :** If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=\frac{-3}{2}$.

Reason (R) : If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors then $|\vec{a}|=1, |\vec{b}|=1, |\vec{c}|=1$ and

$\vec{a}+\vec{b}+\vec{c}=0$, so $|\vec{a}+\vec{b}+\vec{c}|^2=0$.

SECTION - B

This section comprises of very short answer type questions (VSA) of 2 marks each.

21. Find the point(s), at which the function f given by

$$f(x)=\begin{cases}\dfrac{x}{|x|}, x<0\\ -1, x\geq 0\end{cases}$$ is continuous.

22. Find the shortest distance between the lines
$\vec{r}=(4\hat{i}-\hat{j})+\lambda(\hat{i}+2\hat{j}-3\hat{k})$ and
$\vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(2\hat{i}+4\hat{j}-5\hat{k})$.

OR

Find the value of λ, so that the lines $\dfrac{1-x}{3}=\dfrac{7y-14}{\lambda}=\dfrac{z-3}{2}$ and $\dfrac{7-7x}{3\lambda}=\dfrac{y-5}{1}=\dfrac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.

23. If $2f(x^2)+3f\left(\dfrac{1}{x^2}\right)=x^2-1$ $(x\neq 0)$, then find $f(x^2)$.

OR

Prove that $\cos^{-1}\left(\dfrac{12}{13}\right)+\sin^{-1}\left(\dfrac{3}{5}\right)=\sin^{-1}\left(\dfrac{56}{65}\right)$.

24. Find the general solution of the differential equation
$\sec^2 x\cdot\tan y\,dx+\sec^2 y\cdot\tan x\,dy=0$.

25. Evaluate : $\displaystyle\int_0^{\infty}\frac{dx}{(x^2+4)(x^2+9)}$

SECTION - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. If $P(A)=\dfrac{2}{5}$, $P(B)=\dfrac{3}{10}$ and $P(A\cap B)=\dfrac{1}{5}$, then find $P(A'|B')\cdot P(B'|A')$.

OR

Three events A, B and C have probabilities $\dfrac{2}{5},\dfrac{1}{3}$ and $\dfrac{1}{2}$, respectively. Given that $P(A\cap C)=\dfrac{1}{5}$ and $P(B\cap C)=\dfrac{1}{4}$, find the values of $P(C\mid B)$ and $P(A'\cap C')$.

27. Evaluate $\displaystyle\int_1^2\left[\frac{2x-1}{2x^2\cdot e^{-2x}}\right]dx$.

28. Solve the differential equation :
$(1+x^2)\dfrac{dy}{dx}+2xy-4x^2=0$, subject to the initial condition $y(0)=0$.

OR

Find the particular solution of the differential equation $\dfrac{dy}{dx}=\dfrac{x(2\log x+1)}{\sin y+y\cos y}$, given that $y=\dfrac{\pi}{2}$, when $x=1$.

29. Evaluate : $\displaystyle\int\frac{x^2}{(x-1)^3(x+1)}dx$

OR

Find $\displaystyle\int\frac{(x^3+x+1)}{(x^2-1)}dx$

30. Minimize $Z=x+2y$, subject to $x+2y\geq 50$, $2x-y\leq 0$, $2x+y\leq 100$, $x, y\geq 0$.

31. Evaluate : $\displaystyle\int\left[\frac{\sin x}{3+4\cos^2 x}+\frac{\cos x}{3+4\sin^2 x}\right]dx$

SECTION - D

This section comprises of long answer type questions (LA) of 5 marks each.

32. Using integration, find the area of the region bounded by the curves : $y=|x+1|+1$, $x=-3$, $x=3$ and $y=0$.

33. Show that the function $f:(-\infty, 0)\to(-1, 0)$ defined by $f(x)=\dfrac{x}{1+|x|}$, $x\in(-\infty, 0)$ is one-one and onto.

OR

Let $A=R-\{2\}$ and $B=R-\{1\}$. If $f: A\to B$ is a mapping defined by $f(x)=\dfrac{x-1}{x-2}$, then show that f is bijective.

34. If $A=\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$, find A^{-1}.

Hence solve the system of equations;
$x-2y=10$
$2x-y-z=8$
$-2y+z=7$

35. Find the distance between the lines l_1 and l_2 given by

$$l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}),$$

$$l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k}).$$

OR

Using vectors, show that the points $A(-2, 3, 5)$, $B(7, 0, -1)$, $C(-3, -2, -5)$ and $D(3, 4, 7)$ are such that AB and CD intersects at the point $P(1, 2, 3)$.

SECTION - E

This section comprises of 3 case-study/passage-based questions of 4 marks each with sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36. Case-Study 1 : Read the following passage and answer the questions given below.

Suman was doing a project on a school survey, on the average number of hours spent on study by students selected at random. At the end of survey, Suman prepared the following report related to the data.

Let X denotes the average number of hours spent on study by students. The probability that X can take the values x, has the following form, where k is some unknown constant.

$$P(X = x) = \begin{cases} 0.2, & \text{if } x = 0 \\ kx, & \text{if } x = 1 \text{ or } 2 \\ k(6 - x), & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of k.

(ii) What is the probability that the average study time of students is not more than 1 hour?

(iii) What is the probability that the average study time of students is at least 3 hours?

OR

What is the probability that the average study time of students is atmost 2 hours?

37. Case-Study 2 : Read the following passage and answer the questions given below.

In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F, where the room temperature is 50°F. Also, it is given the body temperature at the time of death was normal, *i.e.*, 98.6°F.

Let T be the temperature of the body at any time t and initial time is taken to be 8 p.m.

POLICE LINE DO NOT CROSS • POLICE LINE DO NOT C
E LINE DO
POLICE LINE DO NOT CROSS • POLICE LINE DO NO
POLICE LINE

(i) By Newton's law of cooling, $\frac{dT}{dt} = kf(T)$, where k is constant, then what is $f(T)$?

(ii) What is the body temperature when $t = 0$ and $t = 2$?

(iii) Find the value of $T - 50$ in terms of k.

OR

Find the value of T at any time t.

38. Case-Study 3 : Read the following passage and answer the questions given below.

A differential equation is said to be in the variable separable form if it is expressible in the form $f(x)\,dx = g(y)\,dy$. The solution of this equation is given by $\int f(x)dx = \int g(y)dy + c$, where c is the constant of integration.

(i) If the solution of the differential equation $\frac{dy}{dx} = \frac{ax + 3}{2y + f}$ represents a circle, then find the value of 'a'.

(ii) If $y' = y + 1$, $y(0) = 1$, then find $y(\ln 2)$.

SOLUTIONS

1. (c) : $\because A$ is singular matrix.

$\therefore \quad |A| = 0$

$$\Rightarrow \begin{vmatrix} k & 8 \\ 4 & 2k \end{vmatrix} = 0 \Rightarrow 2k^2 - 32 = 0 \Rightarrow k^2 = 16 \Rightarrow k = \pm 4$$

2. (d) : We have, $y = \log(\cos e^x)$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx}=\frac{1}{\cos e^x}\cdot(-\sin e^x)\cdot e^x \quad\Rightarrow\quad \frac{dy}{dx}=-e^x\tan e^x$$

3. (a) : The direction ratios of the lines are $a_1 = 1$, $b_1 = 2$, $c_1 = 3$ and $a_2 = 2$, $b_2 = 2$, $c_2 = -2$

$$\therefore\quad \cos\theta=\left|\frac{1\cdot 2+2\cdot 2+3\cdot(-2)}{\sqrt{1+4+9}\sqrt{4+4+4}}\right|=0 \quad\Rightarrow\quad \theta = 90°$$

4. (c) : Given that matrix A is singular $\Rightarrow |A| = 0$

$$\Rightarrow \begin{vmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{vmatrix}=0 \Rightarrow 1(-6-2)+2(-3-x)+3(2-2x)=0$$

$$\Rightarrow\quad -8-6-2x+6-6x=0 \quad\Rightarrow\quad -8x-8=0 \quad\Rightarrow\quad x=-1$$

5. (b) : Required probability $= P\{(RBB), (BRB), (BBR)\}$
$= P(RBB) + P(BRB) + P(BBR)$

$$=\frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}+\frac{3}{8}\times\frac{5}{7}\times\frac{2}{6}+\frac{3}{8}\times\frac{2}{7}\times\frac{5}{6}=3\times\frac{5}{56}=\frac{15}{56}$$

6. (d) : Given, $f(x)=\begin{cases}\dfrac{2\sin x-\sin 2x}{2x\cos x}, & \text{if } x\neq 0\\ a, & \text{if } x=0\end{cases}$

$$\text{Now, } \lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{\dfrac{2\sin x}{x}-\dfrac{2\sin 2x}{2x}}{2\cos x}=\frac{2-2}{2}=0$$

Since, $f(x)$ is continuous at $x = 0$.

$$\therefore\quad f(0)=\lim_{x\to 0} f(x)\Rightarrow a=0$$

7. (b) : Here $\vec{A}=\hat{i}+3\hat{j}-7\hat{k}$, $\vec{B}=5\hat{i}-2\hat{j}+4\hat{k}$

$$\overrightarrow{AB}=\vec{B}-\vec{A}=4\hat{i}-5\hat{j}+11\hat{k}$$

Direction cosine along Y-axis $=\dfrac{-5}{\sqrt{16+25+121}}=\dfrac{-5}{\sqrt{162}}$

8. (b) : We have,

Corner points	Value of $Z = 3x + 4y + 270$
$A(30, 0)$	360 (Minimum)
$B(40, 0)$	390
$C(40, 20)$	470
$D(20, 40)$	490
$E(0, 40)$	430
$F(0, 30)$	390

From the table, the minimum value of Z is 360 at the point $A(30, 0)$.

9. (b) : $A(\text{adj } A) = 5I$

We know that $A(\text{adj } A) = |A|\, I = 5I$

∴ $|A| = 5$, I is an identity matrix of order 3

Also, we know that, if A is non-singular matrix, then $|\text{adj } A| = |A|^{n-1}$, where n is the order of matrix A.

∴ $|\text{adj } A| = (5)^{3-1} = 5^2 = 25$

10. (c) : $|\vec{a}+\vec{b}|<1 \Rightarrow |\vec{a}+\vec{b}|^2<1$

$$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}<1 \Rightarrow 1+1+2\vec{a}\cdot\vec{b}<1$$

$$\Rightarrow \vec{a}\cdot\vec{b}<-\frac{1}{2} \Rightarrow |\vec{a}|\,|\vec{b}|\cos\theta<-\frac{1}{2}$$

$$\Rightarrow 1\times 1\times\cos\theta<-\frac{1}{2} \Rightarrow \cos\theta<-\frac{1}{2}$$

$$\Rightarrow -1\le\cos\theta<-\frac{1}{2} \Rightarrow \pi\ge\theta>\frac{2\pi}{3}$$

11. (d) : Since, the given differential equation is not a polynomial in derivative of y.

Therefore, its degree is not defined.

12. (c) : $A^2=A\cdot A=\begin{bmatrix}1&2\\0&1\end{bmatrix}\begin{bmatrix}1&2\\0&1\end{bmatrix}=\begin{bmatrix}1&4\\0&1\end{bmatrix}=\begin{bmatrix}1&2\times 2\\0&1\end{bmatrix}$

$$A^3=A^2\cdot A=\begin{bmatrix}1&4\\0&1\end{bmatrix}\begin{bmatrix}1&2\\0&1\end{bmatrix}=\begin{bmatrix}1&6\\0&1\end{bmatrix}=\begin{bmatrix}1&3\times 2\\0&1\end{bmatrix}$$

Similarly, $A^4=A^3\cdot A=\begin{bmatrix}1&8\\0&1\end{bmatrix}=\begin{bmatrix}1&4\times 2\\0&1\end{bmatrix}$

$$\Rightarrow\quad\Rightarrow\quad A^n=A^{n-1}A=\begin{bmatrix}1&n\times 2\\0&1\end{bmatrix}\ \therefore\ A^n=\begin{bmatrix}1&2n\\0&1\end{bmatrix}$$

13. (c) : The corner points of the feasible region are $O(0, 0)$, $A(3, 0)$, $B(3, 2)$, $C(2, 3)$, $D(0, 3)$.

14. (b) : Let $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k} \Rightarrow (\vec{a}\cdot\hat{i})^2=x^2$

Similarly, $(\vec{a}\cdot\hat{j})^2=y^2$ and $(\vec{a}\cdot\hat{k})^2=z^2$

$$\therefore\quad (\vec{a}\cdot\hat{i})^2+(\vec{a}\cdot\hat{j})^2+(\vec{a}\cdot\hat{k})^2=x^2+y^2+z^2=|\vec{a}|^2$$

15. (a) : The given equation is $\dfrac{dy}{dx}+\dfrac{y}{\sqrt{1-x^2}}=x$,

where $P=\dfrac{1}{\sqrt{1-x^2}}$ and $Q=x$

$$\therefore\ \text{I.F.}=e^{\int P\,dx}=e^{\int\frac{dx}{\sqrt{1-x^2}}}=e^{\sin^{-1}x}$$

16. (b) : Here, $\vec{a}+3\vec{b}=\hat{i}+\hat{j}+2\hat{k}+3(3\hat{i}+2\hat{j}-\hat{k})$
$=10\hat{i}+7\hat{j}-\hat{k}$

and $2\vec{a}-\vec{b}=2(\hat{i}+\hat{j}+2\hat{k})-(3\hat{i}+2\hat{j}-\hat{k})=-\hat{i}+5\hat{k}$

$$\therefore\quad (\vec{a}+3\vec{b})\cdot(2\vec{a}-\vec{b})=(10\hat{i}+7\hat{j}-\hat{k})\cdot(-\hat{i}+5\hat{k})$$
$= 10 \times (-1) + 7 \times 0 + (-1) \times 5 = -15$

17. (c) : Let $P(4, -7, 3)$ be the given point and A be a point on y-axis such that $PA \perp$ to y-axis.

$\therefore \quad A \equiv (0, -7, 0)$

$$\text{Now, } PA = \sqrt{(4-0)^2 + (3-0)^2}$$

$$= \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5 \text{ units}$$

18. (d) : As, there are 9 elements in 3 × 3 order matrix and each element is filled by either 2 or 0 in 2 ways.

$\therefore$ Total number of all the possible matrices of order 3 × 3 with each entry 2 or 0 is 2^9 *i.e.*, 512.

19. (c) : $R = \{(x, y) : y = 3x, x \in A\}$

$\therefore \quad R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$

$\therefore$ Domain of the relation = $\{1, 2, 3, 4\}$

and Range of the relation = $\{3, 6, 9, 12\}$.

A is true but R is false.

20. (a) : Both A and R are true and R is the correct explanation of A.

21. We have, $f(x) = \begin{cases} \dfrac{x}{|x|}, & x < 0 \\ -1 & x \geq 0 \end{cases} \Rightarrow f(x) = \begin{cases} \dfrac{x}{-x} = -1, & x < 0 \\ -1, & x \geq 0 \end{cases}$

$\Rightarrow \quad f(x) = -1 \ \forall \ x \in R$

$\Rightarrow \quad f(x)$ is continuous $\forall \ x \in R$ as it is a constant function.

22. We have, $\vec{a}_1 = 4\hat{i} - \hat{j}, \ \vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k},$

$\vec{b}_1 = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b}_2 = 2\hat{i} + 4\hat{j} - 5\hat{k}$

Now, $\vec{a}_2 - \vec{a}_1 = -3\hat{i} + 0\hat{j} + 2\hat{k}$

$$\text{and } \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = 2\hat{i} - \hat{j} + 0\hat{k}$$

$$\therefore \ (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (-3\hat{i} + 0\hat{j} + 2\hat{k}) \cdot (2\hat{i} - \hat{j} + 0\hat{k}) = -6$$

$$\text{and } |\vec{b}_1 \times \vec{b}_2| = \sqrt{4 + 1 + 0} = \sqrt{5}$$

$$\therefore \text{ Shortest distance} = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right| = \frac{6}{\sqrt{5}} \text{ units.}$$

OR

The given lines are

$$l_1 : \frac{x-1}{-3} = \frac{y-2}{\lambda/7} = \frac{z-3}{2}$$

$$\text{and } l_2 : \frac{x-1}{-3\lambda/7} = \frac{y-5}{1} = \frac{z-6}{-5}$$

Now, $l_1 \perp l_2$ [Given]

$$\therefore \ (-3)\left(-\frac{3\lambda}{7}\right) + \frac{\lambda}{7} - 10 = 0$$

$$\Rightarrow \frac{9\lambda}{7} + \frac{\lambda}{7} - 10 = 0 \ \Rightarrow \ \frac{10\lambda}{7} = 10 \ \Rightarrow \lambda = 7$$

Since for $\lambda = 7$, given lines are at right angles.

$\therefore$ Lines are intersecting.

23. $2f(x^2) + 3f\left(\dfrac{1}{x^2}\right) = x^2 - 1$ ...(i)

Replacing x by $\dfrac{1}{x}$, we get

$2f\left(\dfrac{1}{x^2}\right) + 3f(x^2) = \dfrac{1}{x^2} - 1$ ...(ii)

Solving (i) & (ii), we get

$$9f(x^2) - 4f(x^2) = 3\left(\frac{1}{x^2} - 1\right) - 2(x^2 - 1)$$

$$\Rightarrow \ 5f(x^2) = \frac{3}{x^2} - 2x^2 - 1$$

$$\Rightarrow \ f(x^2) = -\left(\frac{2x^4 + x^2 - 3}{5x^2}\right)$$

OR

$$\text{Let } x = \cos^{-1}\left(\frac{12}{13}\right) \text{ and } y = \sin^{-1}\left(\frac{3}{5}\right)$$

$$\Rightarrow \ \cos x = \frac{12}{13} \text{ and } \sin y = \frac{3}{5}$$

$$\text{Now, } \sin x = \sqrt{1 - \cos^2 x} \text{ and } \cos y = \sqrt{1 - \sin^2 y}$$

$$\Rightarrow \ \sin x = \sqrt{1 - \frac{144}{169}} \text{ and } \cos y = \sqrt{1 - \frac{9}{25}}$$

$$\Rightarrow \ \sin x = \frac{5}{13} \text{ and } \cos y = \frac{4}{5}$$

We know that, $\sin(x + y) = \sin x \cos y + \cos x \sin y$

$$= \frac{5}{13} \times \frac{4}{5} + \frac{12}{13} \times \frac{3}{5} = \frac{20}{65} + \frac{36}{65} = \frac{56}{65}$$

$$\Rightarrow \ x + y = \sin^{-1}\left(\frac{56}{65}\right)$$

$$\Rightarrow \ \cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$$

Hence Proved.

24. We have, $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$$\Rightarrow \ \frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 y}{\tan y} dy \ \Rightarrow \ \frac{d(\tan x)}{\tan x} = -\frac{d(\tan y)}{\tan y}$$

$\Rightarrow$ $\log(\tan x) = -\log(\tan y) + \log c$ (Integrating on both sides)

$\Rightarrow$ $\tan x \tan y = c$

25. Let $I=\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)}$

$$=\frac{1}{5}\left(\int_0^\infty \frac{1}{x^2+4}\,dx-\int_0^\infty \frac{1}{x^2+9}\,dx\right)$$

$$=\frac{1}{5}\left[\left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^\infty-\left[\frac{1}{3}\tan^{-1}\frac{x}{3}\right]_0^\infty\right]$$

$$=\frac{1}{5}\left[\left(\frac{1}{2}\cdot\frac{\pi}{2}-0\right)-\left(\frac{1}{3}\cdot\frac{\pi}{2}-0\right)\right]=\frac{1}{5}\left(\frac{\pi}{4}-\frac{\pi}{6}\right)=\frac{\pi}{60}$$

26. Given, $P(A)=\frac{2}{5}, P(B)=\frac{3}{10}$ and $P(A\cap B)=\frac{1}{5}$

Now, $P(A'\,|\,B')=\frac{P(A'\cap B')}{P(B')}=\frac{P(A\cup B)'}{P(B')}$

$$=\frac{1-P(A\cup B)}{1-P(B)}=\frac{1-[P(A)+P(B)-P(A\cap B)]}{1-P(B)}$$

$$=\frac{1-\left(\frac{2}{5}+\frac{3}{10}-\frac{1}{5}\right)}{1-\frac{3}{10}}=\frac{1-\frac{1}{2}}{\frac{7}{10}}=\frac{5}{7}.$$

and $P(B'\,|A')=\frac{P(B'\cap A')}{P(A')}=\frac{P(B\cup A)'}{P(A')}=\frac{5}{6}$

$\therefore\ P(A'\,|\,B')\cdot P(B'\,|\,A')=\frac{5}{7}\cdot\frac{5}{6}=\frac{25}{42}.$

OR

We have, $P(A)=\frac{2}{5}, P(B)=\frac{1}{3}, P(C)=\frac{1}{2},$

$P(A\cap C)=\frac{1}{5}$ and $P(B\cap C)=\frac{1}{4}$

Now, $P(C\,|B)=\frac{P(B\cap C)}{P(B)}=\frac{1/4}{1/3}=\frac{3}{4}$

and $P(A'\cap C') = P((A\cup C)') = 1 - P(A\cup C)$

$= 1 - [P(A) + P(C) - P(A\cap C)]$

$$=1-\left[\frac{2}{5}+\frac{1}{2}-\frac{1}{5}\right]=1-\frac{7}{10}=\frac{3}{10}.$$

27. Let $I=\int_1^2\left[\frac{1}{x}-\frac{1}{2x^2}\right]e^{2x}dx$

Putting $2x = y \Rightarrow 2dx = dy$

When $x = 1, y = 2$

and when $x = 2$, then $y = 4$

$$\therefore\ I=\frac{1}{2}\int_2^4\left[\frac{2}{y}-\frac{2}{y^2}\right]e^y dy=\int_2^4\left[\frac{1}{y}-\frac{1}{y^2}\right]e^y dy$$

$$\Rightarrow\ I=\left[e^y\cdot\frac{1}{y}\right]_2^4=\frac{1}{4}e^4-\frac{1}{2}e^2=\frac{e^2}{2}\left(\frac{e^2}{2}-1\right)$$

28. We have, $(1+x^2)\frac{dy}{dx}+2xy=4x^2$

$$\Rightarrow\ \frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{4x^2}{1+x^2}$$

This is a linear differential equation of the form

$\frac{dy}{dx}+Py=Q$, where $P=\frac{2x}{1+x^2}$ and $Q=\frac{4x^2}{1+x^2}$

$$\therefore\ \text{I.F.}=e^{\int P\,dx}=e^{\int\frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=1+x^2$$

So, the required solution is given by

$$y(1+x^2)=\int\frac{4x^2}{1+x^2}(1+x^2)\,dx+C$$

$$\Rightarrow\ y(1+x^2)=4\int x^2dx+C\ \Rightarrow\ y(1+x^2)=\frac{4x^3}{3}+C$$

Given that $y(0) = 0$ $\therefore$ $0(1 + 0) = 0 + C$

$\Rightarrow$ $C = 0$

Thus, $y=\frac{4x^3}{3(1+x^2)}$ is the required solution.

OR

We have, $\frac{dy}{dx}=\frac{x(2\log x+1)}{\sin y+y\cos y}$

$\Rightarrow$ $(\sin y + y\cos y)dy = x(2\log x + 1)dx$

On integrating both sides, we get

$-\cos y + y\sin y - (-\cos y)$

$$=2\left[\log x\times\frac{x^2}{2}-\int\frac{1}{x}\times\frac{x^2}{2}dx\right]+\frac{x^2}{2}+C$$

$$\Rightarrow\ y\sin y=x^2\log x-\frac{x^2}{2}+\frac{x^2}{2}+C$$

$\Rightarrow$ $y\sin y = x^2\log x + C$

$\because$ When $x = 1, y = \frac{\pi}{2}$

$$\therefore\ \frac{\pi}{2}\sin\frac{\pi}{2} = 1\cdot\log(1) + C \Rightarrow \frac{\pi}{2} = C$$

∴ $y \sin y = x^2 \log x + \pi/2$ is the required solution.

29. Let $\dfrac{x^2}{(x-1)^3(x+1)} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{(x-1)^3} + \dfrac{D}{x+1}$

$$\Rightarrow\ x^2 = A(x-1)^2(x+1) + B(x-1)(x+1) + C(x+1) + D(x-1)^3 \quad \text{...(i)}$$

Putting $x = 1$ in (i), we get $C = \dfrac{1}{2}$

Putting $x = -1$ in (i), we get $D = -\dfrac{1}{8}$

Putting $x = 0$ in (i), we get $A - B = -\dfrac{5}{8}$ ...(ii)

Putting $x = 2$ in (i), we get $A + B = \dfrac{7}{8}$ ...(iii)

Solving (ii) and (iii), we get $A = \dfrac{1}{8}$ and $B = \dfrac{3}{4}$

$$\therefore\ \int \frac{x^2}{(x-1)^3(x+1)}dx = \frac{1}{8}\int \frac{1}{x-1}dx + \frac{3}{4}\int \frac{1}{(x-1)^2}dx + \frac{1}{2}\int \frac{1}{(x-1)^3}dx - \frac{1}{8}\int \frac{1}{x+1}dx$$

$$= \frac{1}{8}\log|x-1| - \frac{3}{4(x-1)} - \frac{1}{4(x-1)^2} - \frac{1}{8}\log|x+1| + C$$

$$= \frac{1}{8}\log\left|\frac{x-1}{x+1}\right| - \frac{3}{4(x-1)} - \frac{1}{4(x-1)^2} + C$$

OR

Let $I = \displaystyle\int \frac{(x^3+x+1)}{(x^2-1)}dx = \int\left(x + \frac{2x+1}{(x-1)(x+1)}\right)dx$

Now, resolving $\dfrac{2x+1}{(x-1)(x+1)}$ into partial fractions as

$$\frac{2x+1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{x(A+B)+(A-B)}{(x-1)(x+1)}$$

On companing we get A +B = 2 and A – B = 1

Solving we get, $A = \dfrac{3}{2}, B = \dfrac{1}{2}$

$$\frac{2x+1}{(x-1)(x+1)} = \frac{3}{2(x-1)} + \frac{1}{2(x+1)}$$

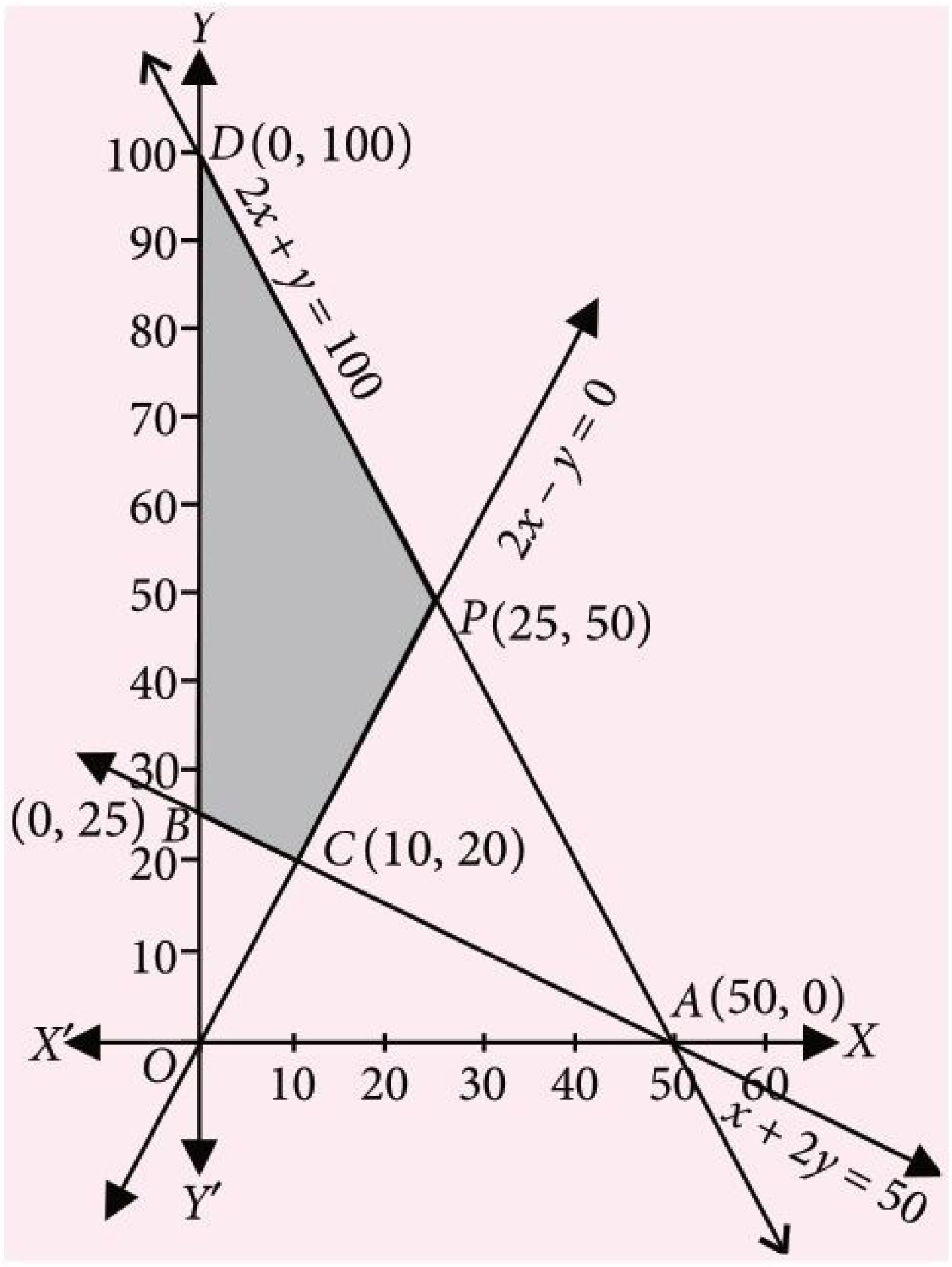
Now, $I = \displaystyle\int \frac{(x^3+x+1)}{(x^2-1)}dx = \int\left(x + \frac{2x+1}{(x-1)(x+1)}\right)dx$

$$= \int\left(x + \frac{3}{2(x-1)} + \frac{1}{2(x+1)}\right)dx$$

$$= \frac{x^2}{2} + \frac{3}{2}\log|x-1| + \frac{1}{2}\log|x+1| + C$$

$$= \frac{x^2}{2} + \frac{1}{2}\log|(x-1)^3(x+1)| + C$$

30. First we draw the lines whose equations are $x + 2y = 50$, $2x - y = 0$ and $2x + y = 100$ respectively. The feasible region is $BCPDB$ which is shaded in the figure.



The vertices of the feasible region are $B(0, 25)$, $C(10, 20)$, $P(25, 50)$ and $D(0, 100)$.
The values of the objective function $Z = x + 2y$ at these vertices are given below.

Corner points	**Value of $Z = x + 2y$**
$B(0, 25)$	50 (minimum)
$C(10, 20)$	50 (minimum)
$P(25, 50)$	125
$D(0, 100)$	200

∴ Z has minimum value 50 at two consecutive vertices B and C.

∴ Z has minimum value 50 at every point of segment joining the points $B(0, 25)$ and $C(10, 20)$.
Hence, there are infinite number of optimal solutions.

31. Let $I_1 = \displaystyle\int \frac{\sin x}{3+4\cos^2 x}dx$, $I_2 = \displaystyle\int \frac{\cos x}{3+4\sin^2 x}dx$

Put $\cos x = t \Rightarrow -\sin x\, dx = dt$

$$\therefore\ I_1 = -\int \frac{dt}{3+4t^2} = -\frac{1}{4}\int \frac{dt}{\left(\frac{\sqrt{3}}{2}\right)^2 + t^2}$$

$$=-\frac{1}{4}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\frac{2t}{\sqrt{3}}+c\left[\because \int\frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$I_1=-\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+c$$

Similarly, $I_2=\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\sin x}{\sqrt{3}}\right)$

$$\therefore\ I=I_1+I_2=\frac{1}{2\sqrt{3}}\left[\tan^{-1}\left(\frac{2\sin x}{\sqrt{3}}\right)-\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)\right]$$

32. Given, $y=|x+1|+1$

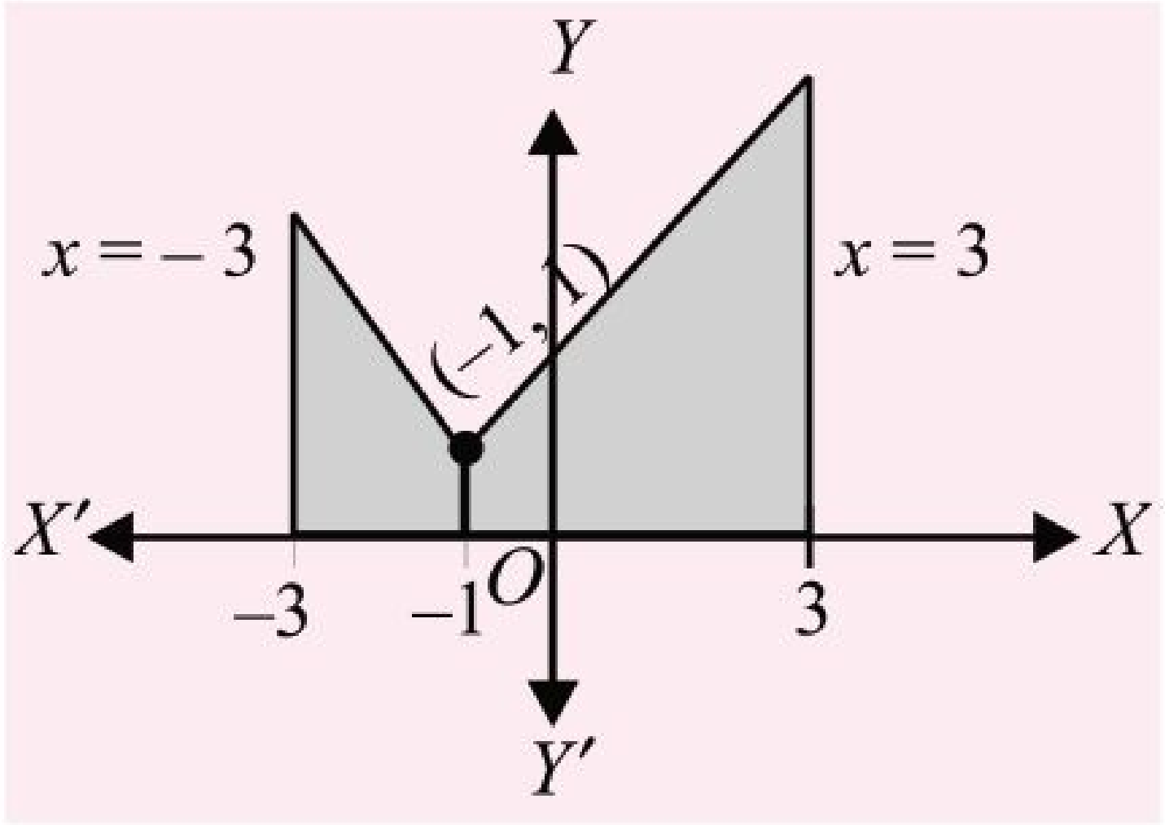
$$\therefore\ y=\begin{cases}x+2\ , & \text{if}\quad x\geq -1\\ -x\ , & \text{if}\quad x<-1\end{cases}$$

We now draw the lines : $y = 0$, $x = 3$, $x = -3$ and

$y = x + 2$ if $x \geq -1$ ...(i)

$y = -x$ if $x < -1$ ...(ii)

Lines (i) and (ii) intersect each other at $(-1, 1)$



$\therefore$ Required area

$$=\int_{-3}^{-1}(-x)dx+\int_{-1}^{3}(x+2)dx$$

$$=-\left[\frac{x^2}{2}\right]_{-3}^{-1}+\left[\frac{x^2}{2}+2x\right]_{-1}^{3}$$

$$=-\frac{1}{2}(1-9)+\frac{1}{2}(9-1)+2(3+1)$$

$= 4 + 4 + 8 = 16$ sq. units.

33. Given, $f(x)=\frac{x}{1+|x|},\ x\in(-\infty,0)\,f(x)=\frac{x}{1-x}$

$(\because\ x\in(-\infty, 0),\ |x| = -x)$

For one-one : Let $f(x_1) = f(x_2)$, $x_1, x_2 \in (-\infty , 0)$

$$\Rightarrow\quad \frac{x_1}{1-x_1}=\frac{x_2}{1-x_2}\ \Rightarrow\ x_1(1-x_2)=x_2(1-x_1)$$

$$\Rightarrow\quad x_1-x_1x_2=x_2-x_1x_2\ \Rightarrow\ x_1=x_2$$

Hence, if $f(x_1) = f(x_2)$, then $x_1 = x_2$ $\therefore f$ is one-one

For onto : Let $f(x) = y$

$$\Rightarrow\quad y=\frac{x}{1-x}\ \Rightarrow\ y(1-x)=x\ \Rightarrow\ y-xy=x$$

$$\Rightarrow\quad x+xy=y\ \Rightarrow\ x(1+y)=y\ \Rightarrow\ x=\frac{y}{1+y}$$

Here, $y \in (-1, 0)$

So, x is defined for all values of y.

Also $x \in (-\infty, 0)$ for all $y \in (-1, 0)$. $\therefore$ f is onto.

OR

We have, $f(x)=\frac{x-1}{x-2}$

For one-one : Let $x, y \in A$ and consider $f(x) = f(y)$

$$\Rightarrow\quad \frac{x-1}{x-2}=\frac{y-1}{y-2}$$

$\Rightarrow\quad (x-1)(y-2) = (x-2)(y-1)$

$\Rightarrow\quad xy - y - 2x + 2 = xy - x - 2y + 2 \Rightarrow x = y$

Thus, $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in A$

So, f is one-one.

For onto : Let y be an arbitrary element of B. Then,

$$f(x)=y\Rightarrow\frac{x-1}{x-2}=y\Rightarrow(x-1)=y(x-2)\Rightarrow x=\frac{1-2y}{1-y}$$

Clearly, $x=\frac{1-2y}{1-y}$ is a real number for all $y \neq 1$.

Also, $\frac{1-2y}{1-y}\neq 2$ for any y, for, if we take $\frac{1-2y}{1-y}=2$, then we get $1 = 2$, which is wrong.

So, f is onto. Hence, f is a bijective.

34. We have, $A=\begin{bmatrix}1&2&0\\-2&-1&-2\\0&-1&1\end{bmatrix}$

$\therefore\quad |A| = 1(-1-2) - 2(-2-0) + 0 = -3 + 4 = 1 \neq 0$

$\therefore\quad A^{-1}$ exists.

Now, adj $A=\begin{bmatrix}-3&-2&-4\\2&1&2\\2&1&3\end{bmatrix}$

$$\therefore\ A^{-1}=\frac{1}{|A|}(\text{adj}\,A)=\begin{bmatrix}-3&-2&-4\\2&1&2\\2&1&3\end{bmatrix}$$

The given equations can be written as

$$\begin{bmatrix}1&-2&0\\2&-1&-1\\0&-2&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}10\\8\\7\end{bmatrix}$$

which is of the form $A'X = B$

$\Rightarrow\quad X = (A')^{-1}B = (A^{-1})'B$

$$\Rightarrow\ \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-3&2&2\\-2&1&1\\-4&2&3\end{bmatrix}\begin{bmatrix}10\\8\\7\end{bmatrix}=\begin{bmatrix}0\\-5\\-3\end{bmatrix}$$

$\Rightarrow\quad x = 0, y = -5, z = -3$

35. Given lines are $l_1:\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$;

$l_2:\vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(4\hat{i}+6\hat{j}+12\hat{k})$

$\therefore$ We have $\vec{a}_1 = \hat{i}+2\hat{j}-4\hat{k}, \vec{b}_1 = 2\hat{i}+3\hat{j}+6\hat{k}$

and $\vec{a}_2 = 3\hat{i}+3\hat{j}-5\hat{k}, \vec{b}_2 = 4\hat{i}+6\hat{j}+12\hat{k}$

So, $\vec{a}_2 - \vec{a}_1 = 2\hat{i}+\hat{j}-\hat{k}$

Also, $\vec{b}_2 = 4\hat{i}+6\hat{j}+12\hat{k} = 2\vec{b}_1 \Rightarrow \vec{b}_1 \,||\, \vec{b}_2$

Hence l_1 and l_2 are parallel lines.

Shortest distance between two parallel lines is,

$$d = \left|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right| \quad \Rightarrow d = \left|\frac{(2\hat{i}+3\hat{j}+6\hat{k})\times(2\hat{i}+\hat{j}-\hat{k})}{\sqrt{2^2+3^2+6^2}}\right|$$

$$\Rightarrow d = \left|\frac{-9\hat{i}+14\hat{j}-4\hat{k}}{7}\right|$$

$$\Rightarrow d = \frac{\sqrt{(-9)^2+14^2+(-4)^2}}{7} = \frac{\sqrt{293}}{7} \text{ units.}$$

OR

Here $\vec{a} = -2\hat{i}+3\hat{j}+5\hat{k}, \vec{b} = 7\hat{i}-\hat{k}$

$\therefore$ Equation of line joining A and B is, $\vec{r} = \vec{a}+\lambda(\vec{b}-\vec{a})$

$\Rightarrow \vec{r} = -2\hat{i}+3\hat{j}+5\hat{k}+\lambda(9\hat{i}-3\hat{j}-6\hat{k})$ ... (i)

Again, $\vec{c} = -3\hat{i}-2\hat{j}-5\hat{k}, \vec{d} = 3\hat{i}+4\hat{j}+7\hat{k}$

$\therefore$ Equation of line joining C and D is, $\vec{r} = \vec{c}+\mu(\vec{d}-\vec{c})$

$\Rightarrow \vec{r} = -3\hat{i}-2\hat{j}-5\hat{k}+\mu(6\hat{i}+6\hat{j}+12\hat{k})$ ... (ii)

Equations (i) and (ii) will intersect, when

$$-2\hat{i}+3\hat{j}+5\hat{k}+\lambda(9\hat{i}-3\hat{j}-6\hat{k}) = -3\hat{i}-2\hat{j}-5\hat{k} + \mu(6\hat{i}+6\hat{j}+12\hat{k})$$

On equating, we get $-2+9\lambda = -3+6\mu$;

$3-3\lambda = -2+6\mu$; $5-6\lambda = -5+12\mu$

$\Rightarrow 9\lambda - 6\mu = -1$; $3\lambda + 6\mu = 5$; $6\lambda + 12\mu = 10$

Solving first two equations, we get $\lambda = \frac{1}{3}, \mu = \frac{2}{3}$ which also satisfy third equation.

Put $\lambda = \frac{1}{3}$ in (i), we get the point of intersection of lines is $\hat{i}+2\hat{j}+3\hat{k} \equiv (1,2,3)$.

36. **(i)** We know that $\Sigma P_i = 1$

Then $0.2 + k + 2k + 3k + 2k + 0 = 1$

$\Rightarrow 8k = 1 - 0.2 = 0.8 \Rightarrow k = 0.1$

(ii) P(Average study time is not more than 1 hour)

$= P(X \le 1) = P(X = 0) + P(X = 1) = 0.2 + 0.1 = 0.3$

(iii) P(Average study time is at least 3 hours)

$= P(X \ge 3) = P(X = 3) + P(X = 4) = 0.3 + 0.2 = 0.5$

OR

P(Average study time is atmost 2 hours)

$P(X = 0) + P(X = 1) + P(X = 2) = 0.2 + 0.1 + 0.2 = 0.5$

37. **(i)** Given, T is the temperature of the body at any time t. Then, by Newton's law of cooling, we get

$\frac{dT}{dt} = k(T-50)$, where k is the constant of proportionality.

$\therefore$ $f(t) = T - 50$

(ii) From given information, we have

At 8 p.m. temperature is 70°F $\quad \therefore$ At $t = 0$, $T = 70°$F

At 10 p.m., temperature is 60°F

$\therefore$ At $t = 2$, $T = 60°$ F

(iii) $\frac{dT}{dt} = k(T-50) \Rightarrow \frac{dT}{T-50} = k\,dt$

On integrating both sides, we get

$\log|T - 50| = kt + \log C \Rightarrow T - 50 = Ce^{kt}$

Clearly, for $t = 0$, $T = 70° \Rightarrow C = 20$

Thus, $T - 50 = 20e^{kt}$

OR

We have $T - 50 = 20\,e^{kt}$

For $t = 2$, $T = 60° \Rightarrow 10 = 20\,e^{2k}$

$$\Rightarrow 2k = \log\left(\frac{1}{2}\right) \Rightarrow k = \frac{1}{2}\log\left(\frac{1}{2}\right)$$

$$\text{Hence, } T = 50 + 20\left(\frac{1}{2}\right)^{\frac{t}{2}}$$

38. **(i)** We have, $\frac{dy}{dx} = \frac{ax+3}{2y+f} \Rightarrow (ax+3)dx = (2y+f)dy$

Integrating

$$\Rightarrow a\frac{x^2}{2} + 3x = y^2 + fy + c$$

$$\Rightarrow -\frac{a}{2}x^2 + y^2 - 3x + fy + C = 0$$

This will represent a circle, if $\frac{-a}{2} = 1 \Rightarrow a = -2$

[$\because$ In circle, coefficient of x^2 = coefficient of y^2]

(ii) $y' = y + 1 \Rightarrow \frac{dy}{y+1} = dx$

Integrating

$\Rightarrow \ln(y+1) = x + c$

Now, $y(0) = 1 \Rightarrow c = \ln 2$

$$\therefore \ln\left(\frac{y+1}{2}\right) = x \Rightarrow y + 1 = 2e^x$$

So, $y(\ln 2) = -1 + 2e^{\ln 2} = -1 + 4 = 3$

◆

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1.	(a)	**2.**	(b)	**3.**	(b)	**4.**	(a)	**5.**	(b)
6.	(c)	**7.**	(a)	**8.**	(a,c)	**9.**	(a,d)	**10.**	(c)
11.	(a,c,d)	**12.**	(a,b,c)	**13.**	(c)	**14.**	(c)	**15.**	(b)
16.	(a)	**17.**	(2)	**18.**	(288)	**19.**	(1)	**20.**	(6)

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. Find the domain of function

$$f(x)=\sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)}+\frac{1}{x^2-36}.$$

(Chinmay, Assam)

Ans. $f(x)=\sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)}+\frac{1}{x^2-36} \Rightarrow f(x)=f_1+f_2$

For domain of f_1, $f_1=\sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)}$

It is defined, if $\log_{0.4}\left(\frac{x-1}{x+5}\right)\geq 0 \Rightarrow \left(\frac{x-1}{x+5}\right)\leq 1$

$\Rightarrow \frac{x-1}{x+5}-1\leq 0 \Rightarrow \frac{-6}{x+5}\leq 0$

$\Rightarrow \frac{6}{x+5}\geq 0 \Rightarrow x+5\geq 0 \Rightarrow x\geq -5$

Also, $\frac{x-1}{x+5}>0 \Rightarrow x\in(-\infty,-5)\cup(1,\infty)$

$\therefore$ Domain of $f_1=(1,\infty)$

For domain of f_2, $f_2=\frac{1}{x^2-36}$

$\Rightarrow x^2-36\neq 0 \Rightarrow x\neq \pm 6$

Hence, domain of $f(x)=(1,\infty)-\{6,-6\}=(1,6)\cup(6,\infty)$

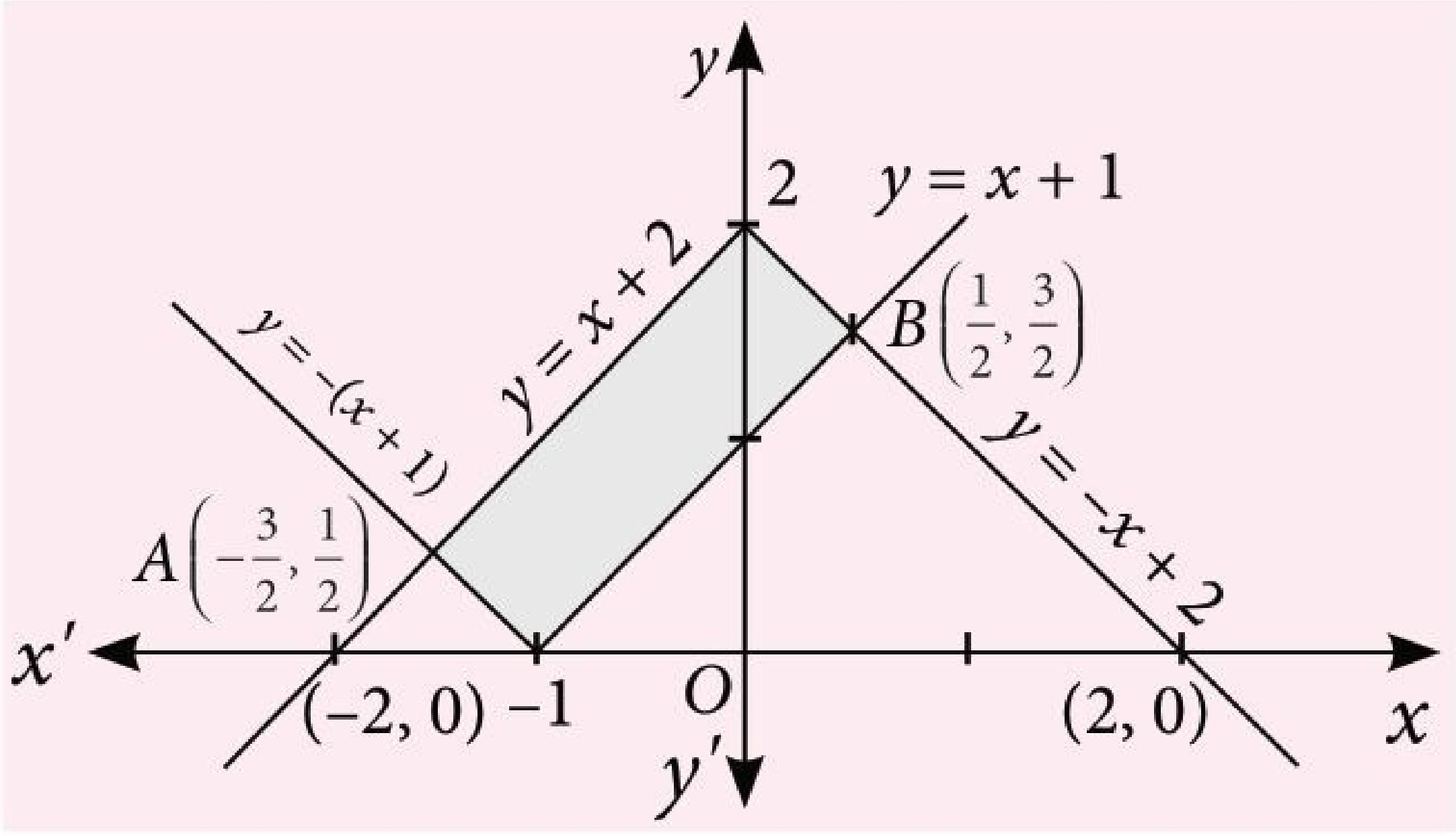
2. Find the area enclosed by the curves $y=|x+1|$ and $y=2-|x|$.

(Avinash, Delhi)

Ans. We have, $y=|x+1|=\begin{cases}-(x+1), & x\leq -1\\ x+1, & x>-1\end{cases}$

and $y=-|x|+2=\begin{cases}x+2, & x\leq 0\\ -x+2, & x>0\end{cases}$

Point of intersection of $y=x+2$ and $y=-(x+1)$ is $A\left(\frac{-3}{2},\frac{1}{2}\right)$ and point of interaction of $y=-x+2$ and $y=x+1$ is $B\left(\frac{1}{2},\frac{3}{2}\right)$.



Required area = area of shaded region

$$=\int_{-3/2}^{-1}\left[(x+2)-(-x-1)\right]dx+\int_{-1}^{0}\left[(x+2)-(x+1)\right]dx+\int_{0}^{1/2}\left[(-x+2)-(x+1)\right]dx$$

$$=\int_{-3/2}^{-1}(2x+3)dx+\int_{-1}^{0}1dx+\int_{0}^{1/2}(-2x+1)dx$$

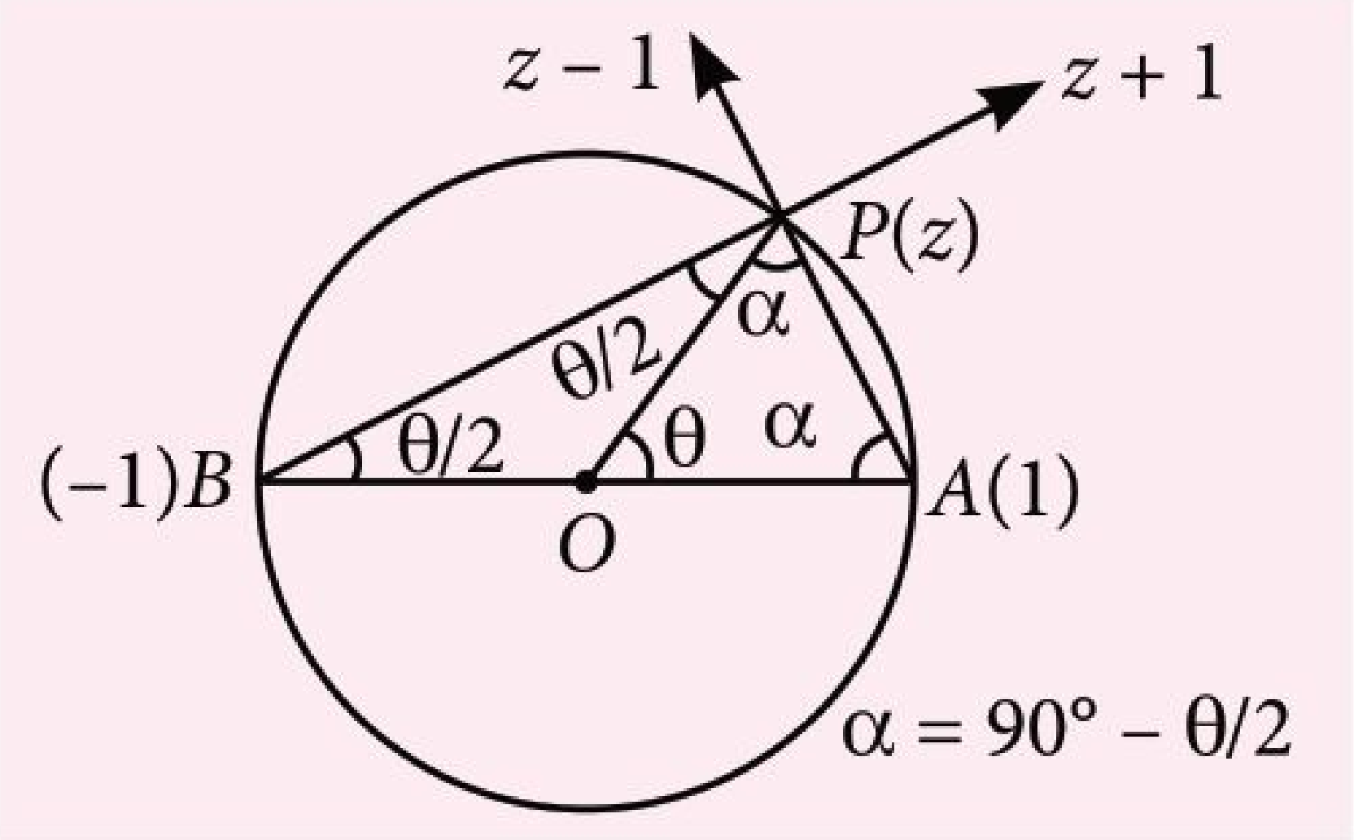
$$=\left[x^2+3x\right]_{-3/2}^{-1}+[x]_{-1}^{0}+\left[-x^2+x\right]_{0}^{1/2}$$

$$=(1-3)-\left(\frac{9}{4}-\frac{9}{2}\right)+1+\left(\frac{-1}{4}+\frac{1}{2}\right)=\frac{3}{2}\text{ sq.units.}$$

3. If z is a non-real complex number lying on the circle $|z|=1$, then show that $\tan\left(\frac{\arg(z)}{2}\right)=\frac{i(1-z)}{1+z}$.

(Sunil, Uttar Pradesh)

Ans. The circle $|z|=1$ meets the real axis at points A and B whose complex representations are 1 and –1 respectively.



If P whose complex representation is z is any point on the given circle and $\theta=\arg(z)$, then $PB=2\cos\left(\frac{\theta}{2}\right)$ and $PA=2\sin\left(\frac{\theta}{2}\right)$

The complex representations of BP and AP are $z+1$ and $z-1$ respectively.

$$\therefore \quad \frac{z-1}{z+1}=\frac{AP}{BP}e^{i(\pi/2)}=i\tan\left(\frac{\theta}{2}\right) \qquad [\because AP\perp BP]$$

$$\Rightarrow \tan\left(\frac{\theta}{2}\right)=\frac{z-1}{i(z+1)}=\frac{i(1-z)}{z+1}$$

$$\Rightarrow \tan\left(\frac{\arg(z)}{2}\right)=\frac{i(1-z)}{1+z}$$

◆◆

MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 6 : Indefinite Integration

Time Taken : 60 Min.

Only One Option Correct Type

1. The integral $\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$ is equal to

(a) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|+C$

(b) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{3}\right)}\right|+C$

(c) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}\right|+C$

(d) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)}\right|+C$

2. If $\int\frac{(x^2+1)e^x}{(x+1)^2}dx = f(x)e^x+C$, where C is a constant, then $\frac{d^3f}{dx^3}$ at $x = 1$ is equal to

(a) $-\frac{3}{4}$ (b) $\frac{3}{4}$ (c) $-\frac{3}{2}$ (d) $\frac{3}{2}$

3. The integral $\int\frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$ is equal to (where C is a constant of integration)

(a) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{5}{4}}+C$ (b) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+C$

(c) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{5}{4}}+C$ (d) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{1}{4}}+C$

4. Evaluate : $\int\frac{x^3}{(x+1)^2}dx$

(a) $\frac{x^2}{2}-2x+3\log|x+1|+\frac{1}{x+1}+C$

(b) $\frac{x^2}{2}+2x+2\log|x+1|+C$

(c) $\frac{x^2}{2}+2x+6\log|x+1|-\frac{1}{x+1}+C$

(d) none of these

5. If $\int\frac{1}{2+\cos x}dx = f(x) + C$, then the value of $f(2\pi/3)$ is

(a) $\frac{-\pi}{2\sqrt{3}}$ (b) $\frac{\pi}{2\sqrt{3}}$ (c) $\frac{\pi}{\sqrt{3}}$ (d) $-\frac{2\pi}{\sqrt{3}}$

6. Evaluate : $\int\frac{3\sin x+2\cos x}{3\cos x+2\sin x}dx$

(a) $\frac{13}{12}x+\frac{5}{13}\log|3\sin x+2\cos x|+C$

(b) $\frac{11}{12}x-\frac{13}{5}\log|3\sin x+2\cos x|+C$

(c) $\frac{12}{13}x - \frac{5}{13}\log|3\cos x + 2\sin x| + C$

(d) None of these

One or More Than One Option(s) Correct Type

7. If $\int \sin 4x \cdot e^{\tan^2 x} dx = f(x) + C$, then the value of $f(\pi/4)$ is

(a) $-e/2$ (b) $-e$ (c) $-2e$ (d) e

8. A curve $g(x) = \int x^{27}(1 + x + x^2)^6(6x^2 + 5x + 4)dx$ is passing through origin, then

(a) $g(1) = \frac{3^7}{7}$ (b) $g(1) = \frac{2^7}{7}$

(c) $g(-1) = \frac{1}{7}$ (d) $g(-1) = \frac{3^7}{14}$

9. If $\int \frac{e^{x-1}}{(x^2 - 5x + 4)} 2x\,dx = AF(x-1) + BF(x-4) + C$ and $F(x) = \int \frac{e^x}{x}dx$, then

(a) $A = -\frac{2}{3}$ (b) $B = \frac{4}{3}e^3$

(c) $A = \frac{2}{3}$ (d) $B = \frac{8}{3}e^3$

10. If $\int \frac{dx}{x^4 + x^3} = \frac{A}{x^2} + \frac{B}{x} + \log\left|\frac{x}{x+1}\right| + C$, then

(a) $A = \frac{1}{2}, B = 1$ (b) $A = 1, B = -\frac{1}{2}$

(c) $A = -\frac{1}{2}, B = 1$ (d) None of these

11. If the primitive of $\frac{1}{x^6 + x^4}$ is $f(x) + \frac{1}{x} + g(x)x^{-3} + c$, then

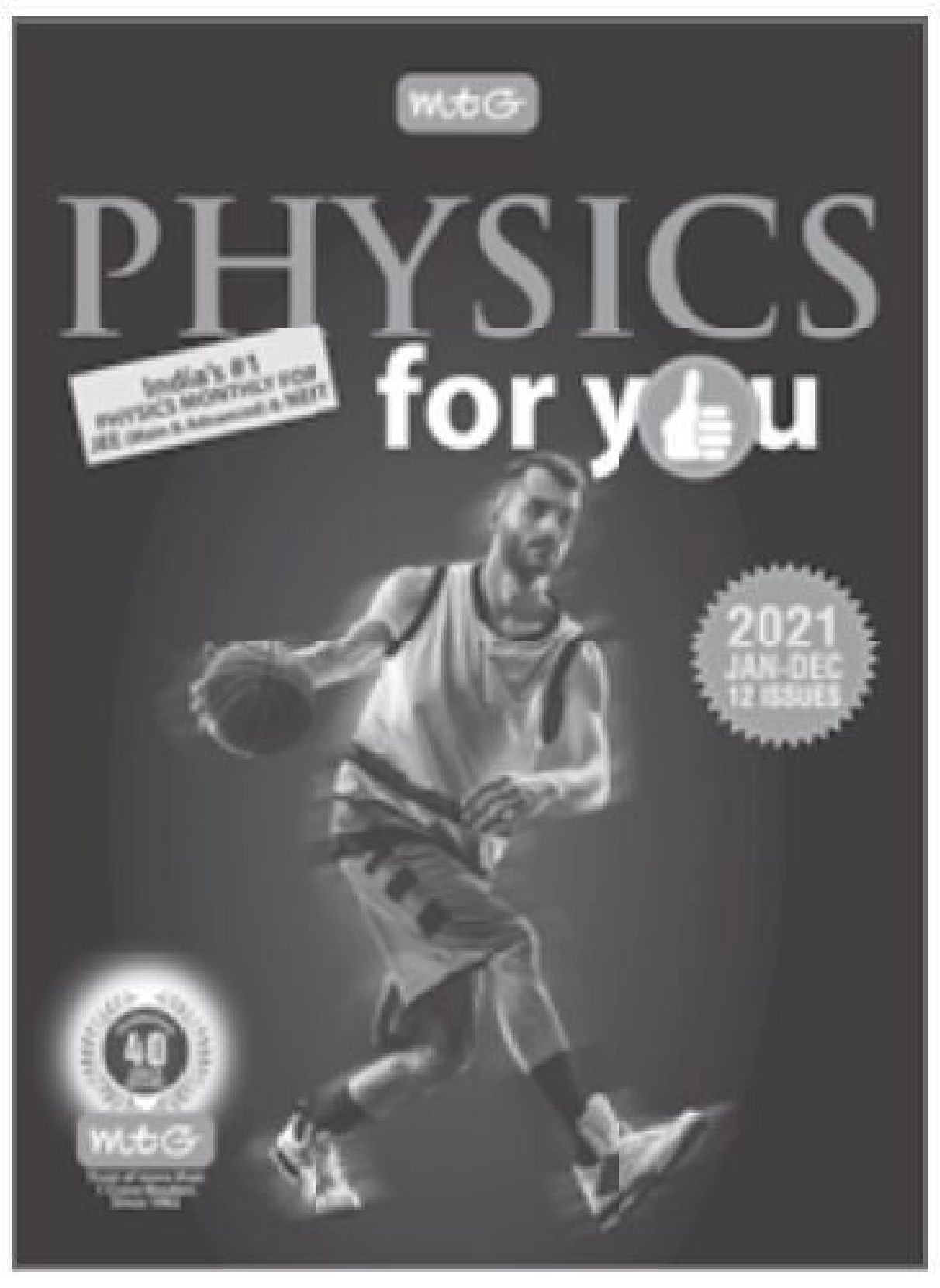
(a) $f(x) = \tan^{-1}x$

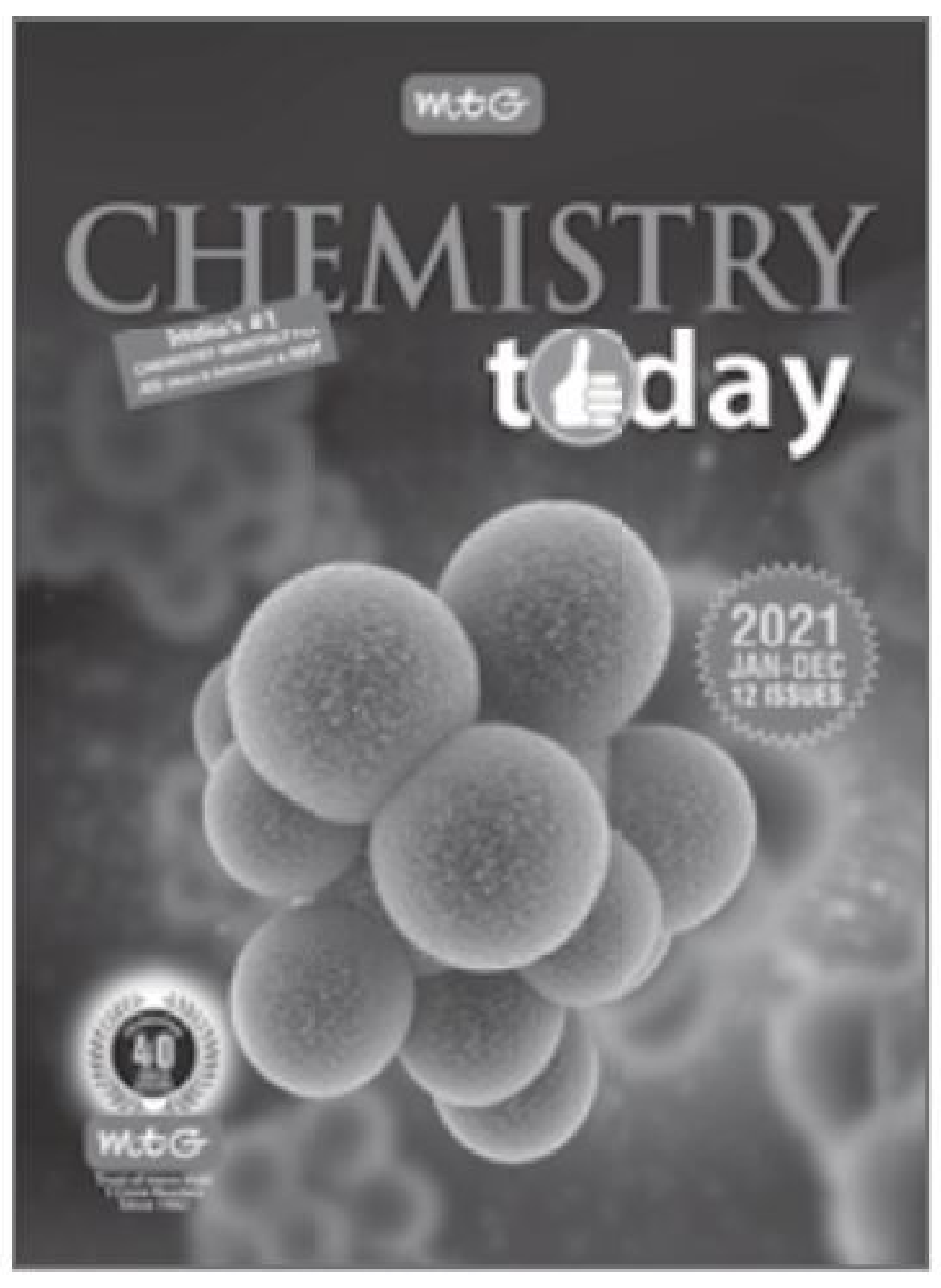
(b) $f(x) = 2\tan^{-1}x$

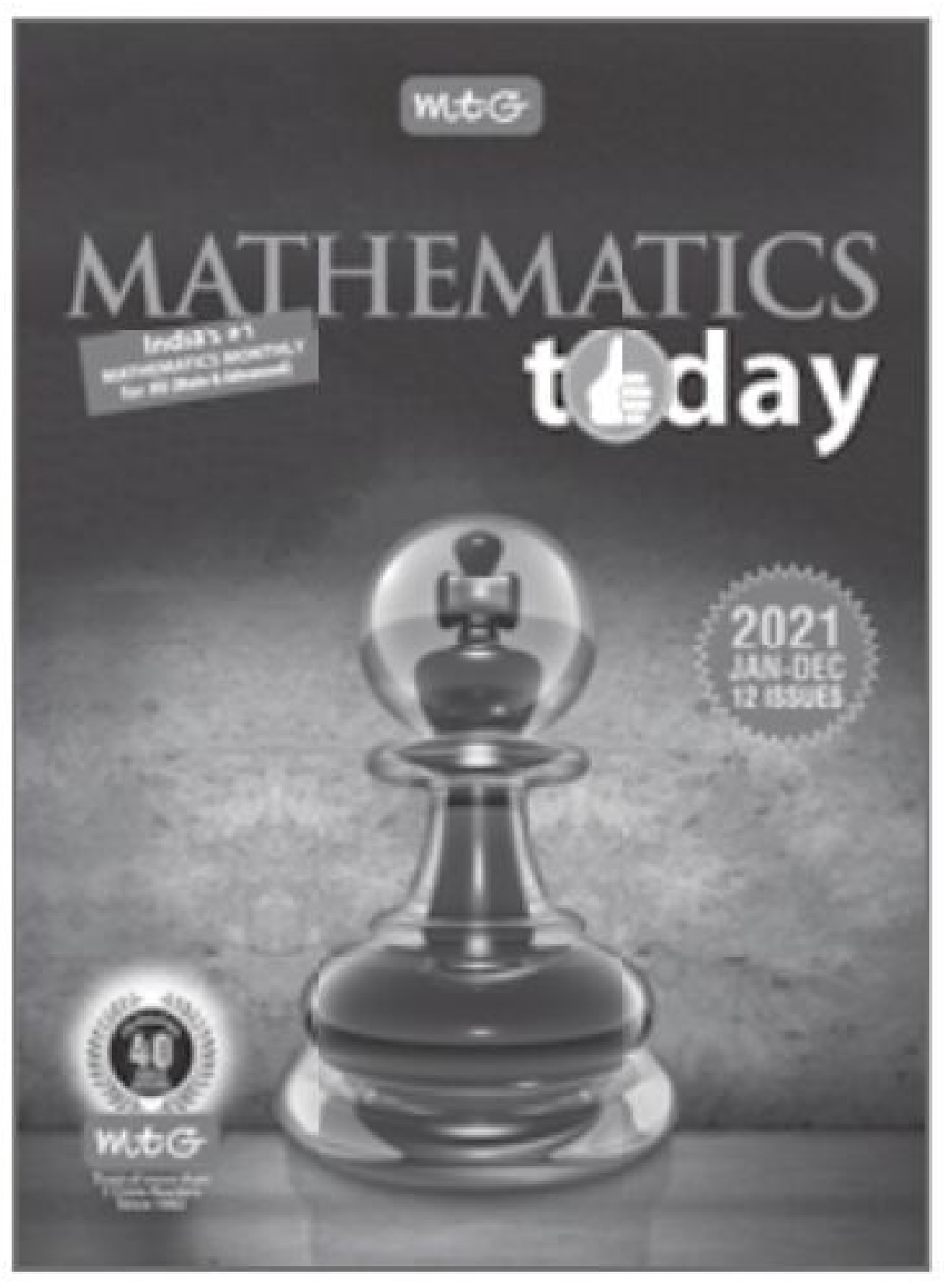
(c) $g(x)$ is a constant function

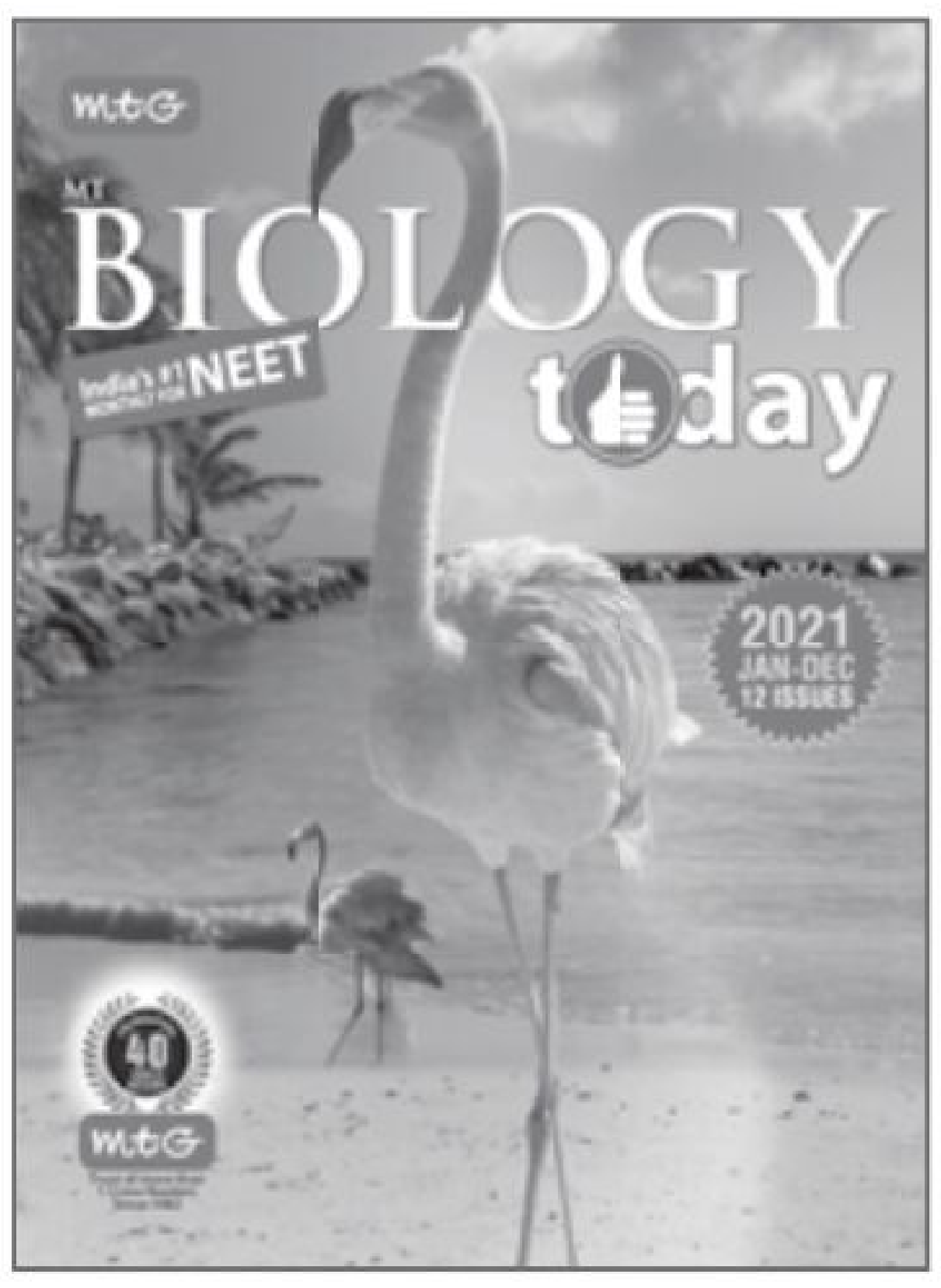
(d) $g(x) = -1/3$

12. If $\int \frac{\left(\frac{1}{x} + \frac{1}{x^2}\right)(x-1)\,dx}{\left(\frac{1}{x^4} + \frac{1}{x^2}\right)\sqrt{(x^4 - x^3 + x^2)(x^4 + x^3 + x^2)}}$ $= \sec^{-1}\{(f(x)\} + c$, then









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(a) Maximum value of $f(x) = -2$
(b) Minimum value of $f(x) = 2$
(c) $f(0)$ is not defined
(d) $f(\pi) < f(e)$

13. $\int\left\{\frac{(\log x-1)}{1+(\log x)^2}\right\}^2 dx$ is equal to

(a) $\frac{x}{x^2+1}+C$
(b) $\frac{\log x}{(\log x)^2+1}+C$
(c) $\frac{x}{(\log x)^2+1}+C$
(d) $\frac{xe^x}{1+x^2}+C$

Comprehension Type

Paragraph for Q. No. 14 and 15

Integrals of class functions following a definite pattern can be found by the method of reduction and recursion. Reduction formulas make it possible to reduce an integral dependent on the index $n > 0$, called the order of the integral, to an integral of the same type with a smaller index. Integration by parts helps us to derive reduction formulas.

14. If $I_n = \int\frac{dx}{(x^2+a^2)^n}$, then $I_{n+1} + \frac{1-2n}{2n}\cdot\frac{1}{a^2}\,I_n$ is equal to

(a) $\frac{x}{(x^2+a^2)^n}$
(b) $\frac{1}{2n\,a^2}\,\frac{1}{(x^2+a^2)^{n-1}}$
(c) $\frac{1}{2n\,a^2}\cdot\frac{x}{(x^2+a^2)^n}$
(d) $-\frac{1}{2n\,a^2}\cdot\frac{x}{(x^2+a^2)^n}$

15. If $I_{n,-m} = \int\frac{\sin^n x}{\cos^m x}dx$, then $I_{n,-m}+\frac{n-1}{m-1}I_{n-2,2-m}$ is equal to

(a) $\frac{\sin^{n-1}x}{\cos^{m-1}x}$
(b) $\frac{1}{(m-1)}\,\frac{\sin^{n-1}x}{\cos^{m-1}x}$
(c) $\frac{1}{(n-1)}\,\frac{\sin^{n-1}x}{\cos^{m-1}x}$
(d) $\frac{n-1}{m-1}\,\frac{\sin^{n-1}x}{\cos^{m-1}x}$

Matrix Match Type

16. Match the following :

Column - I		Column - II	
(P)	If $\int\frac{2^x}{\sqrt{1-4^x}}dx = k\sin^{-1}(2^x) + C$, then $k =$	(1)	2
(Q)	$\int\frac{x^4+1}{x(x^2+1)^2}dx = k\log\lvert x\rvert + \frac{m}{1+x^2}+C$, where C is the constant of integration, then $mk =$	(2)	1
(R)	$\int\frac{dx}{5+4\cos x} = k\tan^{-1}\left(m\tan\frac{x}{2}\right)+C$, then $k/m =$	(3)	$\frac{1}{\log 2}$

	P	Q	R
(a)	3	2	1
(b)	1	2	3
(c)	2	3	1
(d)	3	1	2

Numerical Answer Type

17. Let $k(x) = \int\frac{(x^2+1)dx}{\sqrt[3]{x^3+3x+6}}$ and $k(-1) = \frac{1}{\sqrt[3]{2}}$. Then the value of $k(-2)$ is ______.

18. If $\int\frac{\log x}{x^3}dx = -\frac{\log x}{ax^2}-\frac{1}{bx^2}+C$, then the value of $a^5 + b^4$ must be ______.

19. If $\int\frac{(\sqrt{x})^5}{(\sqrt{x})^7+x^6}dx = a\log\left(\frac{x^\lambda}{x^\lambda+1}\right)+C$, then $a\lambda =$ ______.

20. If $\int\frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx = Ax + B\log(9e^{2x}-4) + C$, then $\lvert 4A\rvert =$ ______.

◆ ◆

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SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

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90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

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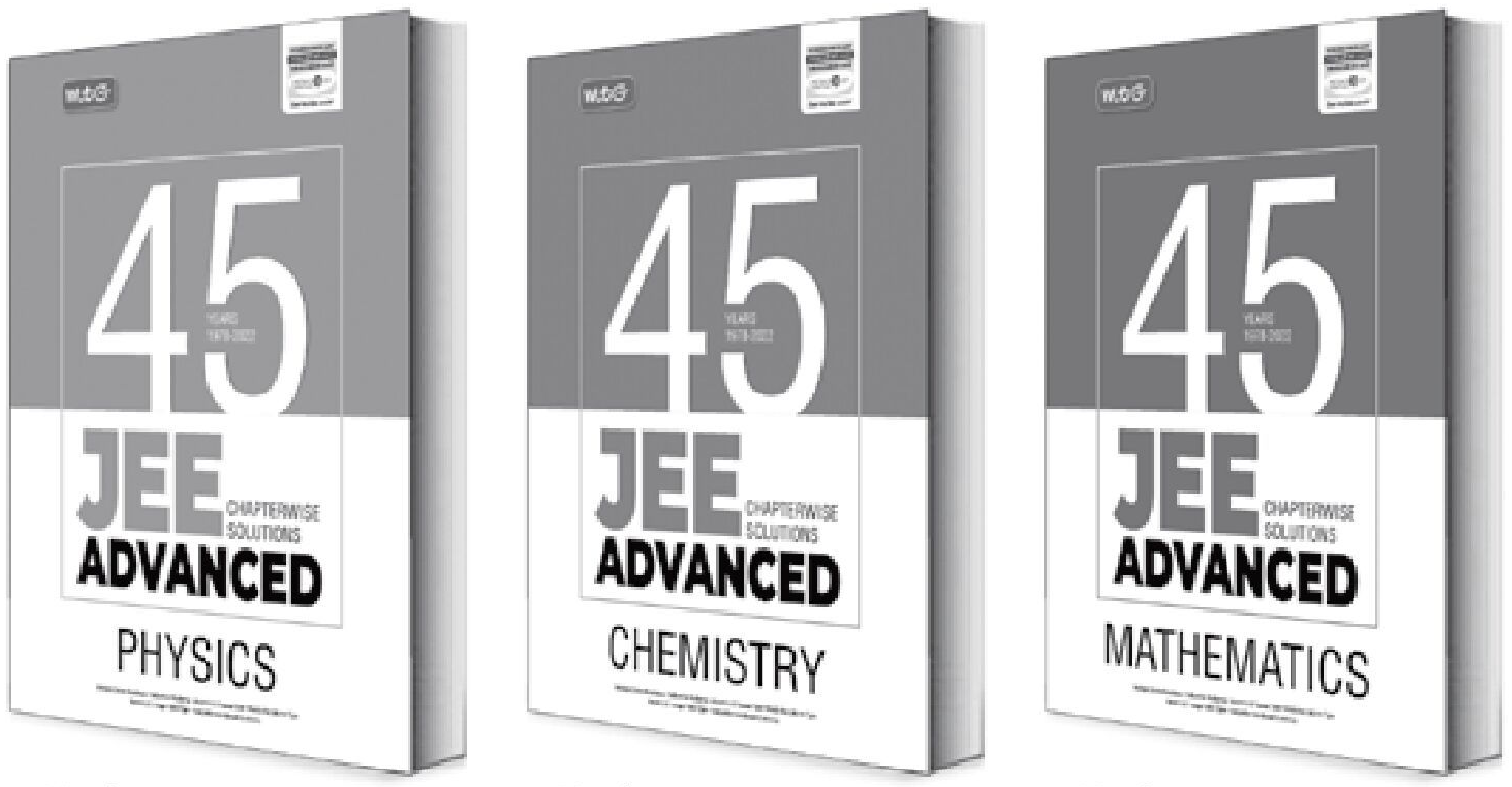
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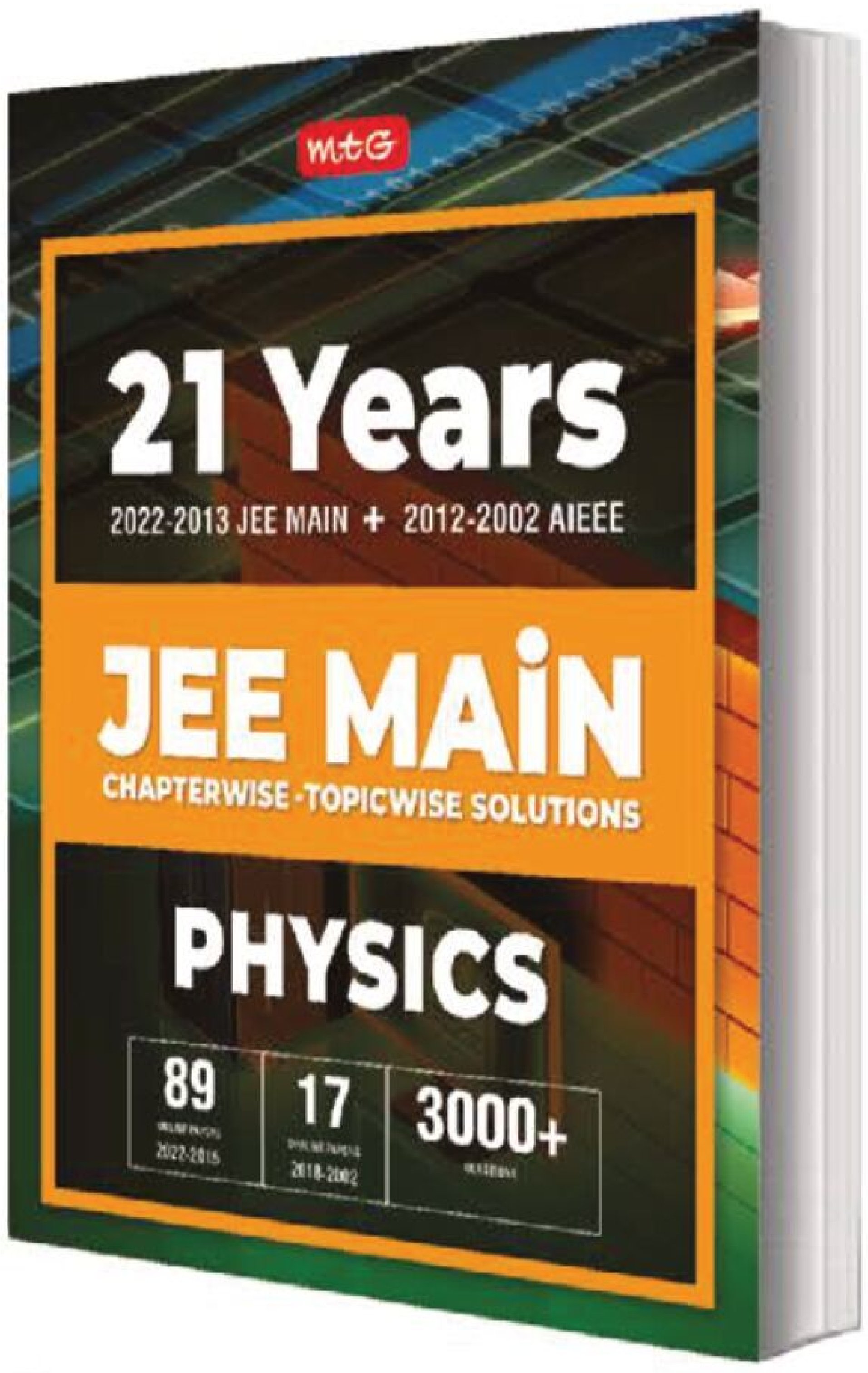
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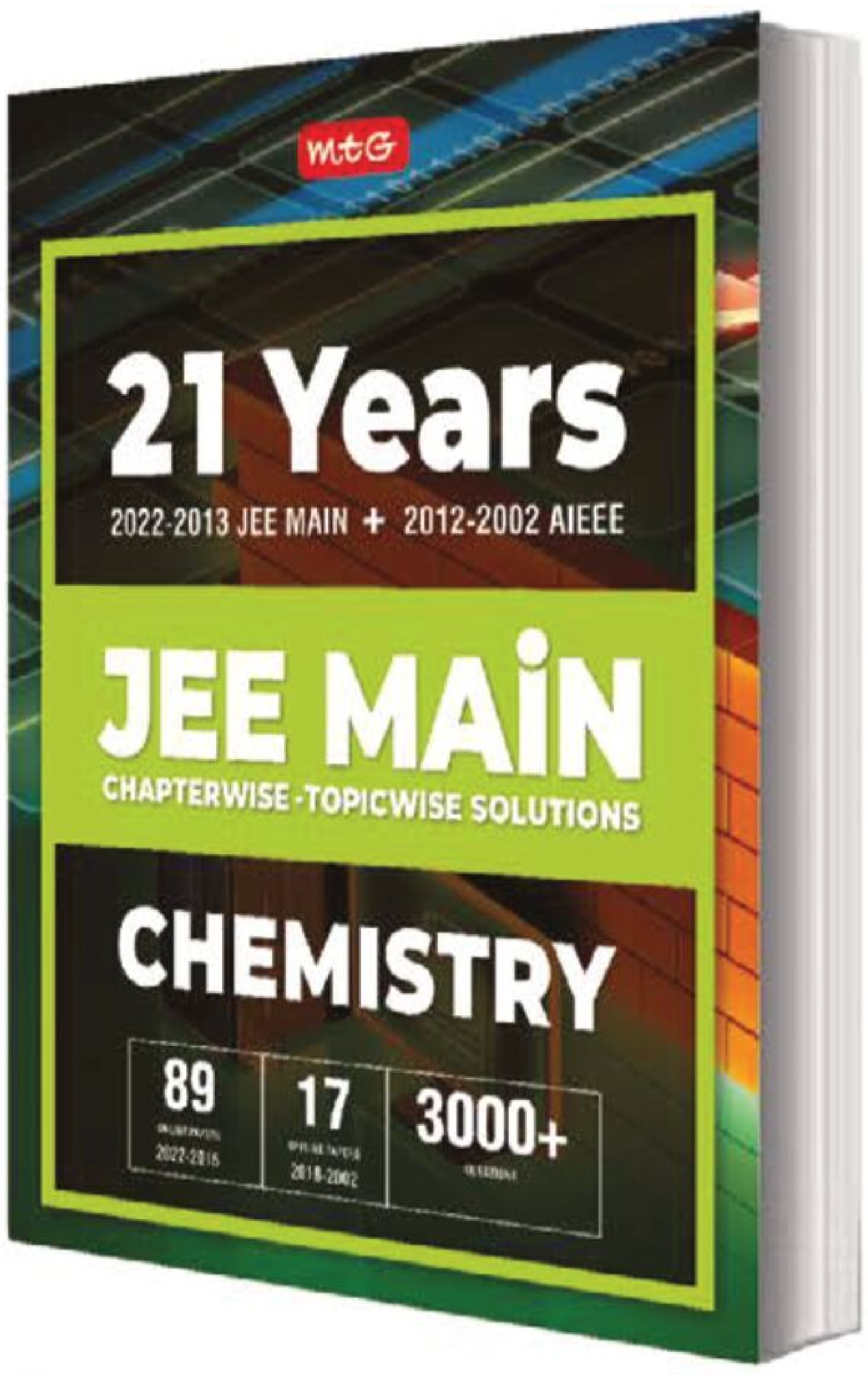
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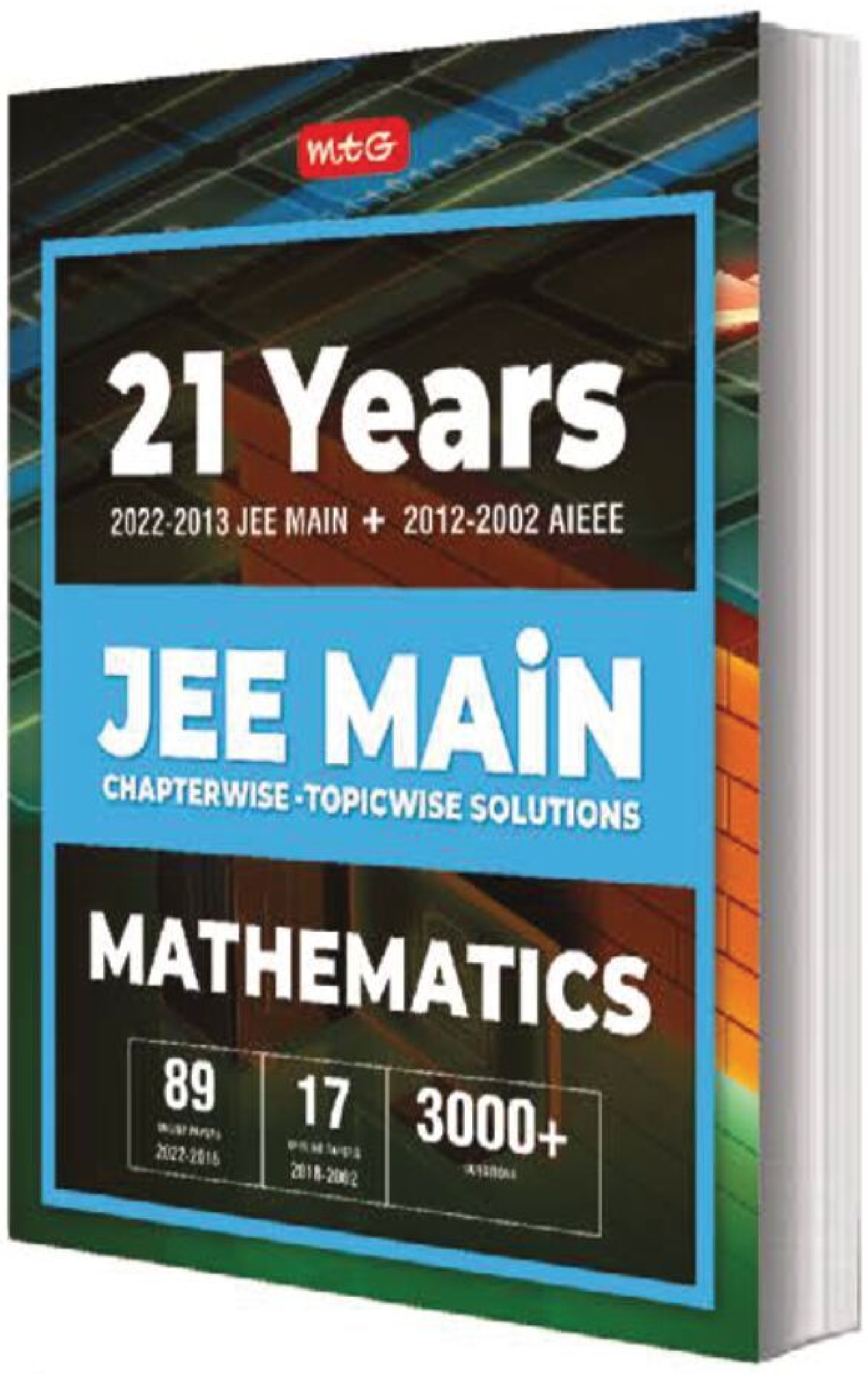
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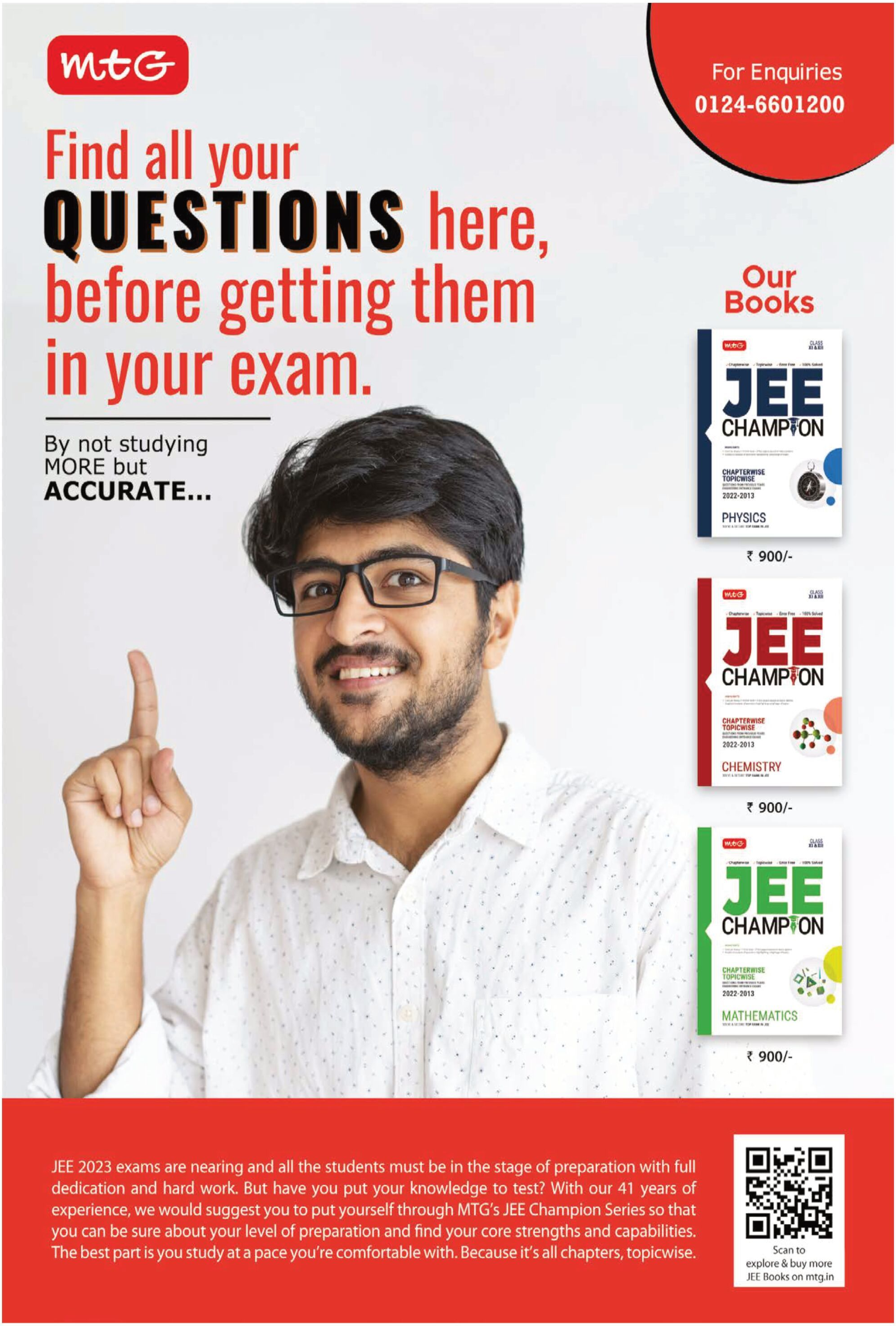

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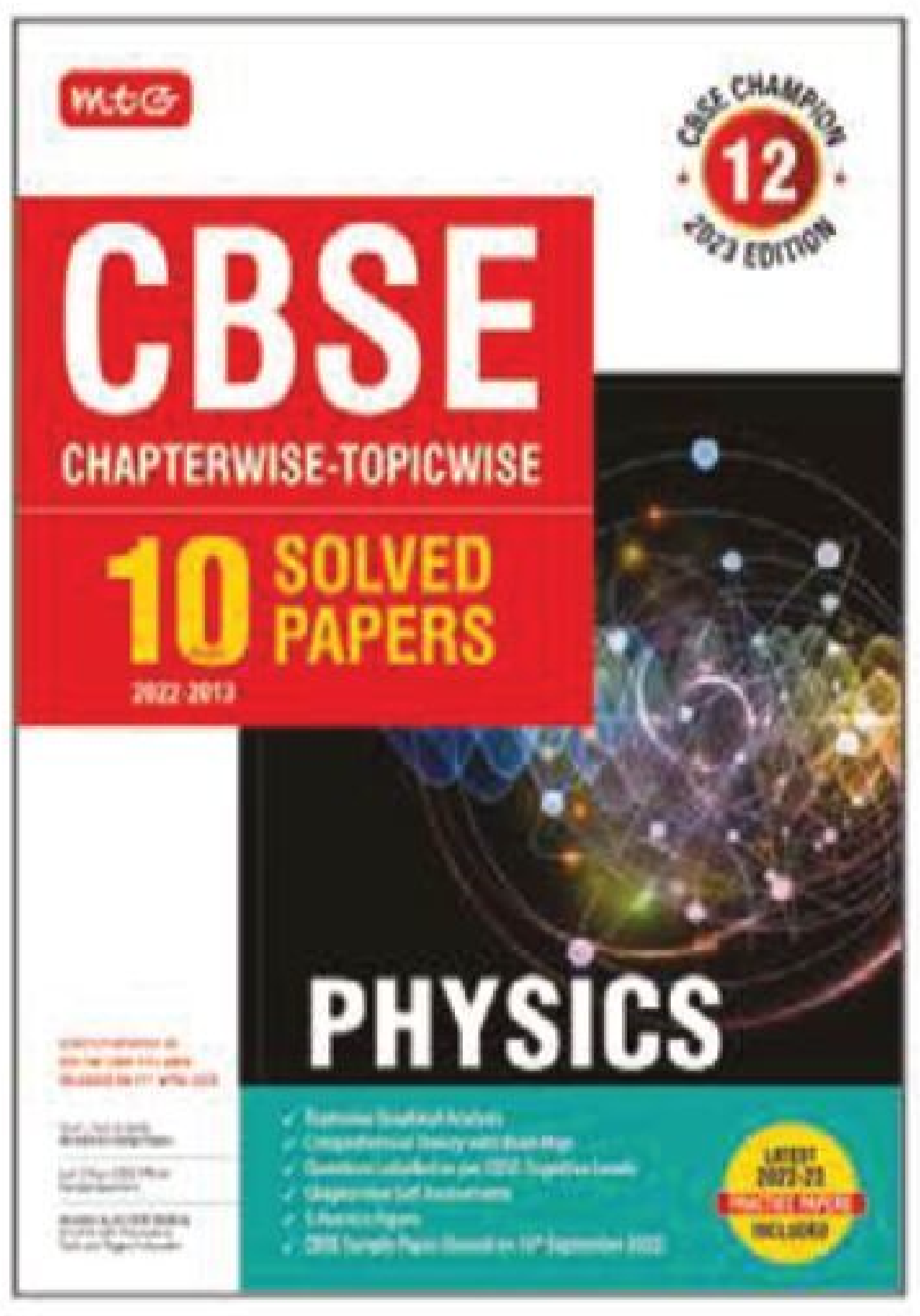
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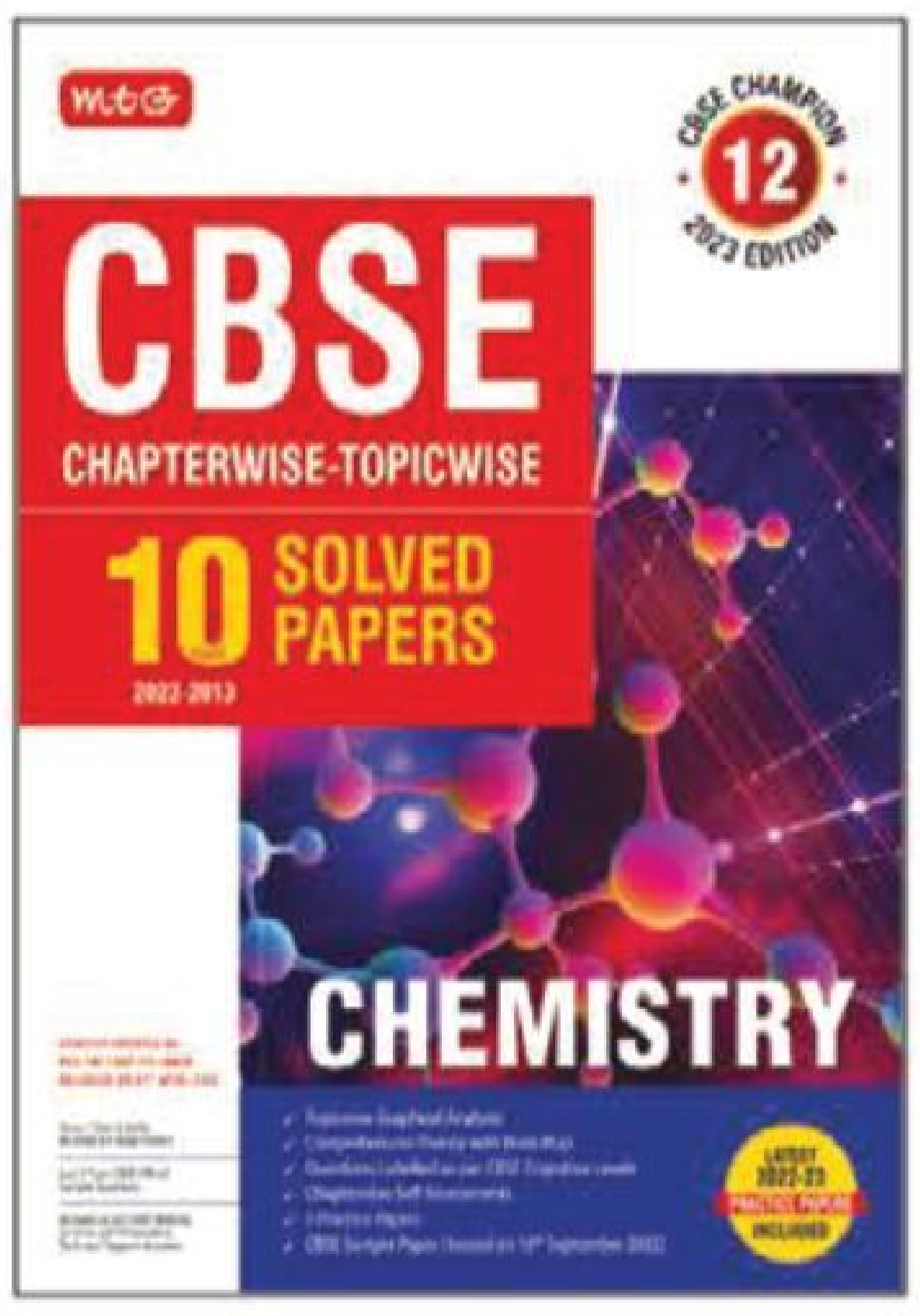
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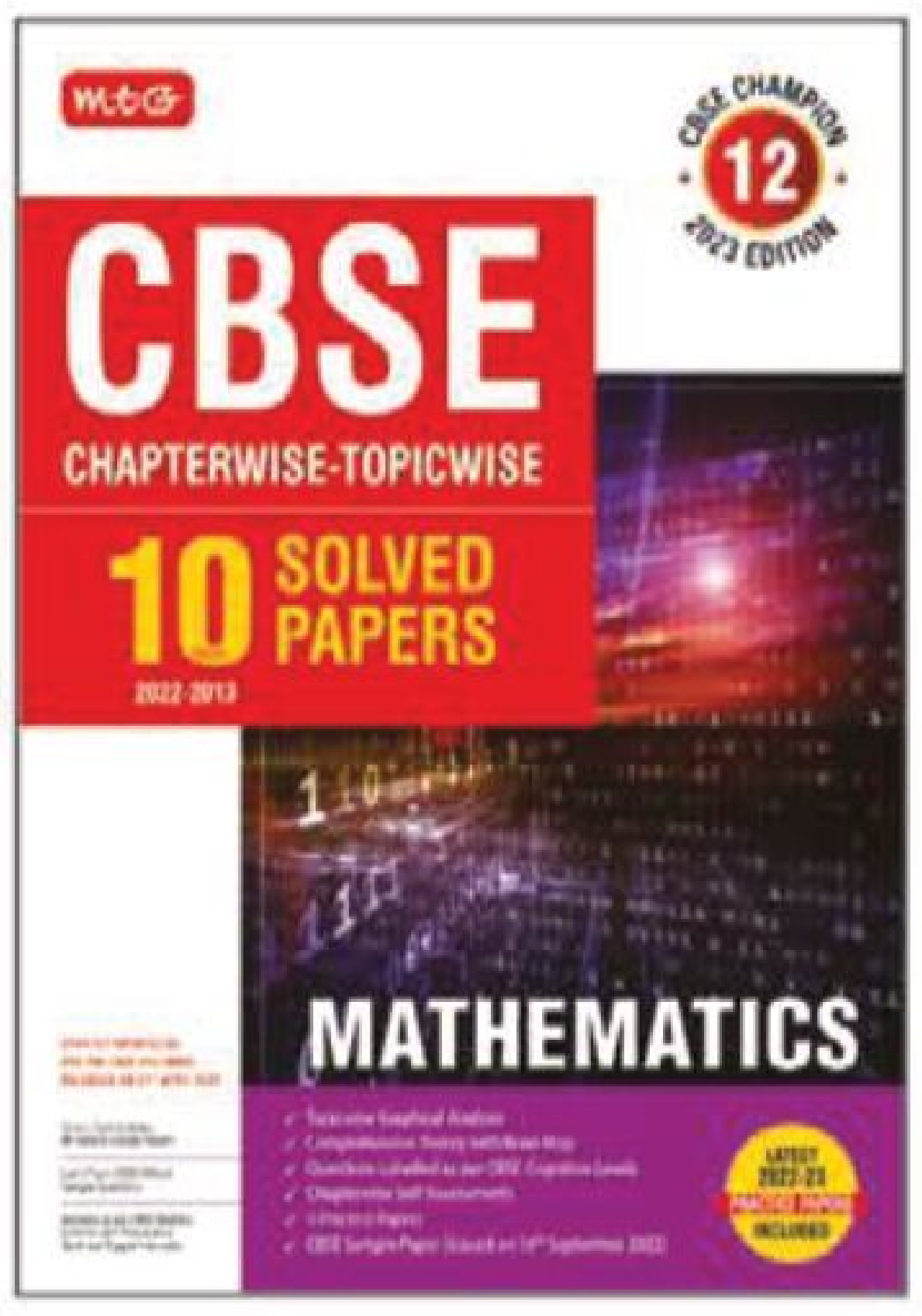
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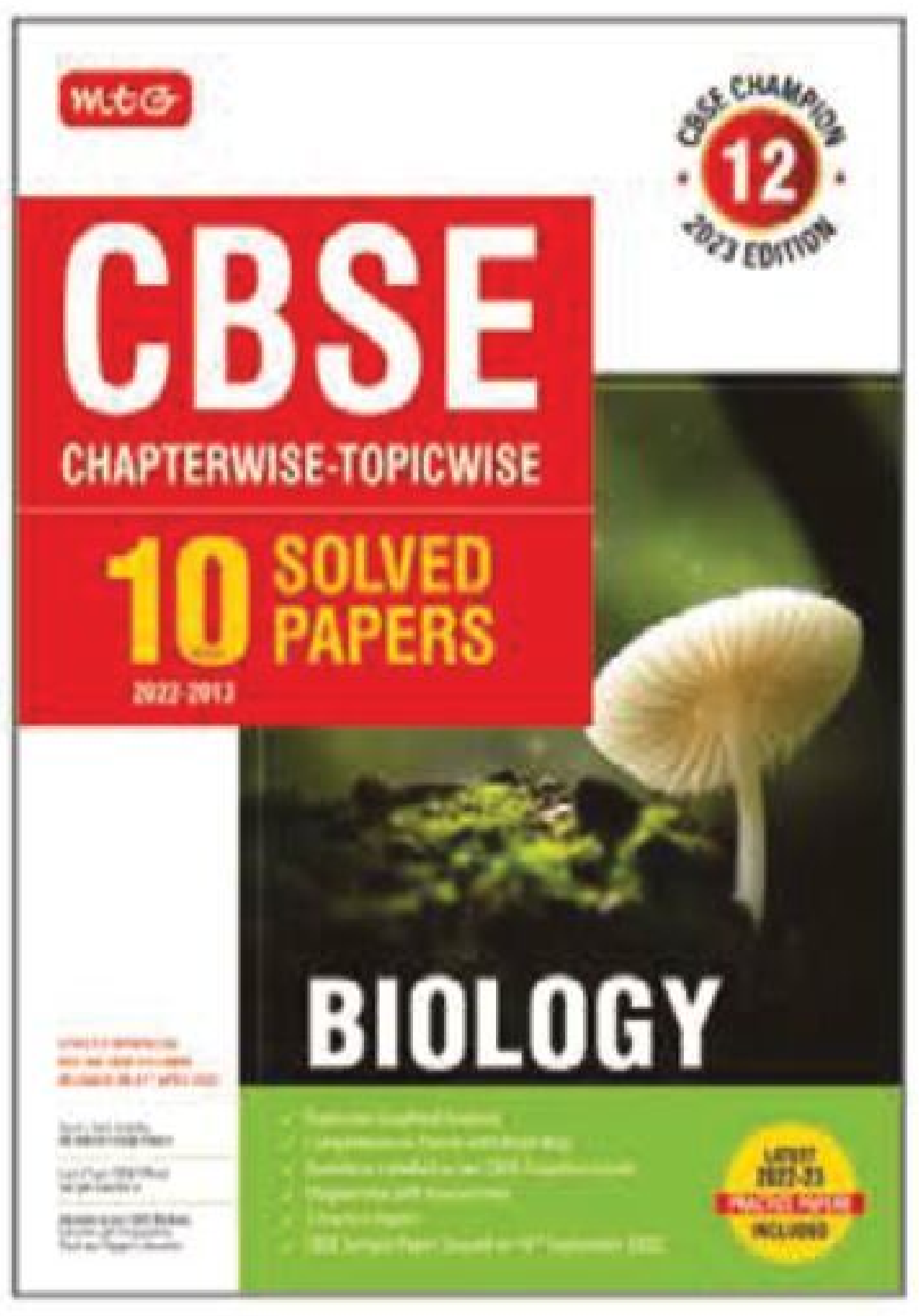
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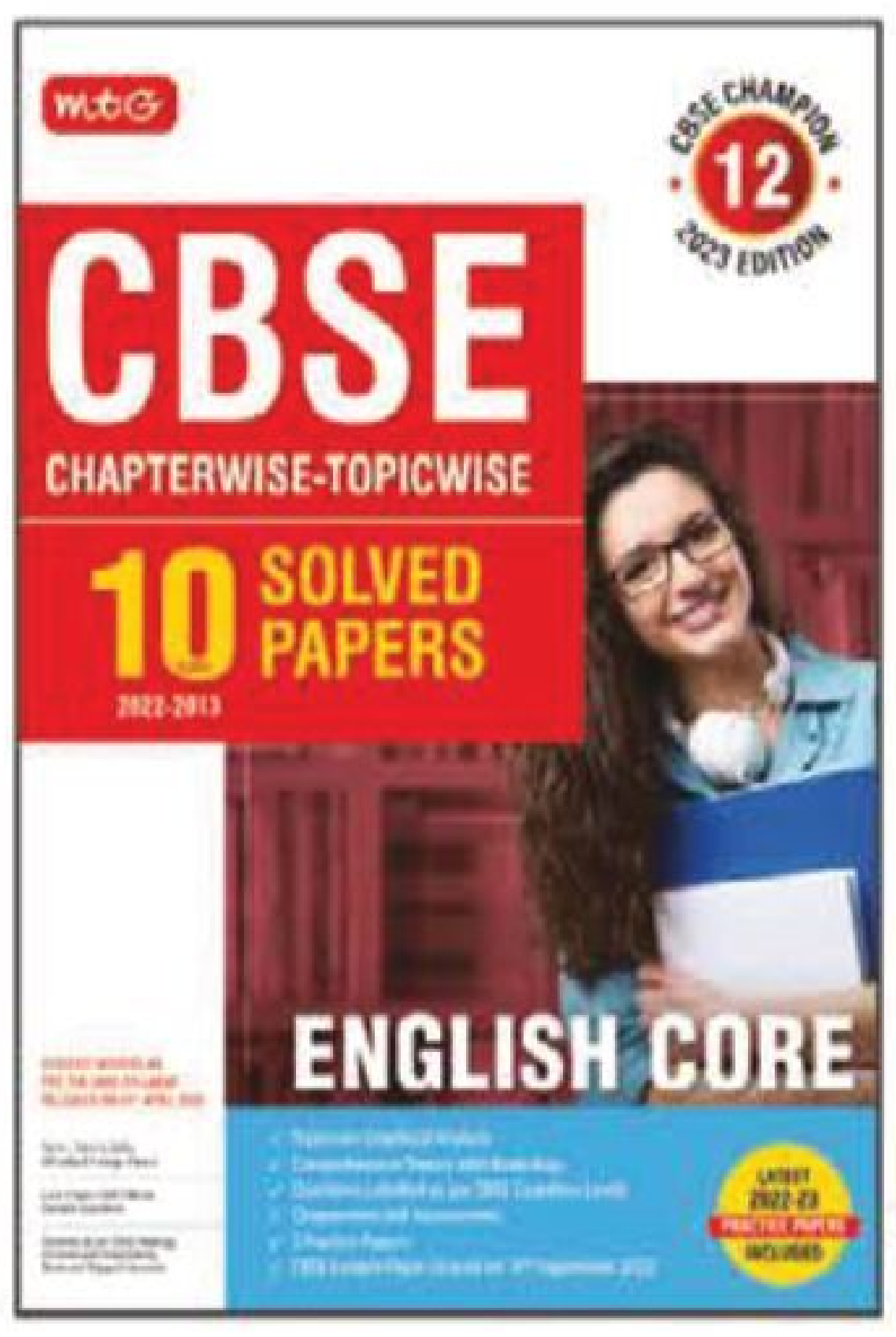
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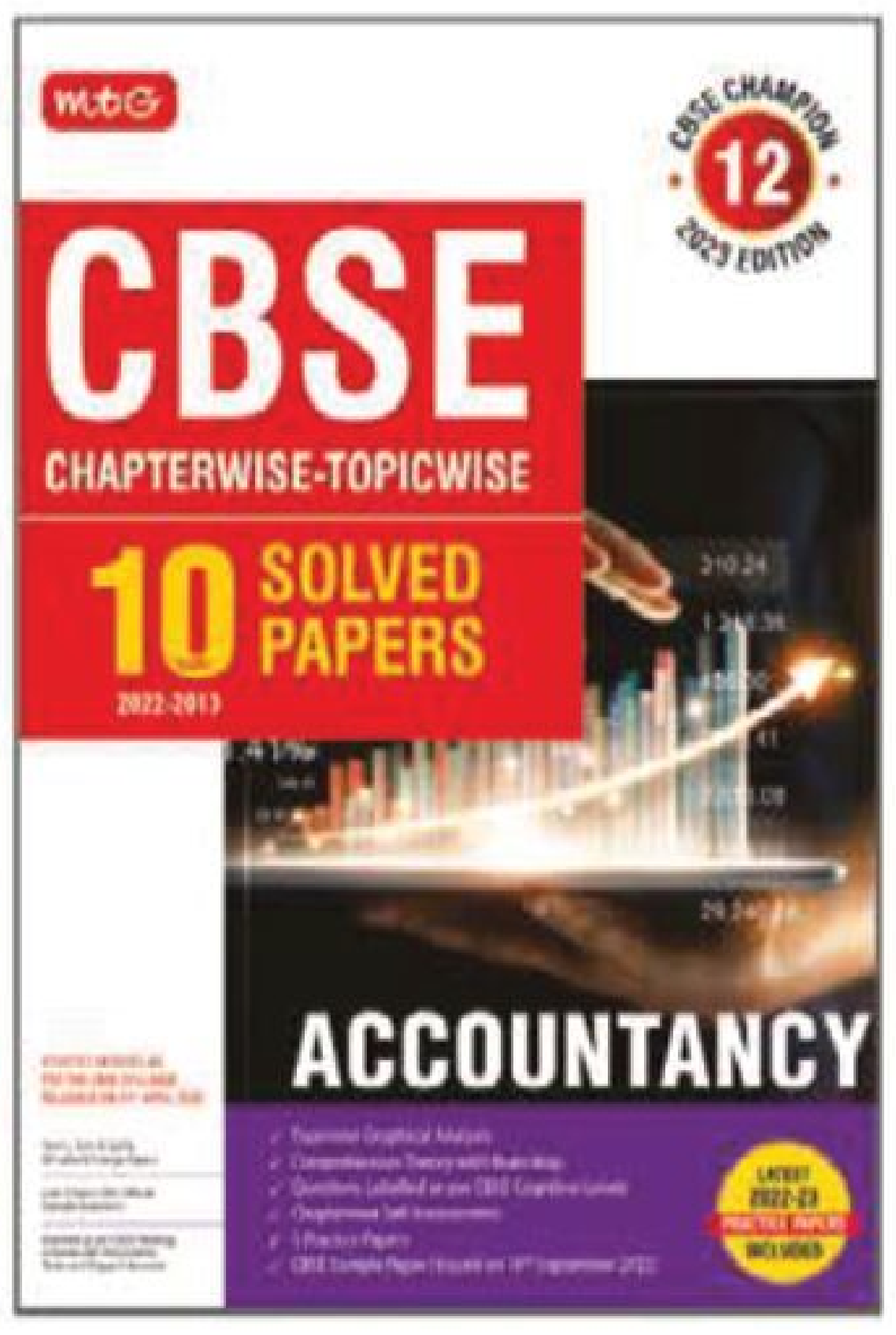
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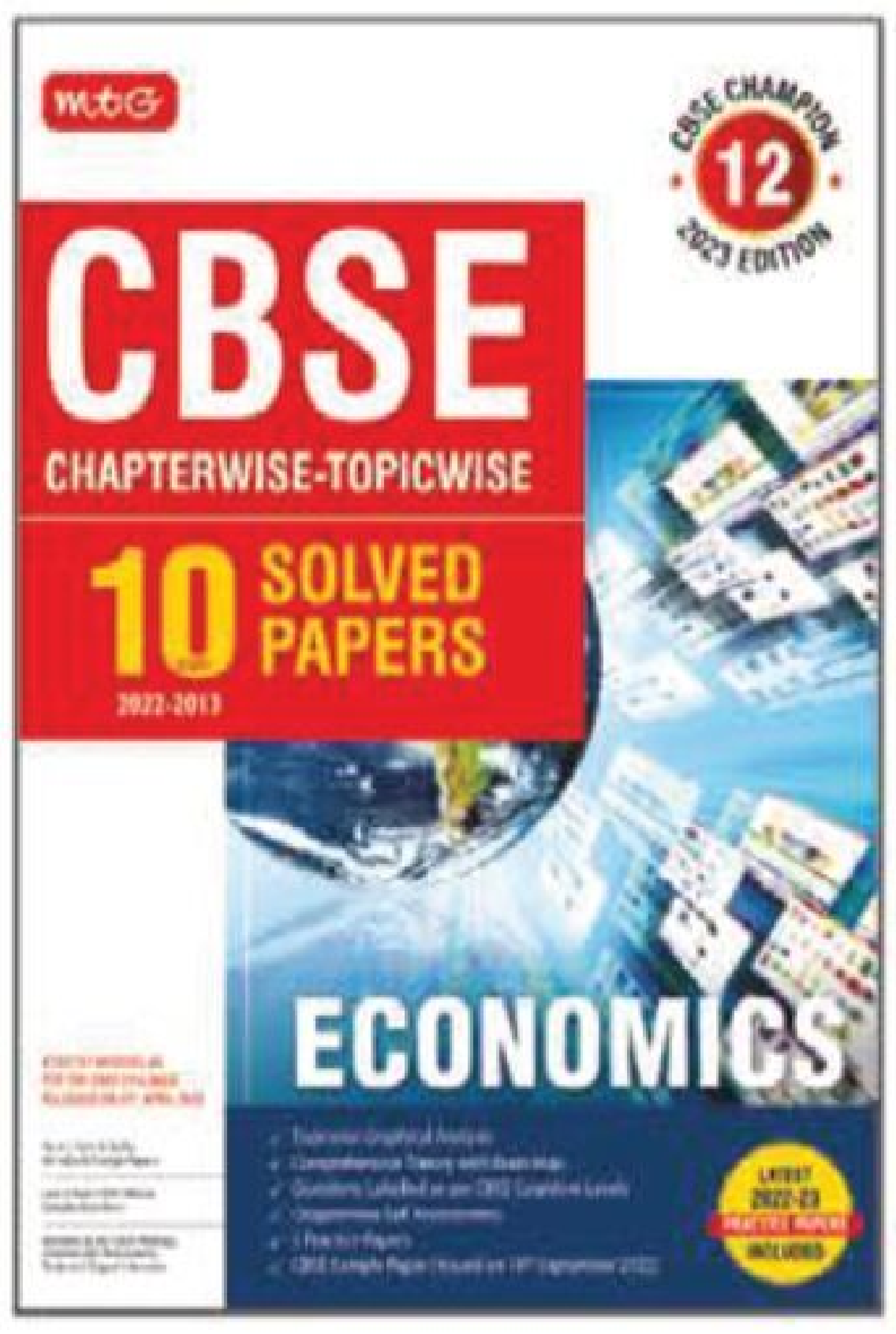
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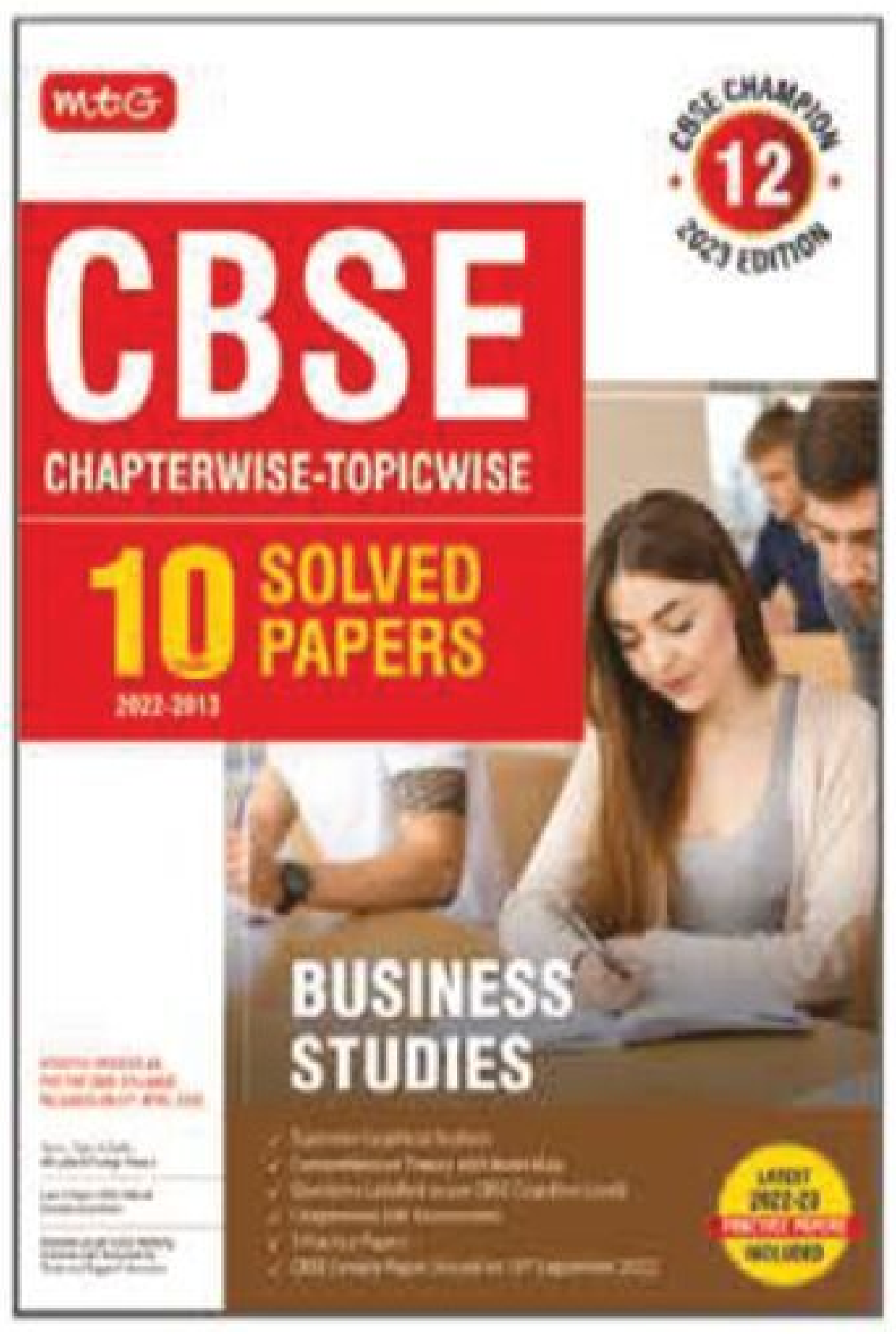
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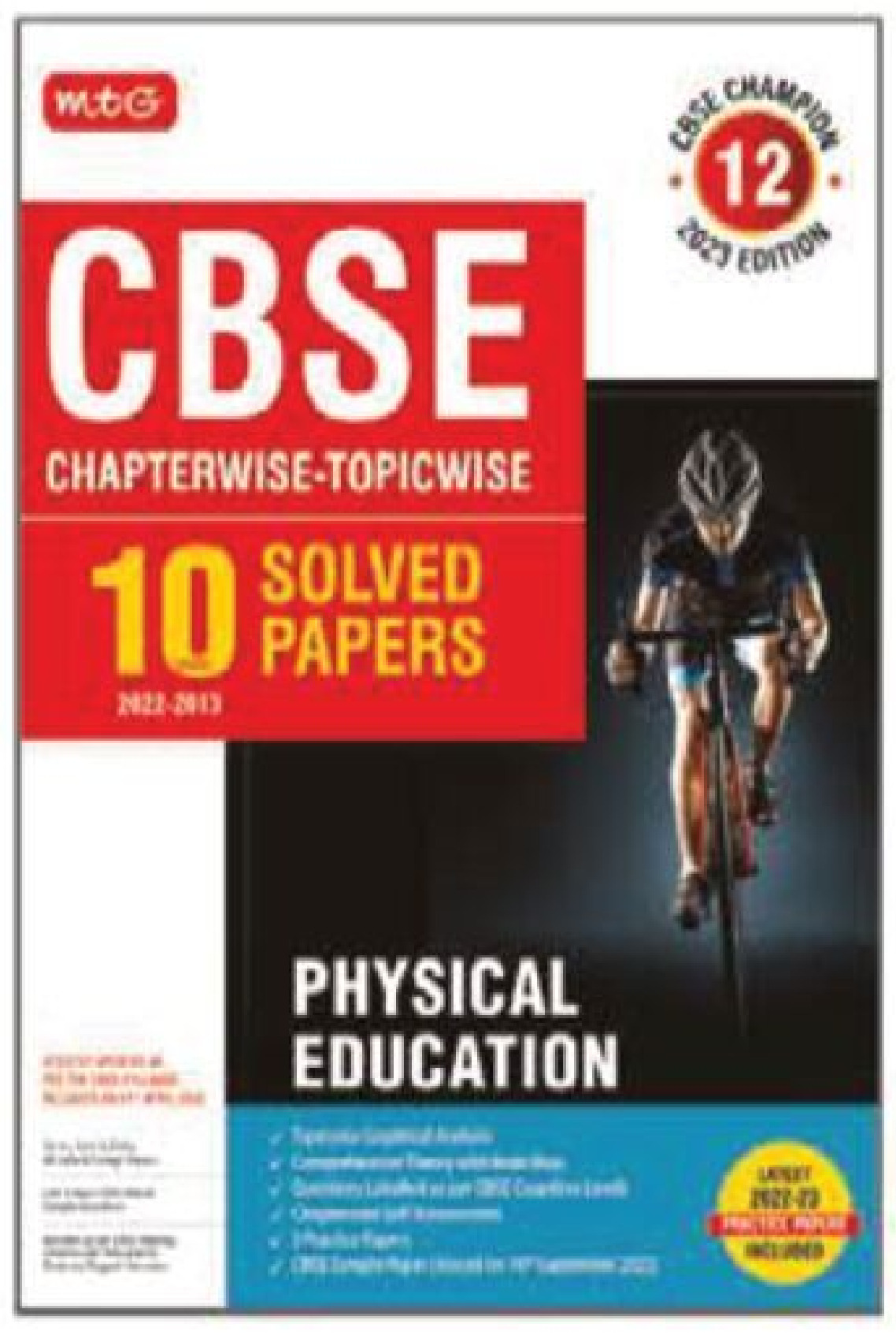
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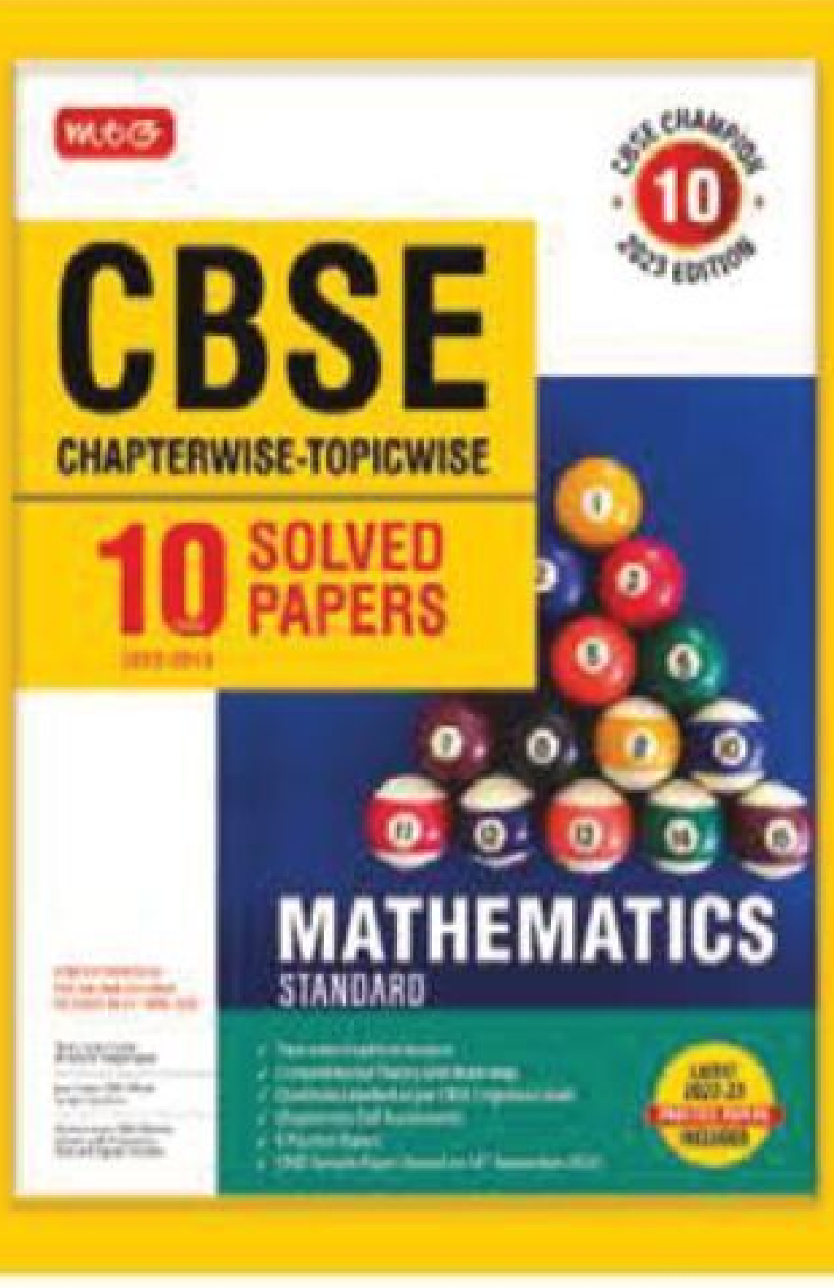
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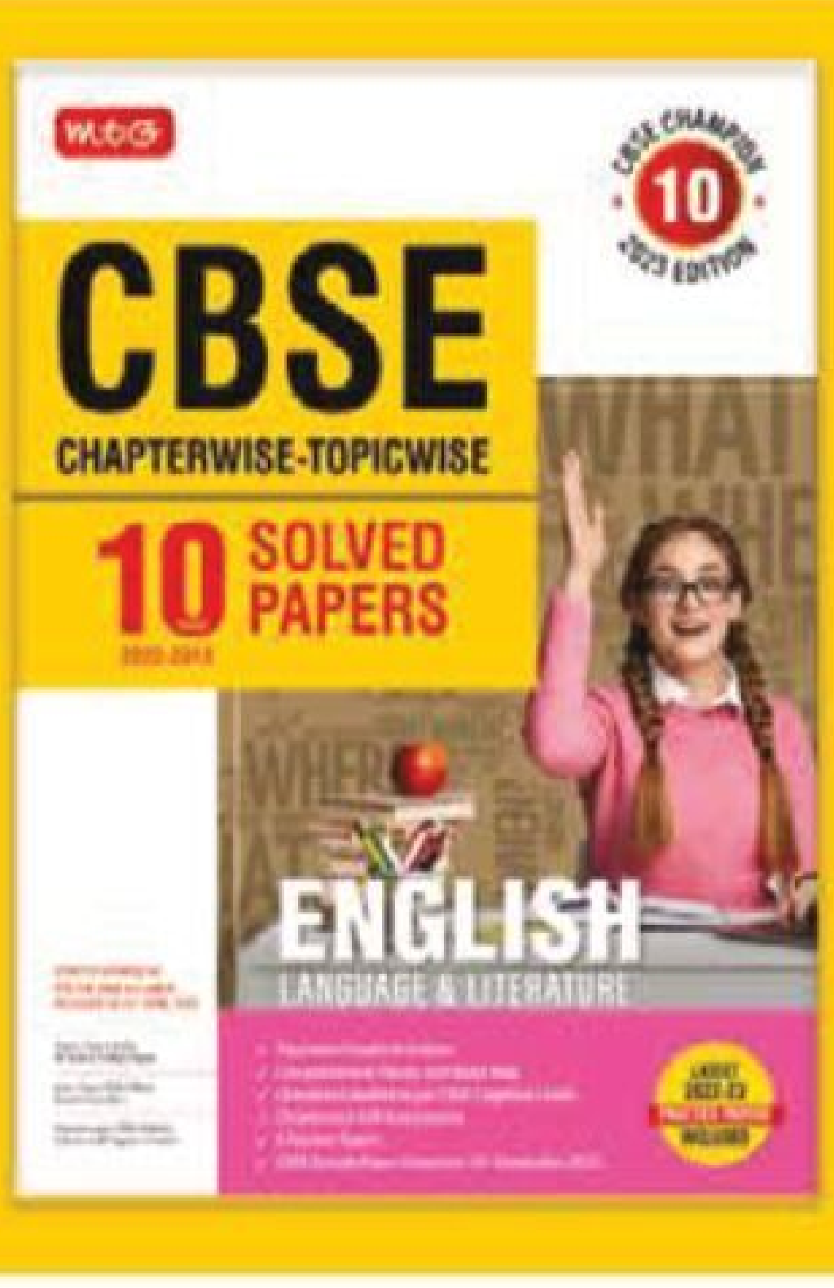
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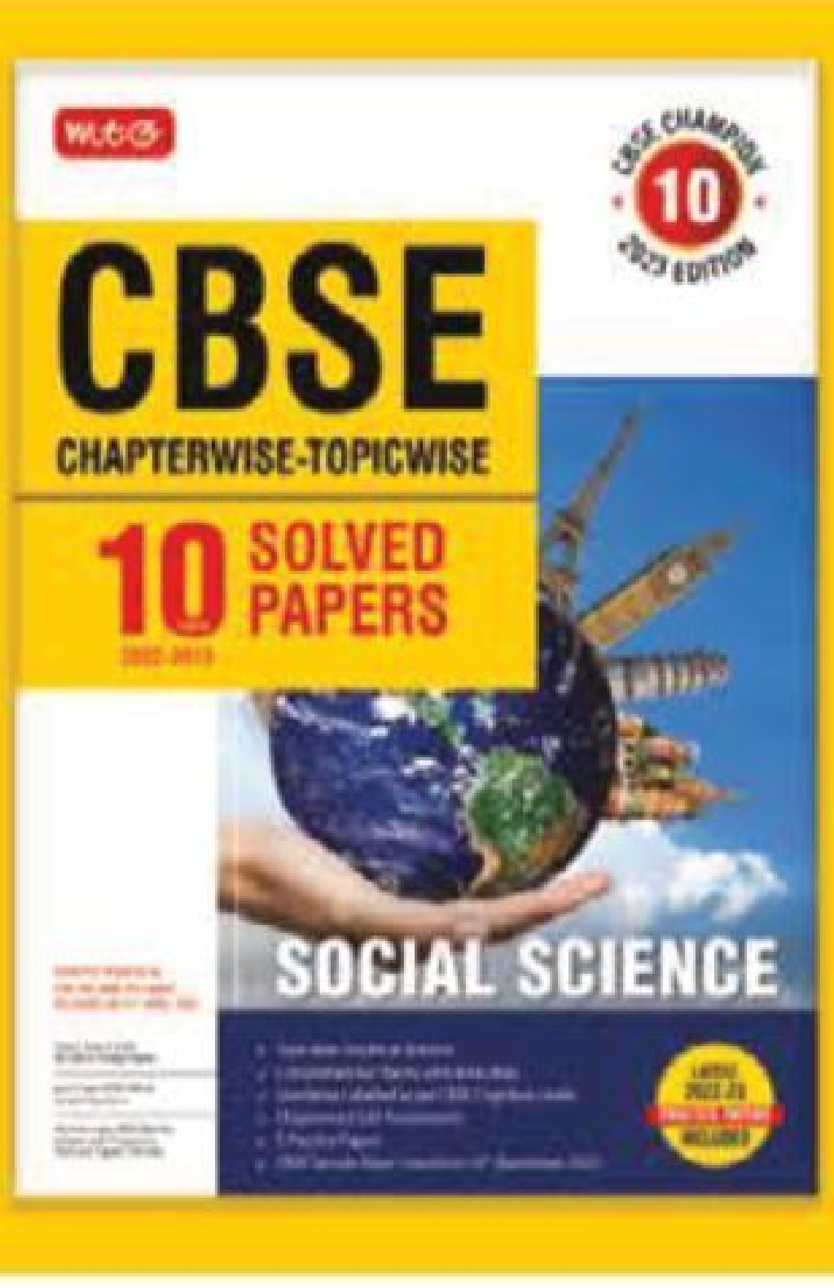
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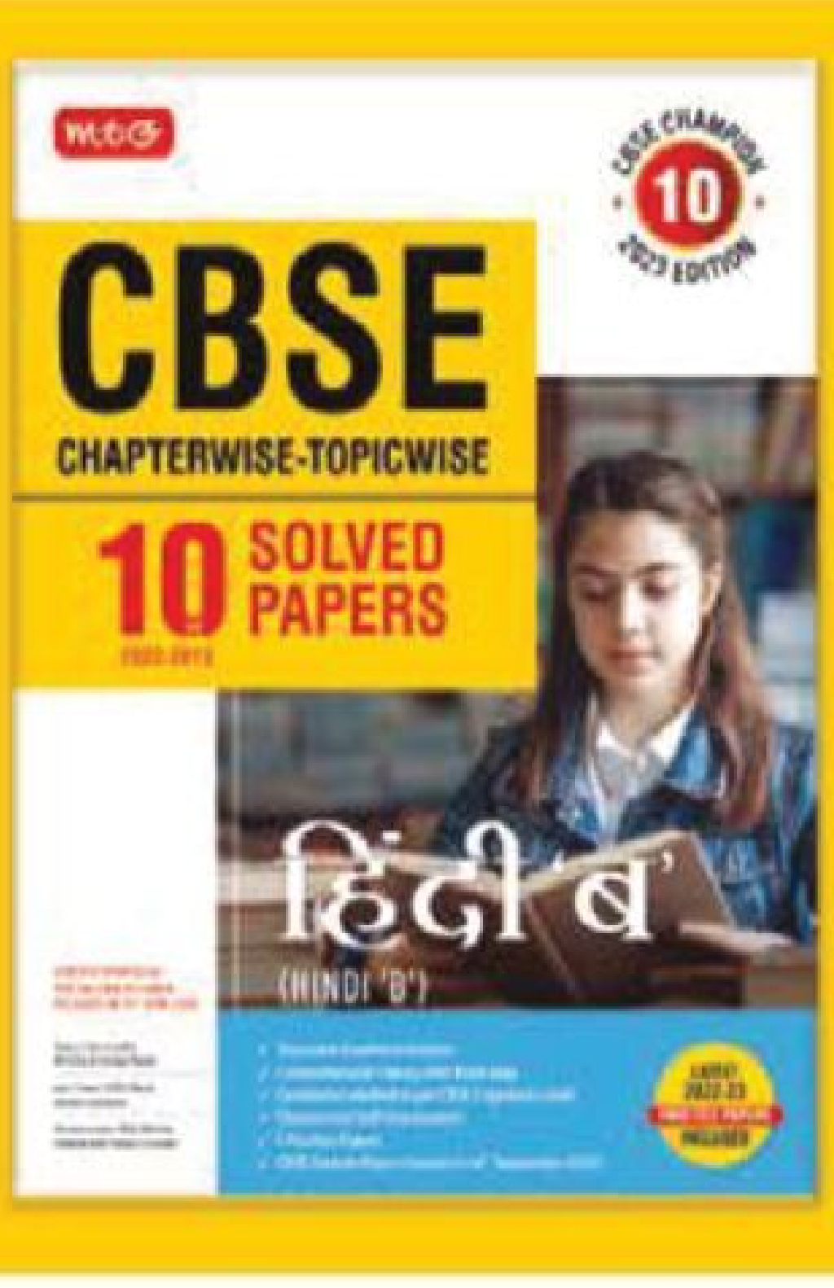
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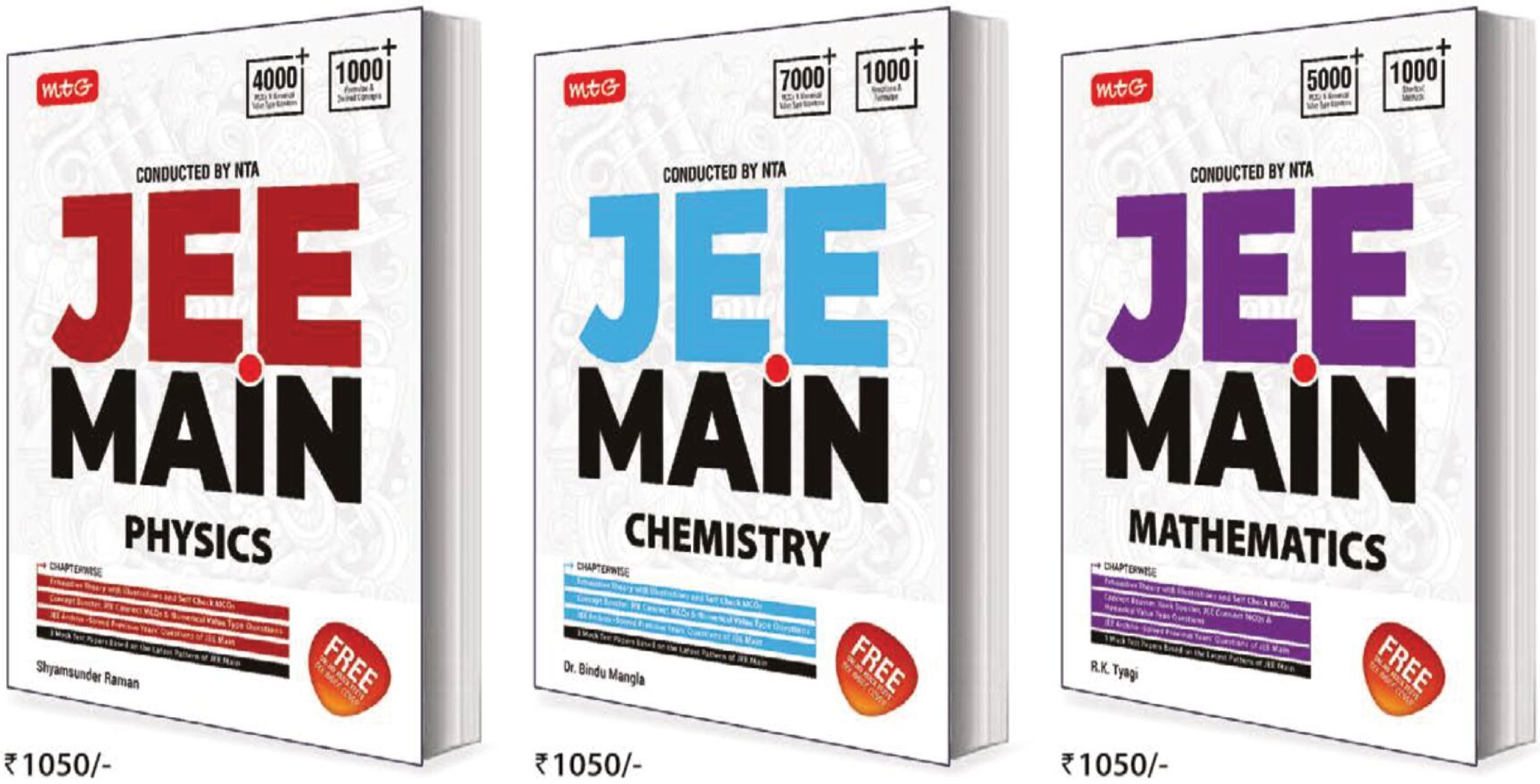
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